TEXT IS CROSS IN THE BOOK


RAINFALL
and RUNOFF

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# RAINFALL

# *and* RUNOFF

EDGAR E. FOSTER

MEMBER, AMERICAN SOCIETY OF CIVIL ENGINEERS; MEMBER, AMERICAN METEOROLOGICAL SOCIETY; MEMBER, AMERICAN GEOPHYSICAL UNION; MEMBER, AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE; FELLOW, AMERICAN GEOGRAPHICAL SOCIETY

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# PREFACE

The purpose that motivated the preparation of this book was two-fold. It was desired to present a reasonably complete picture of the science of hydrology as related to rainfall and runoff, including the more important and well-established principles that have been developed within the past two decades. Second, it was intended to introduce into hydrological practice the use of the methods of statistical analysis. In the combination of these two objectives it is hoped to put the science of hydrology on a logical basis by eliminating guesswork as much as possible, by setting limits within which judgment may be exercised safely, and by introducing means of measuring the numerous variations observed in the occurrence of water in nature.

Developments in the field of hydrology have multiplied tremendously within the past 15 to 20 years, and since these developments have been reported chiefly in current periodicals, it appears justifiable to bring them together in one volume. They have been reported to the public through the publications of many organizations, such as the American Society of Civil Engineers, the American Meteorological Society, the American Geophysical Union, and in many special publications by different agencies interested in hydrological work. These publications represent the widely scattered interests and activities of the investigators.

The use of statistical methods has likewise been developed and extended greatly within the last two decades. It has been used to facilitate research in many fields, such as business, agriculture, and biological work. The theory of probability has been utilized in studying the turbulence of water, thus bringing statistics into a field that has been especially applicable to analysis by methods of exact mathematics. Although a few engineers have adopted in some degree the methods of statistical procedure to determine flood frequencies, the profession as

a whole has limited itself to a few empirical processes or rules, apparently reluctant to adopt fully such methods for hydrology. Yet it has not hesitated to utilize the methods of least squares for adjustment of observations in geodesy.

It is the aim of this book to show the application of statistical methods to many hydrological processes, and to show the applicability of the methods and the reasons in appropriate places. However, in view of the vastness of the fields of both hydrology and mathematical statistics, it is possible only to skim over the surface, and present chiefly the more elementary phases. Methods that are still in the process of development have been avoided and controversial matter, other perhaps than the application of statistical theory, have been omitted. It is hoped that the book will point the way to a more comprehensive use of statistical methods.

An attempt has been made to present a clear exposition of the fundamentals of the hydrology of rainfall and runoff. It has been the author's intention to adhere to those principles which may be universally applicable rather than to undertake a description of regional hydrology, although it has been necessary as a matter of course, to use examples of specific places to illustrate principles and methods. The examples have been drawn for the greater part from the portion of the United States between the Rocky Mountains and the Atlantic Ocean, but this has come from the circumstance that the greater portion of the author's experience was obtained in that area and it should not be inferred that hydrological principles are thus limited.

Of the two objectives mentioned above, the use of statistical methods has been particularly appealing to the author. This appeal was aroused in his collegiate days under the guidance of Professor E. F. Chandler of the University of North Dakota. It has grown with added intensity through the passing years as he became increasingly familiar with the many complex problems of hydrology until no other approach now seems logical. In this use of statistical methods the author has not been entirely alone, however, for an increasing number of investigators are reporting the results of their work in terms of statistical analysis.

Furthermore, since it has long been the opinion of the author that hydrology is the basic science in the utilization of water resources, the application of those principles to economic problems has been emphasized where practicable. Chapter 12 is devoted to a very brief discussion of methods of utilization of the principles.

The author wishes to acknowledge his gratitude to those who have assisted him or encouraged his work. Most of the data used were

obtained from the published records of the U. S. Weather Bureau and U. S. Geological Survey. Free use was made of published works of others; it has been the intention of the author to acknowledge fully each instance through the bibliography and any failure to do so is an oversight. Dean Green of the University of Nebraska has lent encouragement to the completion of this volume, and Professor Evinger has read the manuscript and made valuable suggestions. The publisher has provided opportunity through various reviews to revise and correct the manuscript. The author is particularly indebted to Mr. Karl Jetter of Omaha, Nebraska, whose painstaking and able review of the entire manuscript has added much to its clarity and accuracy. Mr. Jetter also furnished Figure 101.

*Edgar E. Foster*

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# I THE FIELD OF HYDROLOGY

## A. DEMANDS ON WATER RESOURCES

*The Development of the Use of Water Resources.* Since the earliest man drank at a brook or water hole and sought shelter from rain and snow, human life has been closely dependent upon water. In those early days man satisfied his wants when and where he could find water, giving probably little or no thought to the possible limitations of supply and its replenishment. Then as man developed those arts that we call civilization and augmented his number, his dependence upon water increased accordingly and proportionately. At a much later date with the advent of the machine age and the doubling of the world population that followed, the demands for water multiplied and water resources rose in value so that increasingly more efficient use had to be made of them.

Only within comparatively few recent generations, however, has man attempted to analyze the problems involved in the utilization of water resources. It is even more recently that he has made any extensive investigations or intensive studies of the dispersion of water in its several forms over the surface of the earth. It is to those problems that this book will be devoted.

*Navigation.* Navigation was probably the first use that was made of streams, to lighten the burden of travel and promote the exchange of goods. As the needs of civilization broadened, greater use was made of streams as avenues of commerce. Until comparatively recently, rivers and the oceans with which they connected constituted the only practical and efficient means of transportation, and even now they are almost the only main commercial routes in many parts of South America, Africa, and Asia. The Hwang Ho and Yangtze Kiang in China, the Congo and Nile in Africa, and the Amazon and Parana in South America are a few of the most important examples. In Europe extensive use is made of streams for navigation as illustrated by the Volga, the Danube, the Rhine, and the Rhone Rivers. In North

America there are the Mississippi and many of its tributaries, the Hudson, and the Columbia Rivers.

Primitive use of rivers for navigation did not require much consideration of either the hydraulics or hydrology of streams except that some thought was given primarily to the season to insure required depths for shallow-draft vessels. However, modern river traffic requires continuous availability to relatively deep-draft vessels, and in many rivers such as the Ohio and upper Mississippi, complete canalization is required adequately to fill this demand. In such rivers, the cost involved and subsequent efficient operation require extensive study of the hydraulic and hydrologic features of the watershed. Data of stream flow as well as of stages, frequency, volume, and peak discharge of floods are required, and it is desirable also to know thoroughly the related phenomena of and variations in precipitation both as rain and as snow.

*Irrigation.* The practice of irrigation began in the infancy of civilization in Egypt, Mesopotamia, and the Indus River Valley, possibly as early as the fourth millenium B.C. Hammurabi's Code, compiled about 2250 B.C. by the King of Babylon with that name, contained provisions for payment of damages caused by carelessness in irrigating.

At the present time, India is reported by Encyclopedia Britannica (Fourteenth Edition, 1927) to have approximately 40,000,000 acres under irrigation. The United States is said by the same authority to be irrigating approximately 20,000,000 acres, which have probably been increased since that figure was compiled. Some 15 other countries are said to irrigate 1,000,000 or more acres each. In some warm humid regions, where rainfall may be sufficient for other crops and excess water is available, irrigation is practised in the cultivation of rice. In other regions, it is practised because of deficient moisture in the summer growing season. In arid and semiarid regions, regardless of the kind of crops produced, it may be the only source of subsistence for the community.

The hydrologic information required in studying the possibilities of irrigation consists of data of stream flow and its distribution through the year. Data of rainfall and evaporation are also essential since those two factors determine largely the amount of water needed for irrigating in a given locality. In many regions water supply for irrigation is derived from summer melting of snow in mountain areas and hence data of snowfall are important to forecast the available supply. Data of flood flows are necessary in studies for storage reservoirs and in the design of impounding dams. In one region at least, such data are needed to indicate what water supply is available for holders of junior water rights.

*Water Supply.* A supply of water for domestic use has always been an important matter, but not until people began to congregate in large cities did it become a serious problem. Beginning with the growth of densely populated centers in antiquity much labor was expended for an adequate supply as is attested by the many remains of ancient conduits, aqueducts, and other water works. With the rise of the machine age, the existence of present-day industrial cities necessitated a careful search for an adequate water supply. Costly collection and conveyance works impel painstaking investigations of the hydrology of watersheds in order to insure that they can provide the supply. Nor are these investigations limited to watersheds for the large industrial centers, but they must be made by even small towns that desire an adequate and safe water supply, either from surface runoff or ground water. Institutions and isolated industrial plants must likewise make hydrological investigations to insure a water supply suited to and adequate for their diverse purposes.

The most important factors on which data are required are runoff, its distribution and volume and rates of flow. For adequate interpretation of runoff supplementary data of precipitation are required. In the colder or mountainous regions where dependence for water supply is placed on melting snow, data of depths and water equivalent of snow are needed. If reservoirs are constructed, data of evaporation are necessary for study of probable losses from impounded water. Many communities draw on ground water for their supply, and therefore these localities are interested in data pertaining to conditions of ground water.

*Water Power.* The earliest use of water power is unrecorded in history, yet since its simplest development requires some mechanical ingenuity, it must be accepted as a product of civilization. The present widespread use of water power is a comparatively late achievement and is, in fact, a product of the machine age. Rivers and streams produce an appreciable proportion of the power used in the world. The total developed water power in the world was set at 33,000,000 horsepower in 1926 by Voskuil (188). Justin and Mervine (103) state that in 1932 there were in the United States 11,800,000 horsepower of installed hydroelectric capacity in a total capacity of 43,750,000 horsepower. These figures are of central stations only. Since stream flow to be used with an available head is the raw material of an hydroelectric plant, hydrological data are of great importance in water power development.

The hydrological data required for water power development include data of stream flow, both rates and volume, peak flows, annual distribution of runoff. To supplement information of runoff, data of precipita-

tion are needed in both the design and subsequent operation. In regions receiving snow, data of depths and water equivalent of snow cover are needed to form the basis of reservoir operation during winter and spring. Data of evaporation are needed to estimate losses of water from that source.

*Flood Control.* Floods are or may be among the most disastrous phenomena of nature. Man has contended with floods since the dawn of civilization. Hammurabi's Code mentions dikes used for protection against overflow and places on local interests the responsibility for maintenance of the protective works. China has striven against floods since her earliest history. Other countries which contain rivers with variable flow have suffered from the same cause. The increasing population in the United States and its expansion along streams have compelled the Federal Government to assume large portions of the burden of providing flood control where protection is found necessary.

Flood control may be achieved by means of large reservoirs to retain large volumes of runoff for release after the flood, channel rectification or enlargement, occasionally closed conduits and pumping plants for safely passing the water, or by levees high and strong enough to withhold the flood water from the adjacent land. Each locality is virtually a separate problem and must be given individual study.

The most important kind of data required for flood control consists of the magnitudes of flood peaks and times of occurrence when more than one stream on a basin are concerned. This includes not only both stages and discharges but also their frequency, in order to determine the extent of protection required and its economic justification. Supporting data include annual quantities of precipitation, time distribution, rates, areas covered, including depths of snowfall and water equivalent of snow cover. Infiltration of water into soil is the principal deduction from flood flow so that data of infiltration are needed to estimate runoff of storms for design of structure or operation of the plant.

*Drainage.* The objective of drainage is similar to flood control in that it involves the disposal of excess water rather than use of water resources. The purpose of drainage is usually reclamation of arable land by removal of the water standing on or in land making it unsuitable for cultivation. It is provided also in the case of many industrial establishments to prevent detrimental ponding of runoff from heavy rainfall. The data required are similar to those for flood control but particularly important is the intensity of rainfall or rate of short duration.

*Erosion Control.* Although erosion of land surfaces has been a normal geological process since the beginning of the earth, it has been only

within comparatively recent years that artificially induced erosion has come to be recognized as an economic problem of serious proportions demanding remedial measures. The problem has without doubt been materially augmented by increasing economic exploitation of agriculture and lumbering, construction of highways, and perhaps other activities. Erosion is an active process in all climates and all regions, and depends upon many geologic and climatic factors. Rainfall and runoff are probably the most active agents, although wind is also a potent factor in arid and semiarid regions.

Data of amounts and intensity of precipitation, runoff, and infiltration are the types of hydrological information needed in studies of protective measures.

*The Use and Removal of Snow.* In addition to its function as a portion of the annual precipitation as mentioned heretofore, snow has some aspects peculiar to itself. It provides the basis for extensively practised outdoor winter sports such as tobogganing and skiing. Its depth, cover, and texture are important for such sports, and in regions where they are developed to economic proportions, data of those features are compiled and published currently. On the other hand, snow constitutes a severe handicap to most activities in the region where it occurs. Highway and railway traffic are frequently blocked or interrupted by heavy snowfall and considerable expenditures are made annually to effect its removal. Data of depths of snow cover, maximum amounts, and frequency are needed to plan for its removal from highways and streets.

## B. DEFINITION AND SCOPE OF HYDROLOGY

*Definition of Hydrology.* Hydrology is the science that treats of the waters of the earth in their various forms. The progressively extensive use of the water resources of the world in recent times has necessitated far-reaching research and intensive study of water in its natural occurrence and forms. The body of knowledge that has been developed by these investigations and studies is called "hydrology." Hydrology considers water in its natural occurrence as ice, snow, or other forms of the solid state, and as a gas or vapor. It deals with distribution of water under the ground, on the surface, and in the atmosphere, and it treats of all phenomena relating to its natural movements from one locality to another and its changes from one state to another. Hydrology is concerned mainly with physical occurrence or relationships since the chemistry of water and its physical properties are not included within its scope except as chemical phenomena may have some bearing on the occurrence of water.

Hydrology is recognized by the International Association of Scientific Hydrology as one of the earth sciences. The Executive Committee of that organization has made four general divisions of hydrology, namely,

1. Potamology, the science of surface streams
2. Limnology, the science of lakes and still bodies of water
3. Subterranean waters, being the portion of general hydrology that deals with underground water, but as yet has no distinctive name
4. Cryology, the science of ice and snow

*Sources of Data.* The data of hydrology are obtained from many diverse sources; some are obtained by direct observation of the various natural phenomena of water and others are secured from the works of related branches of science. The natural phenomena that involve water are extensive and are best known through constant observation. Any scientific activity that obtains information of water under natural conditions is a source of data for hydrology.

Relatively few data of hydrology are obtained by laboratory or experimental methods. The phenomena involving occurrence of water under natural conditions are usually far too extensive to be reproduced by experimental or laboratory methods. Such phenomena may include storms hundreds of miles in diameter and stream flow on river systems draining thousands of square miles. Furthermore, the complexity of natural phenomena precludes their adequate reproduction so that dependence must be put on intensive observation and collection of data of these phenomena. In hydrology experimental methods are limited to phenomena that can be reproduced on a small scale; for example, evaporation rates are data that have been determined by experiment.

A few of the more important items for which statistics are required are: data of rainfall and snowfall; stream flow under different conditions; flood peaks, including peak flows and volumes of runoff; rates of evaporation and infiltration; depths to ground water. There are, of course, many other phenomena studied in connection with water, and data of all such phenomena are also required. Important supplementary data are those of types and extent of storms, humidity, air masses, temperature of the air; even such phenomena as variation in sun spots and thickness of tree rings have been utilized in the study of long-term variation in precipitation. The aid of a number of sciences is required to obtain the manifold data of hydrology. A few of the more important sciences concerned and their interrelation or overlapping in the field of hydrology are briefly mentioned below.

*Meteorology.* Meteorology is defined as the science of atmospheric phenomena. It is particularly concerned with changes of atmospheric

conditions and the causes thereof. Meteorology includes the study of moisture in the atmosphere, including its forms and precipitation, and thus overlaps a portion of the field of hydrology. Meteorology also treats of temperature, movements of air or wind, solar radiation, and many other atmospheric phenomena which, however, are of interest to hydrology only as they may affect the occurrence, transfer, or precipitation of water. Meteorology has furnished most of the data of rainfall, snowfall, humidity, evaporation, and action of storms, all of which have been of basic importance in hydrologic study. Since moisture is distributed over the earth surface through the atmosphere, meteorology and hydrology are closely related sciences in this common field.

*Climatology.* Climatology is the science of climate, which is defined by Dr. Helmut Landsberg (111) as the average state of the atmosphere at a given place within a specified period of time. To this definition should be added the statement that variations from the average state are also important in describing the climate of a given locality. Climatology considers all continuous or recurring aspects of the atmosphere, whereas hydrology is concerned only with those of or affecting water.

*Geology.* Geology is the science of the constitution and history of the physical earth. Certain of its subdivisions, petrology, structural geology, and physiography, contribute indirectly to hydrology since the nature of the rocks of the earth and land forms have an important bearing on the occurrence and distribution of water. The types of bedrock and soil mantle determine the depth and volume of ground water. The land forms, varying from broad, flat plains to massive mountain ranges affect the movements of the atmosphere and hence the distribution of precipitation. Furthermore, the action of water is a powerful agent in producing geological changes so that the occurrence of water is a common field with hydrology. Geology contributes many data that are necessary to a proper understanding of some of the problems in hydrology.

*Fluid Mechanics and Hydraulics.* In two of its three states water is a fluid, and hence the science of fluids, called "fluid mechanics," is closely related to hydrology. "Hydromechanics," the portion of fluid mechanics that deals with liquids, is conveniently divided into two parts: "hydrostatics," which considers liquids under static conditions, and "hydrodynamics," which treats of liquids in motion. "Hydraulics" is the portion of hydromechanics that covers the phenomena of water under conditions of particular interest to the engineer. It treats of the flow of water in natural streams. This branch of hydraulics has become so important that it is designated by the special name of "potamology" which is the common field of hydraulics and hydrology. The principles

of hydraulics are necessary to explain and analyze movements of liquids encountered in hydrology.

*Mathematics.* Mathematics may be defined as the science of quantities. Since the data of hydrology are practically always quantities of some sort, mathematics is, of course, a necessary tool of hydrology. Furthermore, the data of hydrology are variable to a high degree and they are usually the result of events which are themselves the products of simple constituent phenomena in various combinations. Therefore, although such other branches of mathematics may be employed as may be useful, the theory of probability and the theory of statistics are particularly applicable for analyzing hydrologic data. It will be the particular aim of this book hereinafter to analyze and study hydrology with the aid of the mathematics of probability and statistics wherever appropriate.

## C. TREATMENT OF HYDROLOGIC DATA

*Types of Knowledge.* Two broad types of knowledge gained from observation may be recognized, namely, experimental and historical. These two types require different methods of approach, and since both are existent in the field of hydrology, it is pertinent to discuss them briefly.

*Experimental Knowledge.* Experimental knowledge consists of or is based on experiments which are made to obtain specific data. These experiments are the basis of practically all our knowledge in the realms of physics and chemistry. They can be repeated by any competent observer who can obtain the same data and verify the results secured by his predecessors.

By these experiments, repeated at will, the universality of the laws of nature is established because like causes produce like effects. For example, it has been established that the velocity $V$ of water passing through an orifice varies as the square root of the product of twice the acceleration of gravity $g$ times the head $h$, or expressed as an equation

$$V = \sqrt{2gh}.$$

Since this relationship has been proved correct in one part of the earth at one time, it is accepted as correct for the entire earth for all time. It can, of course, be rechecked by experiment by any competent person.

The unvarying uniformity of like results from like causes is known as the law of causality. There are two current views of this law, somewhat different but essentially the same. The popular view is that a given

causal event is always followed by the same effect. The physical and mathematical view is that causality can be expressed as a relationship, such as

$$V = \sqrt{2gh}.$$

However, given the right hand side of the equation, there results the left hand side as the effect, so that the two views are really two ways of looking at the same thing.

The causes of the action sought in experimental work are usually well known, particularly if the experiment is for verification. In fact a well organized experiment would be limited to definite controlled and measured causes in order to obtain the desired effect. The variations of the results would be limited to observational errors which can be measured and analyzed by the methods of least squares.

*Historical Knowledge.* Historical knowledge consists of those facts that are not derived from experiments, and the desired data therefore cannot be obtained at will. This knowledge is as certain and as authentic as any other if competently observed, but since the event cannot be repeated other observers can not duplicate it for observation or verification.

Nevertheless, accurately recorded historical knowledge does not contravene or negate the laws of causality. The observed event is itself the result of a cause, or more probably a number of concurrent causes operating with different intensities which produced the event at the given time and place. The causes themselves may or may not be known.

Many of the data of hydrology such as events of storms, rainfall, snowfall, floods, are historical in nature because any individual event cannot be repeated under control of an experimenter. Data of precipitation, runoff, and even elevations of water table and condition of the soil moisture are observations of such and such a date and place. They are, therefore, historical data and must be obtained by observation.

*Approach to Hydrologic Data.* The approach to the study of hydrologic data must be varied to fit the nature of the data themselves. Experimental data can be verified by other experiments and compared on a basis approximating uniformity of conditions. Historical data, on the other hand, cannot be repeated for confirmation, and continued observation is necessary for the full analysis and comparison and verification.

If hydrology is to be considered a science, all data, whether experimental or historical, must be precisely defined and verified; they must be properly organized and correlated. Finally the principles must be

shown to be universal in their application since empirical methods alone are not adequate.

The diversity of the data and their sources make this no easy task. The hydrologic events themselves must be analyzed to determine the causes and to measure and classify them. The observational data must be correlated to determine existing relationship together with their limitations and their applicability to other situations. Some of this work can be done by further experiment, and some must be accomplished by continued observation of the natural elements. Some of the relationships are readily apparent while others require powerful tools to determine them. Although there probably will always be a use for observational constants, "coefficient engineering" alone will not place hydrology on a sound scientific basis. Fundamental physical relationships must be found. For these purposes the theory of statistics and the theory of probability can be employed to advantage.

*Nature of Hydrologic Data.* Some of the more important types of data have been given above. The greater proportion of the data consists of variable magnitudes of the various attributes. These magnitudes, regardless of the nature of the attribute, are continuous heterograde variables because they have different values. In other cases the data consist of a number of events in a given unit of time. Such data constitute discrete homograde variables since they are of one value. A number of types of data are listed in Table 1.

TABLE 1. CLASSES OF HYDROLOGIC DATA

| ITEM | DIMENSION | NATURE OF DATA |
|---|---|---|
| Rainfall | Depth | Heterograde graduated |
| Snowfall | Depth | Heterograde graduated |
| Intensity or rates of precipitation | Depth per unit of time | Heterograde graduated |
| Humidity | Mass of water per mass of air, or ratio | Heterograde graduated |
| Evaporation | Depth | Heterograde graduated |
| Rate of Evaporation | Depth per unit of time | Heterograde graduated |
| Stream flow | Volume per unit of time | Heterograde graduated |
| Runoff | Volume per unit of time | Heterograde graduated |
| Flood peaks | Volume per unit of time | Heterograde graduated |
| Frequency | Number per unit of time | Heterograde integral |
| Days with rain or snow | Number per year | Homograde integral |

*Statistical Functions Used in Hydrology.* It is not feasible to give in this work any except a brief mention of the statistical methods applicable to the analysis of hydrological data. In the following paragraphs are listed the various formulas used hereinafter for such analysis, and at appropriate places examples are worked out in detail to show

their application. The reader is referred to the many books on the mathematics of probability and statistics for the basis and development of the formulas.

*Average.* Of the several averages, the arithmetic mean $M$ is the most important. It is expressed as follows:

$$M = \bar{\bar{X}} = \frac{\Sigma X_n}{N}$$

where $\bar{\bar{X}}$ = the arithmetic mean; $X_n$ = the individual values of the series; and $N$ = the total number of items. The arithmetic mean is also referred to as the "first moment" $S_1$ about an arbitrary origin.

*Standard Deviation.* The extent of variation of values about the arithmetic mean is commonly measured by the standard deviation, which is given by the formula

$$\sigma = \sqrt{\frac{\Sigma (x_n^2)}{N}}$$

where $\sigma$ = the standard deviation; $x_n$ = the deviations from the arithmetic mean; and $N$ = the total number of items. The square of the standard deviation is the second moment $U_2$ about an origin placed at the arithmetic mean and is also known as the variance.

*Third Moment about an Arbitrary Base.* The third moment has no special name. It is, however, used to determine the skew, if any, and to compute frequency curves. It is obtained about an arbitrary axis by the formula

$$S_3 = \frac{\Sigma (X^3)}{N}$$

where $S_3$ is the third moment about an arbitrary base, such as zero.

*Moments about the Arithmetic Mean.* In computing frequency curves by statistical methods, the origin is usually put at the arithmetic mean of the series of values; this greatly simplifies both the theoretical treatment and practical use of these methods. The three moments are as follows when shifted to the arithmetic mean:

First moment, $U_1 = O$
Second moment, $U_2 = S_2 - S_1^2$
Third moment, $U_3 = S_3 - 3S_1S_2 + 2S_1^3$

where $U_1$, $U_2$, $U_3$, are the first, second, and third moments, respectively, about the arithmetic mean.

Another method of obtaining the frequency moments is described by Elderton (54) and Forsyth (64). This method, called the "summation method," consists in successive summation of the distribution from the maximum observation to the smallest, and the columns formed by that operation. For example, column (3) is formed by adding the values of column (2) beginning with $o_n$ to each successive class. Column (4) is formed by repeating the process in column (3) and column (5) is obtained in the same manner from (4). It is not easily set in a formula but can be shown best by example as set up in the following tabulation:

| (1) Class | (2) $f(x)$ | (3) 1st | (4) 2nd | (5) 3rd |
|---|---|---|---|---|
| 1 | $o_1$ | $\sum_n^1 o$ | $\sum_n^1(\sum_n^1 o)$ | $\sum_n^1(\sum_n^1\sum_n^1 o)$ |
| 2 | $o_2$ | $\sum_n^2 o$ | $\sum_n^2(\sum_n^2 o)$ | $\sum_n^2(\sum_n^2\sum_n^2 o)$ |
| — | — | — | — | — |
| $n$ | $o_n$ | $o_n$ | $o_n$ | $o_n$ |
| | $S_1 = \sum_n^1 o$ | $S_2 = \sum_n^1(\sum_n^1 o)$ | $S_3 = \sum_n^1(\sum_n^1\sum_n^1 o)$ | $S_4$ |

The mean, $M = \dfrac{S_2}{S_1}$

$$U_2 = 2\frac{S_3}{S_1} - M(1 + M)$$

$$U_3 = 6\frac{S_4}{S_1} - 3U_2(1 + M) - M(1 + M)(2 + M).$$

*Distribution.* An arrangement of a series in some definite order is called a "distribution." There are three recognized distributions which may be used in hydrology, as follows:

Of magnitude, a frequency distribution
Of time, a time series distribution
Of location, a spatial distribution.

*Frequency Distribution.* Frequency distributions assume a number of shapes, but since the number of data is never less than zero the curves are always in the two upper quadrants of the Cartesian coordinate system. The so-called normal frequency curve is bell-shaped with its maximum point on the vertical axis, about which it is symmetrical. It may be represented by plotting the coefficients of the expanded binomial, $(x + y)^n$, about an axis through the midterm.

The distribution of accidental errors about their mean as an origin

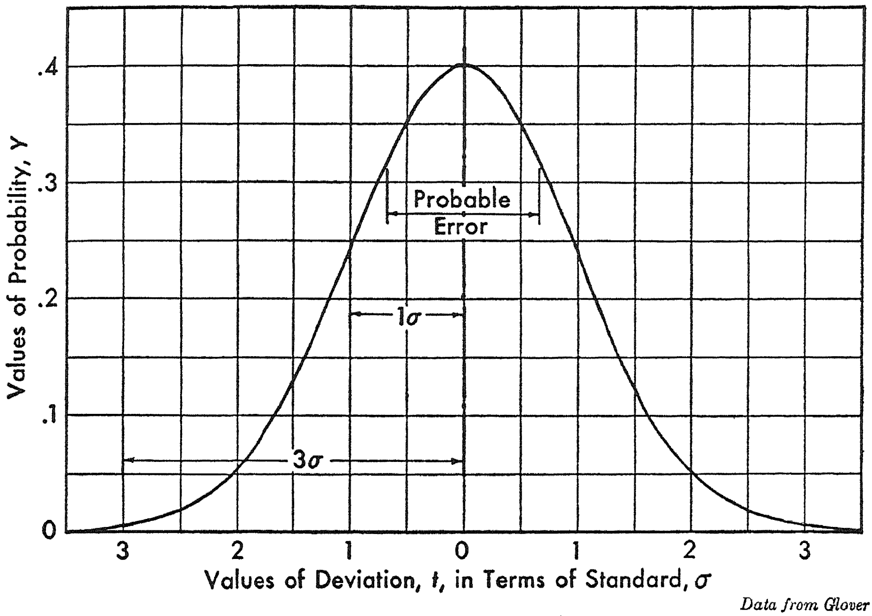



FIGURE 1. Normal Probability Curve, $Y = \theta(t) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$

constitutes the best-known frequency distribution found in engineering practice. Its equation is as follows:

$$Y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(M-X)^2/\sigma^2}$$

Where $Y$ is the probability of an error (or deviation) $x$; $\pi$ is the ratio of diameter to circumference of a circle; $e$ is the base of the natural or Napierian logarithms. The other quantities are as defined heretofore. Figure 1 shows this function plotted in the characteristic shape, with the range of one and three standard deviations and the "probable error."

*Skew Frequency Distribution.* Other frequency distributions, including most of those found in practice, do not exhibit such fine symmetry but are skewed to a variable degree; some distributions in hydrology such as data of daily rainfall are extremely skewed. Distribution functions have been derived by various mathematicians so that there are sufficient formulas to fit practically any well-formed distribution.

A series of skew functions to fit virtually any frequency distribution was derived by Dr. Karl Pearson. Although these functions have been used in statistical work in widely different fields of knowledge, they have only a slight theoretical basis which defines in general terms the

shape of all frequency or distribution curves. The general equation defines a distribution curve so that the probability, $y$, reaches a maximum for some value, $a$, of $x$, and such that as $y$ decreases, $x$ increases. The general and basic equation, as given by Elderton (54) is

$$\frac{1}{y}\frac{dy}{dx} = \frac{x + a}{f(x)}$$

where $y$ is the probability of a value; $x$ is the magnitude or similar attribute; $a$, a constant; and $f(x)$ is some function of $x$.

The function, $f(x)$, is assumed to be one that can be expanded into a power series by Maclaurin's theorem, by the operation of which the general equation becomes

$$\frac{1}{y}\frac{dy}{dx} = \frac{x + a}{c_0 + c_1 x + c_2 x^2 \cdots}$$

where $c$ is an additional constant. This equation includes curves of many shapes. By setting limits to skewness and flatness (called also "kurtosis"), Pearson established certain criteria of types and thereby developed his curves (54). These curves have been used in hydrology to only a limited extent, and are discussed only when applied to some specific problem.

*Binomial Law.* The basic probability function is the binomial law

$$P = C_n^m p^n q^{(m-n)}$$

in which $P$ is the probability that a certain event will happen; $p$ is the probability that any subsidiary event will happen; $q$ is the probability that any subsidiary event will not happen; $m$ is the total number of favorable and unfavorable subsidiary events; $n$ is the number of favorable subsidiary events; and $C_n^m$ is the number of combinations of $m$ things taken $n$ at a time. From this law Dr. Thornton Fry (68) has derived the "normal" probability function substantially as given above.

*Gram-Charlier Series.* The so-called Gram-Charlier series is derived by Dr. Thornton Fry (68) from the normal probability function given above. This series, named after two Scandinavian mathematicians who first developed it, is as follows:

$$P = A_0\phi_0(x) + A_3\phi_3(x) + A_4\phi_4(x) + \cdots$$

where $$\phi_0(x) = \frac{1}{\sigma\sqrt{2}} e^{-\frac{1}{2}(x-m)^2/\sigma^2}$$

$\phi_3(x)$ is the third derivative and, $\phi_4(x)$ is the fourth derivative of the

normal function. $A_0$, $A_3$, and $A_4$ are constants derived from the first four statistical moments. Although this series can be fitted to frequency distributions with considerable skew, it cannot be adapted to such data as daily precipitation or flood peaks. Objections have also been raised against it on theoretical grounds in that it may develop into a sine-wave curve with negative values of probability.

*Logarithmic Transformations.* For distributions of extreme skew, logarithmic transformations of the variate may be resorted to. A logarithmic transformation of the variate substitutes some function of the logarithms of deviations for the deviations themselves. Three such transformations have been developed.

The first such transformation to be stated is that given by Mr. Arne Fisher (59):

$$\phi_0(x) = \frac{1}{n\sqrt{2\pi}} e^{-\frac{1}{2}[(\log x - m)/n]^2}$$

where $\phi_0(x)$ is probability of the deviation $x$; $m$ and $n$ are parameters derived from the statistical moments; the remaining characters are as defined heretofore.

Two other logarithmic transformations were introduced by Slade (169). The first he called the "partly bounded function" because it had an unlimited range in the positive direction:

$$Y = ae^{-c^2[\log d(x+b)]^2}$$

where $Y$ is the frequency of a corresponding value of $x$; and $a$, $b$, $c$, and $d$ are parameters derived from the statistical moments $M_1$, $U_2$, and $U_3$, The second was designated the "totally bounded" function and is as follows:

$$Y = ae^{-p^2c^2(\log d[(x+b)/(g-x)])^2}$$

in which $g$ is a parameter to represent the maximum ordinate and $p$ is a parameter introduced to make the second moment of the curve approximate that of the data. In the introduction of the parameters $p$ and $g$, the function departs from a strictly theoretical basis. The parameter $p$ makes the observed distribution of data the controlling factor, so that the function is virtually empirical and thus loses the guide of the theory of probability. While the introduction of the parameter $g$ is made on nonstatistical grounds, it could be accepted without losing the theoretical basis of the function. The desirability of an upper limiting value is discussed in Chapter 10.

*Poisson's Function.* One other frequency function remains to be mentioned, namely, Poisson's, which is also referred to as the "law of small numbers." It is as follows:

$$P_{(n)} = \frac{e^{-m}m^n}{n!}$$

in which $P_{(n)}$ is the probability of $n$; $e$ the base of the natural logarithms; $m$ the mean; $n$ is a given number of expected events. Fry (68) shows how this function may be derived from the binomial law, which is the usually accepted basis of derivation. Fisher (59) derives the function by showing that when his "semi-invariants" of a statistical distribution are equal, Poisson's function is obtained. (Fisher's semi-invariants correspond to the moments, the first three being identical with $M$, $U_2$, and $U_3$, given previously.)

The theory upon which Poisson's function is based limits its application to distributions of discrete data, that is, it should not be used for data such as daily rainfall but rather to such as number of storms in a year. Its simplicity and convenience of application are distinctive features. Pearson (147) has published tables giving values of $P_{(n)}$ for $n$ ranging from 0 to 37 with $m$ by tenths from 0.1 to 15.0.

*Correlation.* Statistical correlation consists in the measurement of the relationship between two sets of data. Methods for determining the correlation between sets of data having many forms of relationship have been developed, but in this book the discussion is limited to linear correlation, that is, correlation between sets of data having an approximately linear relationship. The correlation coefficient, designated commonly by $r$, may vary from $-1.0$, which value would indicate an inverse mathematical relationship, through zero, indicating a purely chance relationship, to $+1.0$ as for a direct mathematical relationship. The formula for the correlation coefficient is stated in various forms, several of which are given by Reitz (154). A common form given by Reitz and others is as follows:

$$r = \frac{\Sigma(xy)}{N\sigma_x\sigma_y}$$

where $r$ is the correlation coefficient; $x$ and $y$ are the deviations from means of two series of values of correlated attributes; $\sigma_x$ and $\sigma_y$ are the standard deviations and $N$ is the total number of pairs of data, $x$ and $y$.

For greater ease in computation in the case of ungrouped data, that is, not classified into groups of similar magnitudes, this formula may be

reduced to the form

$$r = \frac{\Sigma xy}{\sqrt{(\Sigma x^2)(\Sigma y^2)}}$$

where $x$ and $y$ are the deviations from the means of $X$ and $Y$.

When dealing with large numbers of pairs of data, the work involved by use of the above formula is long and the correlation can be found best by grouping the data. In this case, the formula may be cast into this form:

$$r = \frac{\dfrac{\Sigma(XY)}{N} - \bar{X}\bar{Y}}{\sigma_x \sigma_y}.$$

Where $\sigma_x$ and $\sigma_y$ are the standard deviations of $X$ and $Y$. This form is conveniently used with the so-called correlation table.

*Regression Lines.* Regression lines constitute an important part of correlation; these lines may be defined as the curves which represent the relationship between the two sets of data. With linear correlation, they are straight lines represented by the type equation

$$y = a + mx.$$

However, because of the imperfect relationship usually existing between sets of correlated data, it is necessary to use, in most cases, two equations to represent the regression lines. Thus with sets of data $X$ and $Y$, the regression lines would be as follows:

For regression of $X$ on $Y$: $X = \bar{X} + r\dfrac{\sigma_x}{\sigma_y}(Y - \bar{Y})$.

For regression of $Y$ on $X$: $Y = \bar{Y} + r\dfrac{\sigma_y}{\sigma_x}(X - \bar{X})$.

*Measures of Precision.* A particular advantage of statistical methods is the development and use of measures of precision with which the investigator can estimate the accuracy of his results. There is a considerable array of these measures used in the theory of sampling; many of them, however, serve for special or limited purposes. Only the most generally applicable are listed here but others can be found in the works on statistics listed in the bibliography.

The theory of sampling denotes the use of statistical methods for analyzing relatively small samples from an unlimited "universe" or "population." This theory is particularly applicable to hydrology

because the investigator can deal with only a sample and frequently one unsatisfactorily small. Examples of such samples are the data of annual precipitation for 70 years out of a universe consisting of all the years that precipitation has fallen or will fall or 587 floods on a river out of all the floods that have occurred or will occur under the present climatic conditions.

The first and perhaps most important measure of precision is the standard deviation which is given on page 11 as follows:

$$\sigma_x = \sqrt{\frac{\Sigma (x_n^2)}{N}}.$$

The range of the standard deviation is shown on Figure 1; the area beneath the curve and within these limits is equal to approximately 68 per cent of the total area. This means that 68 out of 100 deviations may be expected to fall within those limits. Likewise within the limits of a value equal to three standard deviations may be expected to fall 997 out of 1000 deviations, which may be taken as a practical certainty.

From the standard deviation (or error) is derived the probable error, thus,

$$PE = 0.6745\sigma_x.$$

The probable error is defined as the error, or deviation, that is just as likely to be exceeded as not. Its range is shown on Figure 1; the area beneath the portion of the curve within the limits of the probable error equals the area without.

The standard deviation of the universe or population is

$$\sigma_m = \frac{\sigma_x}{\sqrt{N}}$$

where $\sigma_m$ is the standard deviation of a large number of samples or a sample with a very large number of data. If only one sample with a limited number of data is available this formula should be used:

$$\sigma_m = \frac{\sigma_x}{\sqrt{N - 1}}$$

where $\sigma_x$ is the standard deviation of the sample. This formula may be used to compute the number of observations required to obtain a mean within specified limits.

The probable error of the mean of population may be estimated by the formula

$$PE_m = 0.6745\sigma_m.$$

The standard deviation and probable error of the coefficient of correlation as obtained by the formula given above are as follows:

$$\sigma_r = \frac{(1 - r^2)}{\sqrt{N}}$$

$$PE_r = 0.6745\frac{(1 - r^2)}{\sqrt{N}}.$$

It should be noted that the foregoing measures of precision are properly applicable only to arrays of data that follow substantially the normal probability law. They should not be used where the distributions possess appreciable skew, and particularly are not applicable to such highly skewed distributions as data of daily rainfall and floods.

In order to determine how well a theoretical function fits an observed distribution, the so-called "goodness-of-fit" test may be used (68, 54). This fit requires computation of a quantity called $\chi^2$ (chi-squared), which may be evaluated by tables computed for the purpose (147).

$$\chi^2 = \Sigma\frac{(f_o - f_t)^2}{f_t}$$

in which $f_o$ represents the observed frequencies, and $f_t$ the theoretical frequencies.

Although not exactly a measure of precision, the coefficient of variation is useful for making comparisons of data upon a non-dimensional basis. The coefficient of variation $CV$ is obtained by dividing the standard deviation by the mean, thus,

$$CV = \frac{\sigma}{M}.$$

*Envelopes.* In some studies of hydrology, it is important to know the maxima or minima of the values of the feature being studied. Observed data from various sources are plotted against a common related value and a curve drawn through the maximum and minimum values to form a presumed envelope of limiting values. Such values are, of course, entirely empirical but do at times furnish a guide in the application of the hydrologic principles.

*Graphic Presentation of Hydrologic Data.* Many hydrologic data may be presented by means of graphs of various sorts. Practically any graph useful in other branches of statistics may find a use in hydrology, but the most commonly used may be classified as follows:

*A*. Line Graphs
  1. On the basis of the coordinate system
    *a*. Rectangular or Cartesian coordinates
    *b*. Logarithmic ruling in one or both directions
    *c*. Probability ruling as one coordinate, with arithmetic or logarithmic ruling as the other
  2. On the basis of use
    *a*. Hydrographs to show a time series
    *b*. Maximum or minimum charts or envelope curves
    *c*. Histograms
    *d*. Band charts
    *e*. Silhouette charts

*B*. Bar or columnar charts

*C*. Maps
  1. Isometric maps (for example, isohyetal maps)
  2. Areal-distribution maps
  3. Statistical maps

Pertinent examples of these graphs are given hereinafter.

*Governing Principles of Hydrology*. The foregoing discussion of statistical principles is not intended to emphasize unduly the importance of those methods, nor to imply that the laws of mathematics are in any way a substitute for those of hydrology. The principles of statistical procedure cannot, of course, control hydrologic activity, but statistical methods enable one to analyze and measure the variations and correlations of hydrologic data, and in the complicated domain of hydrologic activity that function is important. The situation is such that in all hydrologic phenomena the laws of hydrology govern; that is to say, the observed causes are the effects of the operation of hydrologic principles.

# 2 ATMOSPHERIC MOISTURE AND ITS PRECIPITATION

*General Distribution of Water.* Water in one or another of its three states is distributed over the entire surface of the earth. The occurrence of the solid state, ice, is confined to areas and seasons in which the temperature falls below 32.0 F or 0.0 C. Where the temperature remains above that point either the vapor or the liquid or both are present. Water is found through the upper crust of the earth wherever the soil or rock is porous or fissured. The liquid is found over a large portion of the earth's surface, the oceans occupying approximately three-fourths of the global area. The vapor in large although varying quantities permeates the lower strata of the atmosphere. It exists in all regions and at any temperature. Dr. Helmut Landsberg (111) states that even the driest desert has moisture in the air.

*Chemical Properties.* Water is a chemical compound of hydrogen and oxygen; two atoms of the former are united with one of the latter to form one molecule of water, which union is expressed chemically by the symbol $H_2O$. It is a stable chemical compound and has many interesting chemical properties but none need be mentioned here except its power as a solvent. It can dissolve almost all substances in varying degrees, and for this reason pure water is practically never found in nature. Rain water approaches the pure state closely, but it contains small quantities of carbon dioxide and other gases as well as solids in the form of atmospheric dust, which it picks up in its fall.

*Physical Properties.* The physical properties of water are of direct interest to the hydrologist and a number have important effects on its occurrence or status. The freezing point in nature by definition is exactly 0 C or 32 F, and the boiling point is 100 C at sea level. The points serve arbitrarily as accepted measuring points of temperature. The freezing and boiling points under the same conditions from the point of no temperature, the origin of the absolute scale, are respec-

tively, 273.18 C or 459.72 F, and 373.18 C or 639.72 F. The amount of energy, called the "heat of vaporization," required to vaporize one gram of water is high, and at 100 C and a pressure of 760 mm is equivalent to 539 calories. In comparison, 100 calories are required to raise the temperature of one gram of water from 0 C to 100 C. The heat of fusion is the quantity of heat required to melt one gram of ice at 0 C, and is approximately 80 calories. In British thermal units the heat of vaporization is 970 and the heat of fusion is 144 approximately.

Specific heat is essentially the amount of heat required to raise one unit of mass of a given material one unit in temperature. It follows from the definition of the unit of heat that the specific heat of water is 1.0, which value varies slightly with the temperature. This value of specific heat is high as compared with that of many other common substances, and because of this property large bodies of water are effective stabilizers of atmospheric and, indirectly, land temperature. The specific heat of ice is less than one-half that of water.

Water attains its greatest density at 4 C (approximately 39 F), and at this temperature it is taken as 1.0 per cc in the metric system; in English units, usually as 62.4 lb per cu ft. The density of water decreases with temperature above and below 4 C; therefore it becomes slightly lighter as it approaches the freezing point. This characteristic tends to stabilize subsurface temperatures of lake waters, since colder and warmer water will remain on the surface.

The compressibility of water is so small that in all ordinary hydraulic and hydrologic work it may safely be neglected.

*Two Classes of Atmospheric Moisture.* The moisture of the atmosphere may be divided into two classes on the basis of visibility. It may be present as a true vapor or gas, in which case it is invisible and is referred to as humidity. In the second class it consists of minute droplets formed upon condensation and is visible; this class is designated either as clouds or fog, a distinction dependent upon altitude.

*Humidity.* Invisible atmospheric vapor or humidity is a factor of primary importance in meteorology and less directly, an equally important element in hydrology. It has therefore received intensive study from workers in both sciences.

There are three fundamental attributes of humidity subject to measurement. First, there is absolute humidity which is determined by the mass of water vapor in a unit volume of air. Absolute humidity varies as the air expands or contracts and because it is thus variable it is not commonly measured. The second attribute is specific humidity which is defined as the mass of water vapor in a unit mass of air. The

specific humidity for a given mass of air is constant except for real changes in the mass of vapor; that is, specific humidity does not change with volume variations of the air. Because of this constancy, specific humidity is the generally accepted means of defining the moisture characteristics of a given air mass. The third attribute is relative humidity, which is the ratio between actual humidity present and the maximum quantity of vapor that will exist in a given space at any definite temperature.

Occasionally the quantity of humidity is expressed in terms of partial atmospheric pressure, the pressure being expressed as force per unit area or in terms of inches or millimeters of mercury. Since the total pressure of a mixture of gases, such as the atmosphere, is the sum of the partial pressures of each gas, humidity can be expressed in terms of its partial pressure. In works on chemistry, it is sometimes referred to as "aqueous tension" or "vapor tension." However, the designation of a pressure by the term "tension" seems inconsistent.

The total quantity of water vapor that can be held in a unit volume of space is limited but varies with the temperature. When air at a given temperature contains the maximum quantity of vapor it is said to be saturated and the temperature is called the "dew point." Saturation capacity is a unique function of the dew point. Table 2 presents values of the vapor pressure and specific humidity for saturation at the given temperatures.

TABLE 2. MOISTURE CAPACITY OF AIR

| TEMPERATURE Degrees | | VAPOR PRESSURE | | SPECIFIC HUMIDITY |
|---|---|---|---|---|
| *F* | *C* | *Inches of Mercury* | *Millimeters of Mercury* | *Grams per Kg* |
| −30 | −34.5 | 0.0069 | 0.175 | 0.143 |
| −20 | −28.9 | 0.0126 | 0.320 | 0.261 |
| −10 | −23.3 | 0.0222 | 0.561 | 0.460 |
| 0 | −17.8 | 0.0383 | 0.973 | 0.794 |
| 10 | −12.2 | 0.0631 | 1.603 | 1.308 |
| 20 | −6.7 | 0.1026 | 2.606 | 2.13 |
| 30 | −1.1 | 0.164 | 4.165 | 3.40 |
| 40 | 4.4 | 0.247 | 6.275 | 5.12 |
| 50 | 10.0 | 0.360 | 9.15 | 7.46 |
| 60 | 15.6 | 0.517 | 13.13 | 10.72 |
| 70 | 21.1 | 0.732 | 18.60 | 15.18 |
| 80 | 26.7 | 1.022 | 26.0 | 21.2 |
| 90 | 32.2 | 1.408 | 35.8 | 29.2 |
| 100 | 37.7 | 1.916 | 48.7 | 39.7 |

The data of Table 2 were taken from *Psychrometric Tables* by Charles F. Marvin, U. S. Weather Bureau (119), for an atmospheric pressure of 30 inches.

*Effect of Cooling.* Consideration of Table 2 indicates clearly the effect of cooling the atmosphere. If, for example, air containing 10.72 grams of moisture per kilogram has a temperature of 60 F and is cooled to, say 40 F, it must lose 5.6 grams of its vapor (by condensation) since 10.72 grams and 5.12 grams are the capacities at the given temperatures, respectively. If, however, the air is not saturated at a given temperature it can be cooled to its dew point without condensation of its vapor; further cooling would result in condensation.

Condensation from the atmosphere may result from cooling in a number of ways, the principal causes being as follows:

1. Adiabatic expansion on being lifted to higher elevations
2. Contact with a cold surface
3. Mixing with colder air
4. Radiation

Expansion cools air as is well known in physics. Expansion of air under meteorological conditions is achieved primarily by lifting as in moving up a mountain slope, so that lifting and expansion are interrelated processes. That double process is the most effective means for cooling and producing rapid condensation. The remaining three causes result in relatively small amounts of precipitation and fog or clouds. However, since the dew point of air is variable, the same degree of cooling may or may not result in condensation.

*Measurement of Humidity.* Humidity is measured directly only in the laboratory where the amount of moisture can be extracted from a known volume or mass of air and weighed. For observational purposes in meteorological work that process is too slow and cumbersome and recourse is had to measuring it indirectly. Two observable effects of humidity are utilized for its measurement: first, the lowering of temperature by evaporation to the dew point of the surrounding air, and second, the direct effect of atmospheric moisture on some substances such as dry human hair or certain chemical substances.

An instrument called the "psychrometer" is used to determine the depression of temperature to the dew point. It is constructed in various forms which are described in elementary texts on meteorology, such as the work of Dr. Willis I. Milham (136), hence a detailed description is not justified here. Essentially the psychrometer consists of two thermometers, one of which has the mercury bulb covered by a wet muslin and is called the "wet-bulb thermometer." To make the measurement, the muslin cover is fully saturated with clean and, preferably, distilled water, and is thoroughly ventilated by whirling or by a fan until a steady temperature is shown by the wet-bulb thermometer. The

temperatures of both thermometers are read and the difference is computed. The temperature difference and actual air temperature are entered in tables showing relative humidity, vapor pressure at saturation, and temperature of dew point, to obtain the desired values. These tables have been computed and published by the Weather Bureau (119) for actual air temperatures from −40 F to 140 F.

When the actual barometric pressure is known, the specific humidity can be computed by the formula given by Haurwitz (85):

$$q = \frac{621e}{p - 0.379e}$$

where $q$ = specific humidity in grams per kilogram; $e$ = actual vapor pressure; $p$ = actual air pressure. The constant 621 is 1000 times the ratio of molecular weights of water vapor and air. While the psychrometer does not give the actual vapor pressure directly, it can be found for the given dew point in the *Psychrometric Tables.*

The basis of the psychrometer is explained by Dr. W. J. Humphreys (96) by showing that the depression in temperature of the wet-bulb thermometer is attributable to loss of heat by evaporation, the rate of which, in turn, is caused by the difference in the actual humidity and the capacity of the air space for moisture at the dry-bulb temperature. The psychrometric equation is as follows:

$$e = e_s - AB(t - t_w)$$

in which $e$ = the actual vapor pressure; $e_s$ = the saturation vapor pressure at temperature $t_w$; $t$ = the observed dry-bulb temperature; $t_w$ = the wet-bulb temperature; $A$ = a constant dependent upon the specific heat of the free air, the ratio of the molecular weights of water and free air, and the latent heat of vaporization; $B$ = the current barometric pressure.

*Hygrometer.* Since the psychrometer is suitable only for single measurements another instrument, the hygrometer, must be used to obtain continuous records of humidity. The essential part of the hygrometer is some substance having a property that varies with the moisture content of the air. Dry human hair has been used for many years because its length varies with the vapor present. It has been found by Fergusson (56) to be reasonably accurate when used under conditions during which the moisture content of the atmosphere does not change rapidly. When it is desired to measure rapid changes of humidity such as, for example, when making airplane ascents, the hair hygrometer is too sluggish to give accurate results. For this purpose

certain hygroscopic salts, particularly lithium chloride, have been found by Dunmore (50, 51) to be superior. In this type of chemical hygrometer the resistance to an electric current varies as the salt is affected by atmospheric moisture, and the variable current is measured and calibrated to show variations in moisture. Experiments showed that this instrument was appreciably more sensitive to rapid changes in humidity as it was carried to higher elevations by pilot balloon or airplane.

*Variations in Humidity.* Observations show that great variations exist from time to time and place to place in the values of absolute, specific, and relative humidities. Considering the earth as a whole, all phases of humidity vary with latitude. Absolute humidity varies greatly with changes in elevation and to a lesser extent with temperature. Relative humidity is variable at different elevations, but the variations are the result of causes not related directly to altitude. Since relative humidity is principally a function of the air temperature it varies mainly from changes of temperature, and though it usually diminishes with higher elevations the variation is due to a lack of mixing from the surface strata upward. Again, all aspects of humidity vary from day to day as different air masses move in and out of any given locality.

*Variation of Humidity with Latitude.* Since the higher latitudes are in the cold polar regions there is a decrease in both specific and absolute humidity due to the reduced capacity of the air to hold moisture. Likewise there is less evaporation in those regions to replace losses from precipitation. The relationship between latitude and humidity, however, is irregular because of the distribution of land and water masses, prevailing winds, and other causes, but the trend from the equator northward or southward is a continued decrease.

*Variation with Altitude.* Theory requires, and extensive observations show conclusively, that humidity diminishes with elevation above the earth's surface. Because of the decreasing temperature at the higher elevations the capacity of the air space to hold moisture is less. Because of that fact moisture is lost from the higher elevations by precipitation of rain and snow. Furthermore, the source of atmospheric moisture is on the surface of the earth from which moisture is distributed upward by turbulence and convection, so that the greatest concentration should be expected in the lowest stratum. In order to show the variation of actual water vapor present the specific humidity should be studied. Table 3 lists the data of the average specific humidity at various heights observed at several stations. The data were obtained from the *Monthly Weather Review* for the months of January and

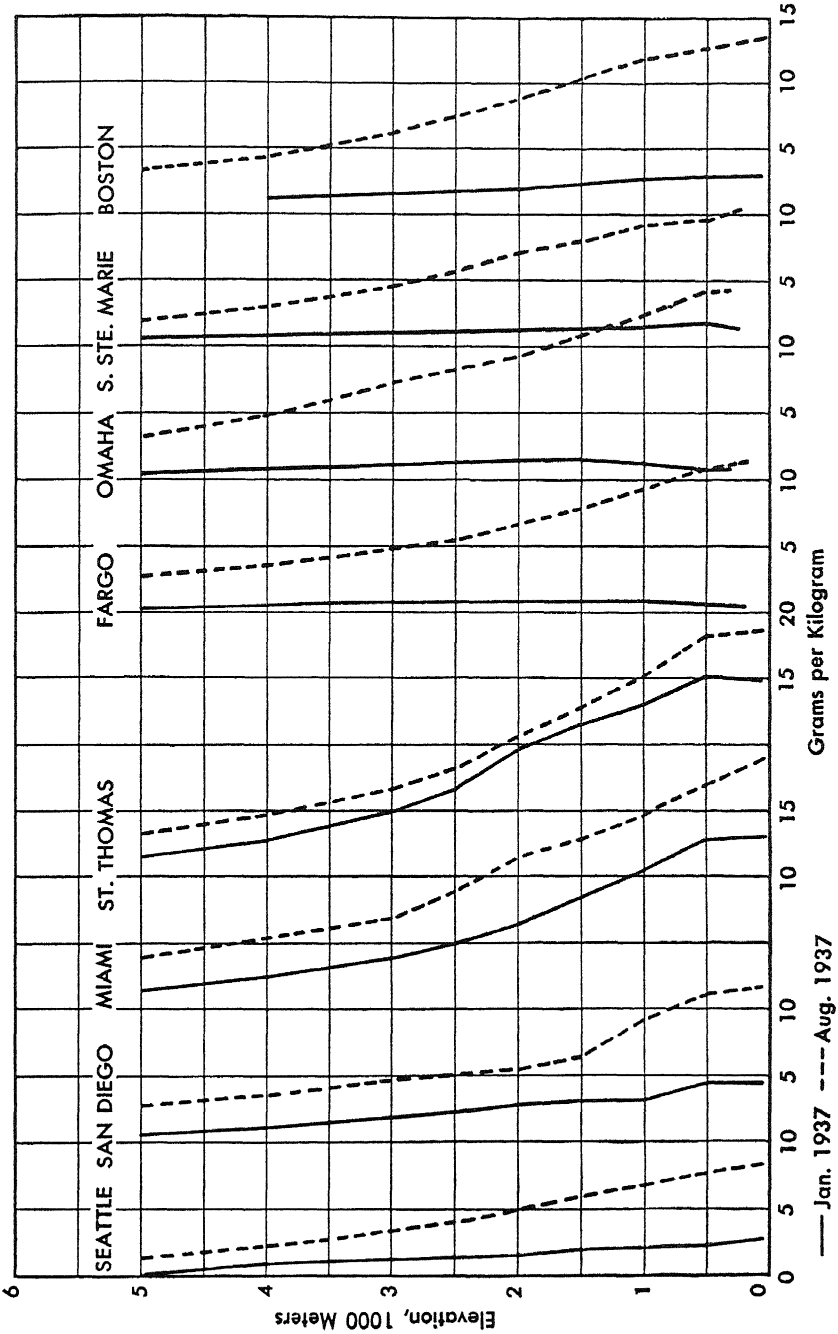



FIGURE 2. Specific Humidity at Various Elevations

August 1937 and may be accepted as typical for the air above those stations. The same data are shown graphically in Figure 2.

Actually the humidity, however, has not the regularity depicted by the average vertical curves shown because the atmosphere may be, and frequently is, made up of layers of air with diverse characteristics of

TABLE 3. SPECIFIC HUMIDITY AT VARIOUS ELEVATIONS IN GRAMS PER KILOGRAM

| ELEVATION *In Meters* | SEATTLE WASH. | SAN DIEGO CALIF. | MIAMI FLA. | ST. THOMAS V. I. | FARGO N. DAK. | OMAHA NEBR. | SAULT STE. MARIE MICH. | BOSTON MASS. |
|---|---|---|---|---|---|---|---|---|
| JANUARY 1937 | | | | | | | | |
| Surface | 2.7 | 4.5 | 13.1 | 14.9 | 0.5 | 0.9 | 1.5 | 2.9 |
| 500 | 2.3 | 4.5 | 12.9 | 15.2 | 0.6 | 0.9 | 1.8 | 2.9 |
| 1000 | 2.1 | 3.8 | 10.5 | 13.0 | 0.8 | 1.3 | 1.6 | 2.7 |
| 1500 | 2.0 | 3.2 | 8.3 | 11.5 | 0.9 | 1.5 | 1.4 | 2.3 |
| 2000 | 1.7 | 2.8 | 6.3 | 9.7 | 0.9 | 1.4 | 1.2 | 1.9 |
| 2500 | 1.5 | 2.4 | 5.0 | 6.6 | 0.9 | 1.3 | 1.1 | 1.7 |
| 3000 | 1.3 | 2.0 | 3.8 | 4.9 | 0.8 | 1.1 | 1.0 | 1.6 |
| 4000 | 0.8 | 1.2 | 2.5 | 2.7 | 0.7 | 0.8 | 0.8 | 1.2 |
| 5000 | 0.1 | 0.6 | 1.4 | 1.5 | 0.3 | 0.4 | 0.6 | .. |
| Number of Observations | 9 | 31 | 30 | 26 | 30 | 31 | 29 | 19 |
| AUGUST 1937 | | | * | | | | | |
| Surface | 8.3 | 11.6 | 18.9 | 18.7 | 11.3 | 14.4 | 10.4 | 13.3 |
| 500 | 7.7 | 11.1 | 17.0 | 18.2 | 10.8 | 14.1 | 9.6 | 12.5 |
| 1000 | 6.7 | 9.2 | 14.6 | 15.1 | 9.3 | 12.4 | 9.2 | 11.2 |
| 1500 | 6.0 | 6.3 | 12.8 | 12.7 | 7.7 | 10.8 | 7.9 | 10.2 |
| 2000 | 5.0 | 5.5 | 11.5 | 10.6 | 6.7 | 9.2 | 7.0 | 8.7 |
| 2500 | 4.0 | 5.0 | 8.8 | 8.1 | 5.4 | 8.2 | 5.6 | 7.5 |
| 3000 | 3.3 | 4.7 | 6.9 | 6.6 | 4.7 | 7.2 | 4.5 | 6.1 |
| 4000 | 2.2 | 3.6 | 5.4 | 4.8 | 3.5 | 4.8 | 3.0 | 4.4 |
| 5000 | 1.3 | 2.9 | 3.9 | 3.4 | 2.7 | 3.2 | 2.0 | 3.3 |
| Number of Observations | 16 | 29 | 23 | 31 | 30 | 31 | 31 | 28 |
| Elevation of Surface Meters | 10 | 10 | 4 | 8 | 274 | 300 | 221 | 5 |

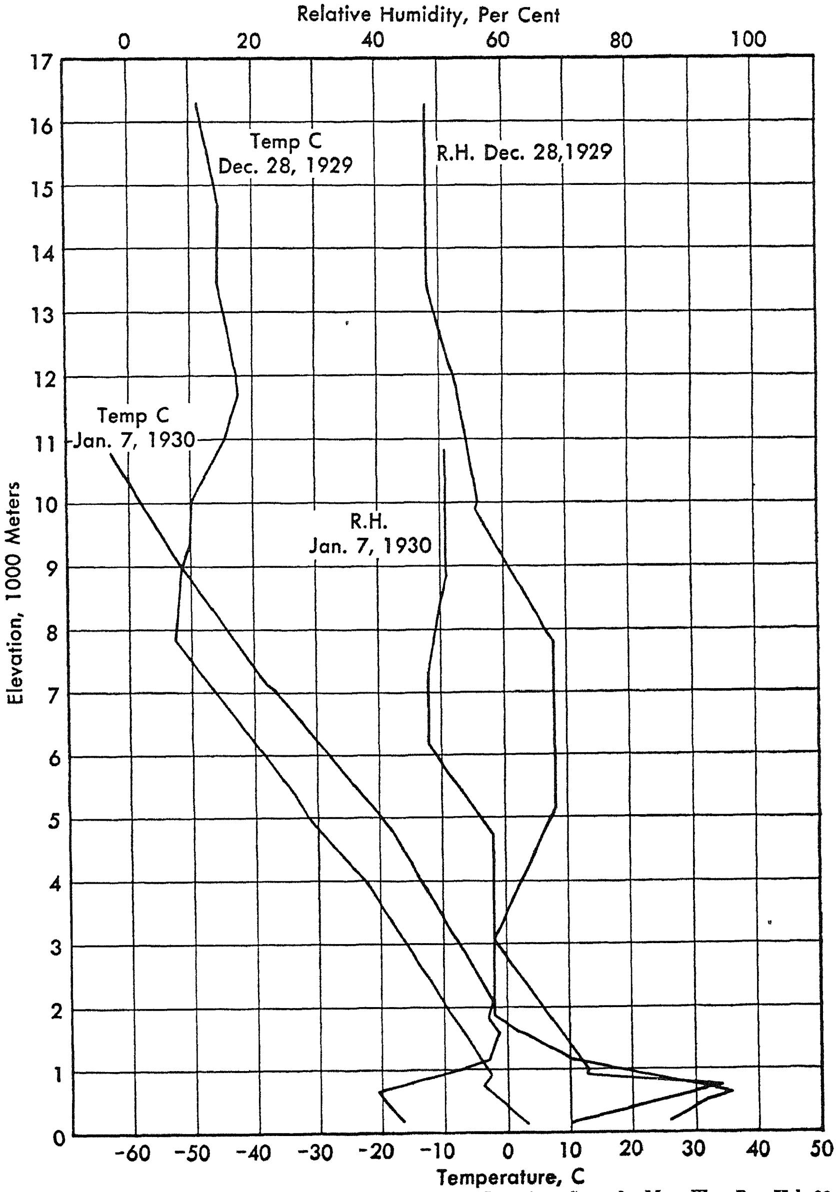
*Data from Coco Solo, Canal Zone; conditions similar to Miami, Fla.

moisture and temperature. The diversity of humidity is best shown by observations made by the electro-chemical hygrometer which responds quickly to changes in moisture. A vertical curve showing the relative humidity on a single flight is given in Figure 3, and other individual flights are illustrated in Figure 6.

*Daily Variations of Humidity.* Since relative humidity is partially dependent upon temperature, it varies considerably during the day. It falls from a maximum in the morning when it may be near saturation to a minimum in the middle of the day when the temperature is highest,

then rising at night as the temperature falls. The nature of this average fluctuation is illustrated by the data in Table 4.

Variations of humidity from day to day are of greater importance since they result from more extensive meteorological activity. Air from different sources moving into a given locality has a continually variable




*Data from Samuels, Mon. Wea. Rev. Vol. 62*

FIGURE 3. Sounding Observations, Davenport, Iowa

humidity, depending upon the characteristics of the region from which it came. Table 5 illustrates the daily variations of humidity expressed as vapor pressure for the four seasons at Boston, Mass.

In March the daily range is from 0.077 to 0.347 inch; in June, 0.211 to 0.638 inch; in September, 0.203 to 0.684 inch; in December, 0.052 to 0.448 inch. It may be noted that the high monthly averages are in the

TABLE 4. DIURNAL FLUCTUATION OF RELATIVE HUMIDITY

| STATION | MEAN ANNUAL TEMPERATURE, °F | MEAN ANNUAL RELATIVE HUMIDITY, PER CENT 8:00 A.M. | Noon | 8:00 P.M. |
|---|---|---|---|---|
| NEW ENGLAND | | | | |
| Boston, Mass. | 50 | 73 | 61 | 71 |
| Eastport, Maine | 42 | 79 | 72 | 79 |
| New Haven, Conn. | 50 | 74 | 62 | 72 |
| Portland, Maine | 46 | 75 | 61 | 72 |
| SOUTHEAST | | | | |
| Key West, Fla. | 77 | 78 | 70 | 77 |
| Miami, Fla. | 75 | 78 | 66 | 74 |
| Tampa, Fla. | 72 | 83 | 58 | 75 |
| NORTH CENTRAL | | | | |
| Bismarck, N. Dak. | 40 | 81 | 56 | 60 |
| Denver, Colo. | 52 | 64 | 40 | 42 |
| Kansas City, Mo. | 55 | 78 | 55 | 69 |
| Omaha, Nebr. | 51 | 79 | 55 | 60 |
| WEST | | | | |
| Pocatello, Idaho | 48 | 68 | 48 | 47 |
| Seattle, Wash. | 51 | 87 | 70 | 67 |
| Red Bluff, Calif. | 62 | 70 | 47 | 41 |

summer. These data should be accepted as only approximate since an inspection of individual values reveals rather frequent repetition of certain values, such as 0.555 and 0.203, which appear more often than would be expected from a purely chance result. It is the type of data to be expected from the lack of sensitiveness of the measuring devices.

It is of interest to note that during March 1936 there occurred the greatest flood of record on many streams in New England due to heavy rain and fast melting of snow, yet the average daily humidity to produce the heavy rain did not range as high as during the summer of the same year. Evidently something in addition to moisture in the air is needed to cause rainfall.

*Correlation of Humidity on Successive Days.* The daily variation of humidity raises a question as to the relationship of the value observed on one day with that obtained on the following day. A study was made of noon humidity expressed in terms of vapor pressure in inches of mercury for the station at Boston, Mass., for January for the six years 1932–37. The observed vapor pressure of each day was paired with the

TABLE 5. DAILY VARIATIONS OF HUMIDITY, BOSTON, MASS., FOUR MONTHS, 1936 (HUMIDITY EXPRESSED AS VAPOR PRESSURE IN INCHES OF MERCURY)

| DAY | MARCH | JUNE | SEPTEMBER | DECEMBER |
|---|---|---|---|---|
| 1 | 0.085 | 0.360 | 0.287 | 0.085 |
| 2 | .098 | .517 | .387 | .180 |
| 3 | .180 | .638 | .555 | .203 |
| 4 | .172 | .417 | .432 | .164 |
| 5 | .203 | .310 | .387 | .108 |
| 6 | .077 | .298 | .432 | .150 |
| 7 | .081 | .387 | .402 | .103 |
| 8 | .081 | .465 | .684 | .077 |
| 9 | .203 | .482 | .555 | .130 |
| 10 | .277 | .499 | .517 | .277 |
| 11 | .247 | .536 | .517 | .247 |
| 12 | .347 | .555 | .661 | .219 |
| 13 | .143 | .482 | .310 | .136 |
| 14 | .136 | .465 | .247 | .113 |
| 15 | .237 | .536 | .334 | .150 |
| 16 | .237 | .347 | .555 | .164 |
| 17 | .334 | .277 | .555 | .266 |
| 18 | .310 | .499 | .448 | .057 |
| 19 | .322 | .555 | .373 | .108 |
| 20 | .203 | .638 | .387 | .310 |
| 21 | .277 | .517 | .465 | .113 |
| 22 | .150 | .211 | .536 | .054 |
| 23 | .089 | .334 | .616 | .052 |
| 24 | .172 | .387 | .555 | .150 |
| 25 | .203 | .373 | .203 | .195 |
| 26 | .211 | .517 | .266 | .150 |
| 27 | .237 | .616 | .360 | .228 |
| 28 | .136 | .448 | .334 | .180 |
| 29 | .180 | .298 | .228 | .113 |
| 30 | .256 | 0.277 | 0.203 | .187 |
| 31 | 0.334 | .... | .... | 0.448 |
| Average | 0.201 | 0.448 | 0.426 | 0.165 |

observation of the following day and the pairs of data were grouped into classes of 0.05 inch each. The figures in the small squares of Table 6 are the number of days so paired in each class. The correlation was computed by means of the correlation table as described by Reitz (154). Since the data are grouped the correlation coefficient can be computed by this form of the equation:

$$r = \frac{\frac{1}{N}\Sigma XY - \bar{X}\bar{Y}}{\sigma_x \sigma_y}.$$

The values to be substituted in the above equation are computed by the formulas immediately following Table 6 from values taken from the table.

TABLE 6. CORRELATION OF HUMIDITY ON SUCCESSIVE DAYS

| $SY$ | $S$ | $Y^2f(Y)$ | $Yf(Y)$ | $Y$ | $f(Y)$ | HUMIDITY ON FIRST DAY .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | HUMIDITY ON SECOND DAY |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 168 | −42 | 240 | −60 | −4 | 15 | 7 | 3 x | 3 | — | 1 | 1 | — | — | — | .05 |
| 135 | −45 | 288 | −96 | −3 | 32 | 1 | 7 | 12 | 3 x | 3 | 5 | 1 | — | — | .10 |
| 158 | −79 | 240 | −120 | −2 | 60 | 2 o | 12 | 20 | 11 x | 8 | 3 o | 1 | 1 | 2 | .15 |
| 13 | −13 | 24 | −24 | −1 | 24 | — | 3 o | 8 | 4 | 2 x | 2 | 2 | 2 | 1 | .20 |
| 0 | −36 | 0 | 0 | 0 | 23 | 3 | 4 | 4 o | 7 o x | 3 o | 1 | 1 | — | — o | .25 |
| −18 | −18 | 10 | 10 | 1 | 10 | — | 3 | 3 | 3 x | 1 | — | — o | — o | — | .30 |
| −30 | −15 | 36 | 18 | 2 | 9 | — | 1 | 6 | — x | 2 | — | — | — | — | .35 |
| 33 | 11 | 36 | 12 | 3 | 4 | — | — | — | — | — | — | 2 | 1 x | 1 | .40 |
| 8 | 2 | 64 | 16 | 4 | 4 | — | — | 1 | — | 1 | — x | 2 | — | — | .45 |
| 467 | | 938 | −144 | | 181 | | | | | | | | | | |
| | | | | | 181 | 13 | 33 | 57 | 28 | 21 | 12 | 9 | 4 | 4 | $f(X)$ |
| | | | | | | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 | $X$ |
| | | | | | −235 | −52 | −99 | −114 | −28 | 0 | 12 | 18 | 12 | 16 | $Xf(X)$ |
| | | | | | 911 | 212 | 297 | 228 | 28 | 0 | 12 | 36 | 36 | 64 | $X^2f(X)$ |
| | | | | | | −35 | −55 | −77 | −32 | −22 | −27 | 7 | −1 | −2 | $T$ |
| | | | | | 467 | 140 | 165 | 154 | 32 | 0 | −27 | 14 | −3 | −8 | $TX$ |

$$\bar{X} = \frac{-235}{181} = -1.298 \qquad \bar{Y} = \frac{-144}{181} = -0.796$$

$$\sigma_x^2 = \frac{911}{181} - (-1.298)^2 = 3.35 \qquad \sigma_y^2 = \frac{938}{181} - (-.796)^2 = 4.55$$

$$\sigma_x = 1.830 \qquad \sigma_y = 2.133$$

$$\frac{1}{n}\Sigma XY = \frac{467}{181} = 2.581 \qquad r = \frac{2.581 - (-1.298)(-.796)}{(1.830)(2.133)} = 0.397.$$

After classifying the pairs of data as shown in Table 6, the first step is to compute the values in the columns at the left and bottom of the grid. Both sets of columns are computed in the same manner so that only one description is needed. The values under $f(Y)$, the distribution of data, are the totals of the classes on the same line. Under $Y$ the middle class is selected as an axis and is designated $O$, and the values above and below are the number of rows from the axis. $Yf(Y)$ and $Y^2f(Y)$ are obtained by multiplying $f(Y)$ by the first and second powers of $Y$ as shown. The values under $S$ are obtained in the following manner: the numbers in class grid on the same line with a given value under $S$ are multiplied by the value of $X$ in the respective columns; all products for each line are summed up algebraically and the total is the value under $S$. For example, take the value of $-42$ on the first line:

| | | |
|---|---|---|
| 7 times | $-4$ = | $-28$ |
| 3 times | $-3$ = | $-9$ |
| 3 times | $-2$ = | $-6$ |
| 1 times | $0$ = | $0$ |
| 1 times | $1$ = | $1$ |
| Total | | $-42$ |

The values under $SY$ are the simple products as indicated. The columns are totaled as shown.

The columns for the $X$'s are computed in a similar manner. The totals for both sets of columns are used to calculate the correlation coefficient as shown in the equations on page 33.

The standard error of the correlation coefficient may be computed as follows:

$$\sigma_r = \frac{1 - r^2}{\sqrt{N}} = \frac{1 - (-0.397)^2}{\sqrt{181}} = \frac{0.846}{\pm 13.45} = \pm 0.063.$$

The correlation coefficient may now be stated as

$$r = 0.397 \pm 0.063.$$

This value of $r$ is high enough to show some correlation but extensive studies of statistics in many fields indicate that it is not high enough to be materially significant. Moreover, it must be interpreted in the light of other information available. Usually the same type of weather prevails over Boston for two, three, or more days in succession so that the correlation found is very likely the result of the persistence of the same type of air. Because of this situation and the rather low correlation that was found, it can be concluded that there is no causal relationship between the humidity values of successive days. To use an expression of Fry's (68), the daily values of humidity may occur "individually at random" or "collectively at random."

*Equations of Regression Lines.* In order to show the relationship between the lines of regression and the distribution of the correlated data as shown in Table 6, the equations of the lines are computed. For the correlation of the humidity of the first day on that of the second day as $Y$ on $X$, the type equation is

$$Y = \bar{\bar{Y}} + r\frac{\sigma_y}{\sigma_x}(X - \bar{\bar{X}}).$$

From Table 6:

$$\begin{aligned}
\bar{\bar{Y}} &= -0.796 \text{ class units} = 0.185 \text{ inch} \\
\sigma_y &= 2.133 \text{ class units} = 0.107 \text{ inch} \\
\bar{\bar{X}} &= -1.298 \text{ class units} = 0.160 \text{ inch} \\
\sigma_x &= 1.830 \text{ class units} = 0.092 \text{ inch} \\
r &= 0.397.
\end{aligned}$$

Substituting in the type equation

$$Y = 0.185 + 0.397\left(\frac{0.107}{0.092}\right)(X - 0.160)$$

which reduces to

$$Y = 0.111 + 0.462X.$$

The line defined by the above equation is a straight line through the best values of the squares in Table 6 marked by the small circles, which squares are the centroids of the values in the columns under humidity on the first day. The equation is the best determination of values of humidity on the first day if values for the second are given.

For the regression of $X$ on $Y$, the type equation is

$$X = \bar{\bar{X}} + r\frac{\sigma_x}{\sigma_y}(Y - \bar{\bar{Y}}).$$

Substituting the values in the preceding paragraph, we have

$$X = 0.160 + 0.397\left(\frac{0.092}{0.107}\right)(Y - 0.185)$$

which reduces to

$$X = 0.97 + 0.342Y.$$

Like the final equation in the preceding paragraph, this represents the best values of $X$ when values of $Y$ are given. It represents a straight line drawn through the group of squares marked by the small crosses.

*Frequency of Humidity.* The data of daily humidity for the four summer months June to September 1932–37 at Boston were used to determine the frequency of different values of daily humidity. A fundamental requirement of data to be used for the determination of frequency curves is independence, that is, such data should not be dependent upon one another to any appreciable degree. Since low correlation of the data of humidity at Boston has been found, and even that is explainable on grounds other than interdependence, this requirement has been met and the frequency can be computed from the data at hand. The data were grouped as before into classes of 0.05 inch of vapor pressure each. The observed number of days with humidity in the various classes of magnitude are listed as $f(X)$ in Table 7, and the distribution was used to plot the histogram shown in Figure 4. The distribution is evidently an approximation of the normal frequency function shown in Figure 1 and for that reason the frequency curve was computed by the equation for the normal law:

$$P_n = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x/\sigma)^2}$$

where $P_n$ is the probable number of days in any given class of humidity; $x$, the deviation from the mean; the other symbols as used heretofore.

The class limits of 0.05 inch each are given in columns 1 and 2; these classes are then designated by a number $X$ in column 3, and the remainder of the work is based on the average group number and the deviations therefrom. The number of days with vapor pressure in each class is given in column 4; the total number of daily observations is 732 and is designated $S_1$.

The statistical moments are computed by the summation method as described in Chapter 1. To compute the values in column 5 one adds successively, beginning at the bottom, each value in column 4; the top figure in column 5 should equal the total of $f(X)$. The total of column 5

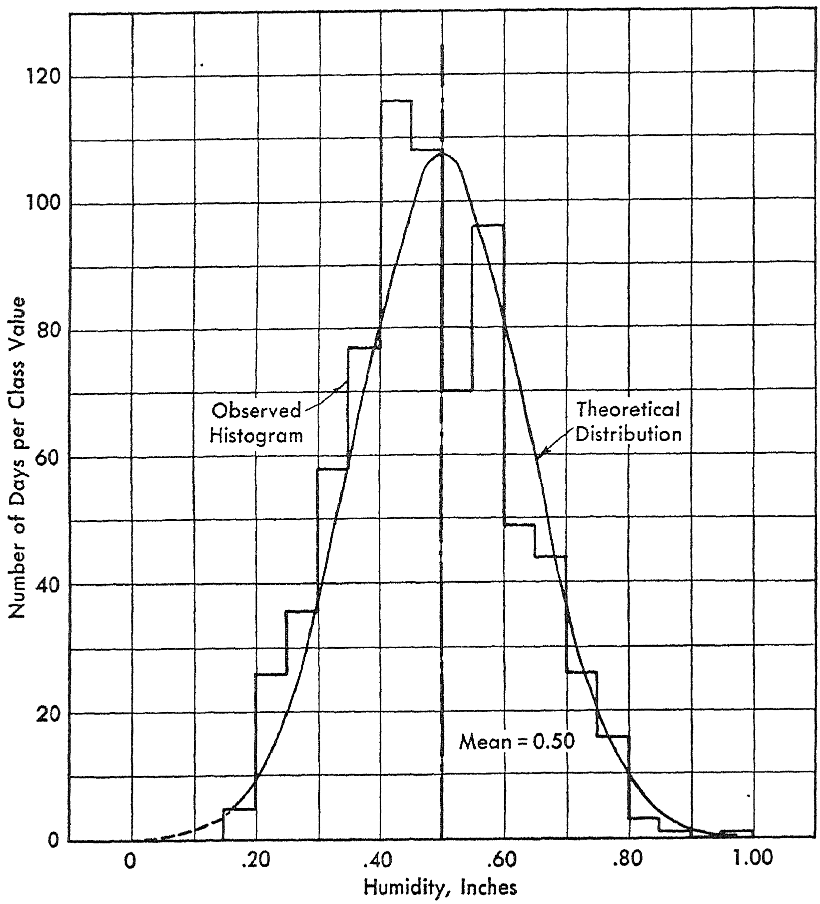



FIGURE 4. Frequency of Daily Humidity Values, Boston, Mass.

is designated $S_2$. The same process is used to compute $S_3$ in column 6, the top value of which should equal the total of column 5. The moments are computed from $S_1$, $S_2$, and $S_3$ by the formulas given in Chapter 1 as follows:

$$M = \frac{S_2}{S_1} = \frac{5226}{732} = 7.14 \text{ class units}$$

$$U_2 = 2\frac{S_3}{S_1} - M(1 + M)$$

$$= \frac{2(24005)}{732} - 7.14(1 + 7.14) = 7.46$$

$$\sigma = \sqrt{7.46} = 2.73 \text{ class units.}$$

Columns 7 to 10 are used to compute the theoretical distributions

given in column 11. The deviations of the class midpoints $X$ from the mean $M$ are given in column 8. In order to utilize the probability tables the deviations must be expressed in units of the standard deviation $\sigma$; this is done in column 9, the values of which are used to enter tables of the probability integral, such as those of Glover (71), to obtain values for $Z$ in column 10. The total number of observations,

TABLE 7. FREQUENCY OF HUMIDITY VALUES, BOSTON, MASS.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| CLASSIFICATION Inches From | To | $X$ CLASS UNITS | $f(X)$ | $S_2$ | $S_3$ | MID-POINT $X'$ | $x = X' - 7.0$* | $\frac{x}{\sigma}$ | $Z$ | $f_t(X)$ |
| 0.15 | 0.20 | 1 | 5 | 732 | 5226 | 0.5 | −6.5 | −2.38 | 0.0235 | 6.3 |
| .20 | .25 | 2 | 26 | 727 | 4494 | 1.5 | −5.5 | −2.02 | .0519 | 14.0 |
| .25 | .30 | 3 | 36 | 701 | 3767 | 2.5 | −4.5 | −1.65 | .102 | 27.4 |
| .30 | .35 | 4 | 58 | 665 | 3066 | 3.5 | −3.5 | −1.28 | .176 | 47.2 |
| .35 | .40 | 5 | 77 | 607 | 2401 | 4.5 | −2.5 | −0.92 | .261 | 70.0 |
| .40 | .45 | 6 | 116 | 530 | 1794 | 5.5 | −1.5 | −.55 | .343 | 92.0 |
| .45 | .50 | 7 | 108 | 414 | 1264 | 6.5 | −0.5 | −.184 | .392 | 106.0 |
| .50 | .55 | 8 | 70 | 306 | 850 | 7.5 | 0.5 | .184 | .392 | 106.0 |
| .55 | .60 | 9 | 96 | 236 | 544 | 8.5 | 1.5 | .55 | .343 | 92.0 |
| .60 | .65 | 10 | 49 | 140 | 308 | 9.5 | 2.5 | .92 | .261 | 70.0 |
| .65 | .70 | 11 | 44 | 91 | 168 | 10.5 | 3.5 | 1.28 | .176 | 47.2 |
| .70 | .75 | 12 | 26 | 47 | 77 | 11.5 | 4.5 | 1.65 | .102 | 27.4 |
| .75 | .80 | 13 | 16 | 21 | 30 | 12.5 | 5.5 | 2.02 | .0519 | 14.0 |
| .80 | .85 | 14 | 3 | 5 | 9 | 13.5 | 6.5 | 2.38 | .0235 | 6.3 |
| .85 | .90 | 15 | 1 | 2 | 4 | 14.5 | 7.5 | 2.75 | .00909 | 2.4 |
| .90 | .95 | 16 | 0 | 1 | 2 | 15.5 | 8.5 | 3.11 | .00317 | .8 |
| .95 | 1.00 | 17 | 1 | 1 | 1 | 16.5 | 9.5 | 3.48 | .00094 | .2 |
| Totals | | | 732 | 5226 | 24005 | | | | | 729.0 |

* $M$ taken equal to 7.0 to simplify the computations.

732, is divided by $\sigma$, then multiplied by the values in column 10 to obtain the theoretical distribution, $f_t(X)$, the curve of which is shown in Figure 4.

It was noted above that the mean and standard deviations were expressed in units of the class interval, 0.05 inch. Some correction is necessary to obtain the mean and standard deviation in terms of inches. The origin used in the computation of the moments is the next class below 1, which is the class with limits 0.10 to 0.15, or more accurately the origin is at the upper limit of this class which is 0.150. Then to obtain the mean in inches, the mean in class units must be multiplied by the value of the class unit in inches, and 0.150 added to the product. On this basis the mean in inches is as follows:

$$M = 7.0(0.05) + 0.150 = 0.500 \text{ inch.}$$

The conversion of the standard deviation from class units to inches is simpler since it is already based on the arithmetical mean; it is necessary only to multiply it by the value of the class unit in inches to obtain the standard deviation in the same units:

$$\sigma = 2.72(0.05) = 0.136 \text{ inch.}$$

Scrutiny of Figure 4 shows that the histogram of observed values is slightly skewed so that it does not fit closely the theoretical curve. This conclusion is supported by a goodness-of-fit test (not included herewith) which also indicated a poor fit with many negative values on one side. However, the rough approximation to the normal probability curve indicated by the histogram shows that the frequency of humidity can be computed by theoretical methods. Since it is mainly a matter of skew, other functions are available to provide a more accurate computation if the results would justify the labor.

TABLE 8. MEAN MONTHLY RELATIVE HUMIDITY

| STATION | RELATIVE HUMIDITY IN PERCENTAGE | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Jan.* | *Feb.* | *Mar.* | *Apr.* | *May* | *June* | *July* | *Aug.* | *Sept.* | *Oct.* | *Nov.* | *Dec.* |
| NEW ENGLAND | | | | | | | | | | | | |
| Boston, Mass. | 68 | 66 | 65 | 65 | 67 | 67 | 73 | 71 | 71 | 68 | 70 | 68 |
| Eastport, Maine | 75 | 74 | 74 | 74 | 75 | 79 | 81 | 81 | 79 | 76 | 76 | 76 |
| New Haven, Conn. | 70 | 70 | 68 | 65 | 66 | 69 | 69 | 72 | 72 | 69 | 69 | 70 |
| Portland, Maine | 69 | 68 | 67 | 64 | 67 | 69 | 71 | 73 | 73 | 71 | 71 | 69 |
| SOUTHEAST | | | | | | | | | | | | |
| Key West, Fla. | 80 | 76 | 73 | 71 | 72 | 74 | 72 | 72 | 75 | 76 | 76 | 78 |
| Miami, Fla. | 74 | 72 | 70 | 69 | 72 | 74 | 73 | 73 | 76 | 75 | 71 | 74 |
| Tampa, Fla. | 74 | 72 | 70 | 66 | 67 | 72 | 74 | 75 | 75 | 73 | 71 | 74 |
| NORTH CENTRAL | | | | | | | | | | | | |
| Bismarck, N.Dak. | 76 | 75 | 69 | 59 | 57 | 63 | 57 | 71 | 61 | 64 | 72 | 76 |
| Denver, Colo. | 53 | 52 | 50 | 49 | 49 | 44 | 46 | 46 | 45 | 47 | 49 | 53 |
| Kansas City, Mo. | 71 | 68 | 63 | 61 | 62 | 62 | 59 | 60 | 61 | 61 | 64 | 70 |
| Omaha, Nebr. | 71 | 71 | 63 | 59 | 60 | 62 | 59 | 62 | 63 | 62 | 67 | 73 |
| WEST | | | | | | | | | | | | |
| Pocatello, Idaho | 75 | 71 | 61 | 56 | 41 | 40 | 35 | 36 | 40 | 53 | 64 | 74 |
| Seattle, Wash. | 82 | 78 | 74 | 63 | 68 | 67 | 66 | 69 | 69 | 80 | 83 | 82 |
| Red Bluff, Calif. | 73 | 67 | 61 | 55 | 47 | 38 | 32 | 34 | 40 | 47 | 61 | 73 |

*Mean Monthly Relative Humidity.* Humidity in the United States varies from month to month with the changing seasons. Table 8 presents data from several localities to illustrate the monthly changes.

The foregoing data were taken from the *Climatic Summaries of the United States* for the year 1930.

*Precipitable Moisture.* A factor of considerable importance to hydrologists and meteorologists is the amount of precipitable moisture

in the atmosphere from the earth's surface to the upper limit of water vapor. The amount of moisture is measured in terms of depth of water in the same units as rainfall. A large quantity of precipitable moisture would indicate the presence of moist and probably warm air, and the possibility of heavy rainfall would be present.

Attempts have been made to measure the total moisture present by means of what is termed the turbidity of the atmosphere. Changes occur in the atmospheric turbidity because of variations in moisture, dust, or smoke. Since moisture in the air is the principal variable factor causing turbidity, which is a measurable element, it can be used to measure the total water content of the atmosphere on the assumption that causes of turbidity other than water are negligible. Values for quantities of total atmospheric water obtained by turbidity measurements were published at one time in the *Monthly Weather Review.*

Other methods, more reliable, but more cumbersome and slower, have been devised for computing the atmospheric moisture from the observed pressure and specific humidity of aerological soundings. Solat (170) presented a method in which the computation was made in accordance with the integral equation

$$M_w = \frac{1}{g}\int_{P_h}^{P_o} q\,dp$$

where $M_w$ = total mass of water vapor
$g$ = the gravity constant
$dp$ = increment of pressure

$P_o$ and $P_h$ are the atmospheric pressures at the surface and upper limit of moisture, respectively, and $q$ is specific humidity.

For the purpose of this sort of computation the upper limit of the moisture may be taken as 5 kilometers. The amount of vapor above that height is small, so that the amount below that elevation may be taken as practically the total precipitable water without appreciable error.

*Distribution of Atmospheric Moisture.* Moisture is distributed over all the earth in variable quantities throughout the lower portion of the atmosphere below an elevation of approximately 10 kilometers. The quantity present at any one point depends upon thermal conditions, geographical location, and altitude of the point. It is found in the air over the poles and over the hottest tropical deserts. It is greatest over the main sources of supply, the tropical oceans, where according to Landsberg (111) it may constitute as much as 4 per cent of the

total volume of air at the surface. The percentage decreases over the continental land areas, although as Landsberg points out, much more vapor would be found over large tropical deserts such as the Sahara than over the polar regions.

*Visible Atmospheric Moisture.* The visible form of atmospheric moisture which results from condensation is designated "fog" if it lies on the earth's surface and "cloud" if floating above. Fog is classified as to origin. If fog is formed when the ground is cooled by radiation it is called radiation fog. If formed when moist air passes over a cool surface it is called advection fog.

Clouds on the other hand have not been classified by origin since distinction on that basis is difficult to make. They have, however, been classified as to form, activity, and position.

*Classification of Cloud Forms.* Clouds are now classified in accordance with the International System which has been adopted by international agreement. This system contains ten principal classes placed in four groups according to the usual elevation of occurrence. These groups and classes are as follows:

| | |
|---|---|
| High level | Cirrus |
| | Cirrocumulus |
| | Cirrostratus |
| Medium level | Altocumulus |
| | Altostratus |
| Low level | Stratocumulus |
| | Stratus |
| | Nimbostratus |
| Low level with high vertical development | Cumulus |
| | Cumulonimbus |

There is no precise definition of the type forms, but on the contrary, each may blend gradually into other types. The cumulus form is marked by towering heights of dense, isolated, or disconnected cloud with a horizontal base. The stratus forms are continuous layers of cloud at a constant elevation. The cirrus forms are light, feathery, clouds at high elevation. Two of the above forms are rain-bearing, namely, the nimbostratus and the cumulonimbus.

*The Hydrologic Cycle.* The process through which a drop of water would pass from the ocean to the land, its precipitation and return to the ocean is termed the hydrologic cycle. Four separate primary steps or phases may be readily distinguished, namely, evaporation, distribu-

tion, precipitation, and runoff. However, not all water completes this cycle as is seen in the study of the various steps, and occasionally some of the steps are omitted entirely.

*Evaporation in the Hydrologic Cycle.* Evaporation is the designation of the process by which a liquid becomes a vapor or gas. The evaporation of water occurs from land and ocean surfaces, but it is evaporation from the latter that constitutes the first phase of the hydrologic cycle because the ocean is the greatest source of moisture for precipitation. Holzman (86) insists that evaporation from ocean surfaces supplies virtually all the precipitable water for the United States. However, since evaporation from ocean surfaces is a remote factor in human economy, it has not received the interest of hydrologists that evaporation from land areas has.

*Precipitation.* Precipitation is deposition of atmospheric moisture and is perhaps the most important phase of the hydrologic cycle if one phase may be considered more important when all are so interrelated and interdependent. In so far as moisture is concerned, it is the first phase to be readily and directly perceived. For this reason it has received much study.

The first step in precipitation is condensation, the incipient stages of which have been discussed above as fog and clouds.

*Runoff in the Hydrologic Cycle.* Runoff is the final phase of the hydrologic cycle and is the returning of the surplus of precipitation to the ocean. The precipitated water is disposed of by three means: evaporation, absorption by the soil or infiltration into the ground, and flow overland into channels of streams whence it returns to the sea. That which seeps into the ground is called underground water, a portion of which later percolates into the streams to join the surface flow. Both overland and underground waters are termed runoff.

*Incompleteness of the Hydrologic Cycle.* It is probable that only a relatively small proportion of the total number of drops evaporated from the ocean make the complete cycle on schedule. Much of the moisture is precipitated again into the ocean before it reaches land. Not all vapor that is carried overland is precipitated; some may be carried again out to sea. A portion of the precipitation is evaporated shortly after it has fallen and a large proportion is only temporarily retained in the soil to be evaporated directly or transpired through growing vegetation. Some of the runoff does not reach the ocean. As pointed out by Holzman (86), much of the water vapor from land evaporation is carried out to sea.

Nevertheless the concept of the hydrologic cycle constitutes a useful framework with which to correlate or unite the various processes

affecting the changes and distribution of water. For this reason and in spite of its deficiencies it is adhered to in the ensuing discussions.

*Forms of Precipitation.* Atmospheric moisture may be precipitated in two states, solid and liquid, the two outstanding forms being rain and snow.

*Rainfall.* Rainfall is the most extensive form of precipitation as it occurs in all parts of the earth except perhaps the polar areas and a few of the driest deserts. Rain is derived from atmospheric vapor which is cooled to dew point and condensed to fog or cloud, commonly falling as free drops of water. Other forms are: *glaze,* rain or fog which freezes in a thin coating of ice on cold surfaces; *mist* and *drizzle,* small droplets which may be deposited by striking a surface or may fall as light rain. The full course of events, however, from visible condensation to falling drops is not fully understood although it is usually conceded that some nuclei such as particles of dust are needed to initiate the formation of droplets too heavy to be sustained by the air. The important feature to the hydrologist is that the drops do form and fall. It is interesting to note in passing that investigators (69) have found that the distribution of these nuclei through a unit of space is in close agreement with what would be expected from Poisson's frequency function.

*Snowfall.* The second most important form of precipitation is the solid form of snow. In this case condensation occurs at temperatures below freezing and the moisture is condensed into flat, hexagonal crystals, except "graupel" which is a form of snow in soft pellets. Since snow is formed at temperatures below freezing, it occurs only in the cooler latitudes and higher altitudes. Since it is solid and may lie on the ground for considerable periods of time, it produces many phenomena of interest to the hydrologist and engineer not presented by rainfall.

*Sleet and Hail.* Sleet and hail are two solid forms of precipitation. Sleet is formed of small solid droplets of ice or frozen rain, having been solidified on its descent through a stratum of cold air. Hail is also ice but is formed irregularly and in appreciably larger particles which appear oftentimes to be made of concentric layers of clear ice and soft white snow-like material; it occurs only in thunderstorms. Sleet and hail usually furnish only small proportions of the total precipitation.

*Dew.* Dew is a form of precipitation in which the moisture is brought in vapor form to the object on which it is deposited. The object must be cooled to dew point, usually by radiation on clear cool nights. Dew is commonly dissipated by evaporation in the morning so that it forms a negligible form of precipitation. However, Landsberg (111) states that dew forms an appreciable portion of the precipitation in certain dry regions such as Palestine.

*Frost and Rime.* Frost and rime are depositions of water vapor on cold surfaces. Frost is formed in the same manner as dew except that the surface must be cooled to a temperature below freezing. Rime is formed by deposition by the wind of fog particles or vapor on cold objects. Neither frost nor rime constitute a measurable portion of the precipitation.

*Measurement of Precipitation.* The measurement of precipitation constitutes an important function of both meteorologists and hydrologists. Much time and study has been given to the observation of precipitation and to the instruments and to their location and capacity for obtaining accurate data. The hydrologist should be familiar with these instruments and their functions in order to appraise properly the observed data.

The data to be obtained consist of measured quantities of rain for given periods and the determination of rates of precipitation. Quantities and rates are closely interrelated, since the total quantity is simply a summation of the rates for unit increments of time, that is,

$$P_T = \int_o^T t\frac{dp}{dt}$$

where $p$ and $t$ are unit increments of precipitation and time respectively, and $P$ the total precipitation for time $T$. Since rate involves time, measurement of the rate requires instruments that measure time as well as quantity.

*Standard 8-inch Gage.* The standard gage used for measurement of precipitation is the open cylindrical receptacle having, in the United States, an inside diameter of 8 inches and a catchment area of approximately 50.4 square inches. The principal parts are the receiver; the measuring tube with an inside diameter of 2.53 inches and length of 20 inches; the overflow attachment which is a container 8 inches in diameter and which incloses the measuring tube; and the measuring stick which is 24 inches long, with a scale graduated to 0.1 inch. The precipitation is caught in the receiver from which it passes through a funnel-shaped bottom into the measuring tube. Measurement of the depth of precipitation collected in the tube is made by inserting the measuring stick to the bottom of the tube. The diameter of the measuring tube is fixed so that, taking into account the cross section of the measuring stick, the depth of water in the tube is exaggerated 10 times to facilitate reading of the measurement, which is taken to the nearest 0.1 inch on the rule, being equivalent to 0.01 inch of rain. The measuring tube can, therefore, catch only 2.0 inches of precipitation without

overflowing into the overflow container. All overflow is carefully measured in the tube.

The type of gage measures only the total precipitation between readings which normally are made once a day. Occasionally an observer may make some additional observations during a storm, but they are seldom sufficient to give an adequate concept of the rate of precipitation. For the latter purpose a recording gage is required.

*The Tipping-Bucket Gage.* There are several types of recording gages in general use of which the tipping-bucket gage is perhaps the most used. This gage has five principal parts as follows:

1. Tripod support
2. Collector and case
3. Tipping bucket and frame
4. Measuring tube
5. Clock mechanism

The tipping bucket is connected through an electric circuit with the quantity recording mechanism and the clock provides means for recording time.

Rain water, being caught in the collector, passes through the funnel-shaped bottom and falls into the tipping bucket. The bucket is divided symmetrically into two compartments, each of which has a capacity equivalent to 0.01 inch of rain entering the area of the 12-inch circular opening of the collector. When full it tips and drops the charge into the measuring tube and at the same time raises the opposite half of the bucket into position to catch the water.

This tipping of each bucketfull of water causes a discrepancy by not recording the full quantity of the catch of precipitation as measured from the measuring tube. For rain of moderate intensity, the discrepancy can be corrected without appreciable error by distributing the difference proportionately to the total catch. However, it is not satisfactory for measuring the high rates in subtropical or tropical regions.

*The Fergusson Weighing Gage.* The Fergusson gage was designed to obtain a continuous record, not only of the rain but also of forms of precipitation such as snow, hail, and sleet, which cannot be obtained by the tipping-bucket gage. Precipitation is caught in a collector and passed into a copper vessel called the receiver, which rests on a scale platform. The weighing is done by a spring scale, and the movement of the scale or receiver platform is transmitted through a series of levers to a recording mechanism. The time and movements of the scale are recorded on a chart mounted on a clock-operated cylinder. This gage can operate on a daily, semi-weekly, or weekly basis.

*The Marvin Float Rain Gage.* The Marvin rain gage is designed to measure rain only, as it cannot operate in freezing weather. The precipitation is caught in an 8-inch collector and passes through a pipe into a receiver which is of one-half the sectional area of the collector. The variation in the water surface is transmitted by a float and chain to the recording mechanism, which is actuated by a clockwork. This gage is designed to operate on a weekly basis.

*Totalizers.* For measurement of precipitation in places accessible only with great difficulty, gages known as totalizers have been devised to hold the catch for periods as long as a year or more. This gage is essentially an 8-inch receiver mounted on a receptacle of sufficient capacity to contain the expected catch for the given time, such as a month, six months, or a year. In usual practise these gages are read only at the intervals of time for which they are designed and therefore they cannot be utilized to obtain data of rates of precipitation or of precipitation for short intervals of time.

*Errors in Observation of Precipitation.* As in other types of observations, those made of precipitation are subject to errors of various kinds and from various sources. There are first the accidental errors in measurement of the catch. These errors, however, are small when compared with the daily variations of precipitation and are compensating so that they may usually be neglected. A more serious type of error is that due to faulty observation, a type that is sometimes designated as blunders. Faulty observation may involve errors of appreciable size and in storms of relatively large intensity may vitiate important conclusions.

*Constant Errors.* Constant errors are more serious than those discussed above because they are accumulative. They arise from a variety of sources, such as inaccurate measuring devices, improper exposure, or other continuing causes. The failure of the tipping bucket to operate fast enough to catch heavy rainfall is a good example of constant error; fortunately in this instance it can be compensated to a reasonable degree of accuracy.

*Errors of Improper Exposure.* Proper exposure and location of the rain gage with respect to adjacent objects are essential for good records of precipitation. If improperly exposed the catch may be augmented by water blown off adjacent trees, shrubbery, or nearby buildings, or by splashing from surfaces too close to the receiver. On the other hand, overhanging trees or sheltering walls or buildings may prevent the appropriate quantity of rain from entering the receiver. The gage should be located away from such objects so that the distance at least equals the height of the object.

*Evaporation of the Catch.* When the catch is left in the gage for a day or more it is subject to loss from evaporation. This loss is not usually accounted for in gages read daily, although there is a possibility of an appreciable loss of catch from a light rain when the gage is allowed to stand through a hot day. In cool weather or for heavy rains the loss would not be great but it is accumulative. In the use of totalizers some means, such as a cover of oil, are taken to prevent or materially reduce evaporation.

*Effect of Wind.* Wind is the great destroyer of accuracy of precipitation data and hence a location of a gage subject to high winds should be

TABLE 9. DEFICIT IN CATCH OF RAINFALL DUE TO WIND

| WIND VELOCITY | | DEFICIT OF TRUE CATCH |
|---|---|---|
| *Meters per Second* | *Miles per Hour* | *Per Cent* |
| 0 | 0 | 0 |
| 2 | 4.47 | 4 |
| 4 | 8.94 | 10 |
| 6 | 13.41 | 19 |
| 8 | 17.88 | 29 |
| 10 | 22.4 | 40 |
| 12 | 26.8 | 51 |
| 14 | 31.3 | 62 |
| 16 | 35.8 | 71 |

avoided. Even if the gage is properly exposed with respect to immediate surroundings, the wind interrupts a proper catch by blowing upward over the edge and thereby carrying a portion over and beyond the receiver. The stronger the wind the greater is the loss and reduction of the recorded precipitation. In the case of strong winds an appreciable portion of the precipitation may be lost.

Many investigators have reported on the difference of catch between a gage placed in an exposed position above ground and one located in pits or otherwise protected from wind.

Dr. H. Koschmieder (110) reported some experiments that he made in comparing the results between a gage set upon a post in the usual mode of exposure and a gage set in a pit. The top of the pit gage was put level with adjacent land surface and the portion of the pit surrounding the gage was covered with an iron grating flush with the top of the gage to prevent objectionable aerial turbulence around it. The results of the experiments indicated clearly that there was an appreciable deficit in the catch of the upright gage and that the deficit varied with the velocity of the wind. The deficits in catch for various velocities of wind are given in Table 9.

Riesbol (157) stated that the standard type of gage used by

Hydrologic Division, Soil Conservation Service, which was a non-recording standard type of the Weather Bureau, consistently caught less precipitation by storms and by months than did comparably exposed shielded gages. The standard gage frequently caught less by more than 10 per cent and in light showers the catch was less by a considerably higher percentage.

The need for proper exposure and the effect of wind turbulence was thoroughly studied by Dr. C. F. Brooks (24). He stated that experiments made in this country and Europe indicated that the unshielded 8-inch gage even when only moderately exposed to the wind caught about 5 per cent less than the true precipitation.

TABLE 10. RECEPTACLES OF RAINFALL, STORM OF MAY 1935

| RECEPTACLE<br>*Kind* | RAINFALL CAUGHT<br>*Inches* |
|---|---|
| Water tank 8 by 2 feet | 18 (estimated) |
| Water tank 8 feet diameter | 24 |
| Standard rain gage and bucket 12 inches deep | 24 |
| 3-gallon can in yard | 6 or 8 |
| Barrel in yard | 4 |
| Half-bushel measure, 12 inches diameter by 14 inches deep | 12 |
| Bucket, 10 inches diameter by 12 inches deep | 9 |
| Coffee can in yard | 4 |
| Oil can in yard | 8 |
| Pail | 4 |
| Can 8½ inches deep | 8½ (overflowed) |
| Stock tank | 15 |
| Can in yard | 8⅓ |
| Can beside road | 7 |

*Miscellaneous Measurements of Precipitation.* Because of the limited coverage of available rain-gage stations, not all precipitation data can be obtained from properly constructed and exposed gages. Not infrequently the most intense portion of a storm centers over an area where there are no gages and the greatest rainfall is not measured, or at best is caught in some vessel that may act by chance as a receiver. The value of each such measurement must be judged individually by the investigator. In some such cases he can obtain adequate and satisfactory data whereas in others he may obtain only good guesses. The probable error of such data, of course, will be appreciably larger than that of data of standard gages, but this reduced accuracy merely diminishes without destroying the usefulness of the results.

To illustrate the variety of receptacles which may function as emergency rain gages, there is given in Table 10 a list of those found by the hydrographers of the State Engineer's Office, Denver, Colo., after the memorable storm of May 30, 1935.

In contrast with the above amounts of rainfall, the U. S. Weather Bureau reported for May 30, 1935, a maximum one-day rainfall of 3.00 inches which was caught at Rush, Colo., on the southern edge of the above storm. Three inches of precipitation in 24 hours is not sufficient to cause flood, yet the largest inundation of record was rolling down the tributaries of the South Platte and Republican Rivers on the following day. It is evident that without those chance-placed receptacles that functioned as gages, no adequate information of precipitation could have been obtained.

In another storm, May 10–11, 1942, covering a much smaller area near Lincoln, Nebr., similar data were gathered to show conclusively that heavy precipitation had fallen although records from official stations indicated rainfall of only moderate intensity. The maximum record of 14 inches was estimated from rainfall in four separate stock-watering tanks, each giving that amount; two pails yielded 12 and 11.5 inches each; another stock tank, two cans, and a pail each showed a measurement of 10 inches; other miscellaneous measurements ranged down to 1.0 inch. The maximum record obtained by a Weather Bureau station was 3.15 inches at Crete, Nebr.

These are not isolated instances. Discussing the flood of June and July 1932, in Texas, Dalrymple (45) and his colleagues found "much reliable information that the maximum precipitation was about 35 inches instead of 20.3 inches, the maximum measured by an official gage." White (191) found similar results for the New England storm of September 1932; however, many of the data for this storm came from rainfall stations of the Massachusetts Dept. of Public Health and are equal in accuracy to data of the Weather Bureau. Examples could probably be repeated indefinitely but enough have been given to show that valuable data of precipitation can and often must be obtained from other sources than established rainfall stations.

*Measurement of Snowfall.* The measurement of snowfall is attended with considerably more difficulty than that of rainfall. Although snowfall may be measured by the regular 8-inch non-recording gage, the turbulence of the wind affects the catch so adversely that the records are not reliable. The relative permanency of snowfall as compared to rainfall on the ground allows other means of measurement. Where runoff is used intensively enough to justify the cost, the temporary storage of the winter precipitation permits its measurement by the making of regular snow surveys. The different methods of snow measurement appear to justify full treatment and are consequently considered further in Chapter 7.

*Relation Between Precipitation and Atmospheric Temperature.* It is a common experience that a rain storm causes a reduction in atmospheric temperature. It is particularly noticeable after a heavy shower or thunderstorm on a hot summer day. The effect of clouds in reducing temperature is widespread and is a material factor in reducing the temperatures of the moist portions of the tropics. Messrs. Finch and Trewartha (57) point out that the maximum mean monthly temperatures of tropical rain forests range close to 80 F, and that this relatively low average temperature as compared with the dry tropics must be ascribed to the prevailing mantle of rain-bearing clouds.

It is of interest to note what correlation may exist between temperatures and rainfall. In order to gain some conception of the relationship between precipitation and temperature in the middle latitudes, three studies were made of annual averages for as many localities in the central portion of the United States.

The data for the first study were obtained by the Weather Bureau at Bismarck, N. Dak. The correlation was computed by the equation in the form

$$r = \frac{\sum XY}{\sqrt{\sum (X)^2 \sum (Y)^2}}.$$

Table 11 gives all the data and the computations.

$$r = \frac{-228.734}{\sqrt{(32.95)(16.79)}} = -0.41.$$

The probable error,

$$e_r = .6745\, \frac{(1 - (-.41)^2)}{\sqrt{66}} = \pm .069.$$

Therefore,

$$r = -0.41 \pm .069.$$

A second study was made in the same manner for Omaha, Nebr. The correlation between annual precipitation and temperature was found to be $-0.33 \pm 0.075$.

The third study dealt with data of mean annual temperature and precipitation of an area rather than a locality. The data for the basin of the Cedar River above Cedar Rapids, Iowa, were computed and furnished by Mr. L. C. Crawford, District Engineer, U. S. Geological Survey, Iowa City, Iowa. The computation for correlation was made

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TABLE 11. CORRELATION OF ANNUAL TEMPERATURE AND PRECIPITATION, BISMARCK, N. DAK

| YEAR | PRECIPITATION *Inches* | TEMPERATURE *Fahrenheit* | $X = P - M_p$ | $Y = T - M_t$ | $XY$ | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 1875 | 27.52 | 35.3 | 11.26 | −5.7 | −64.182 | 126.7876 | 32.49 |
| 76 | 30.92 | 37.5 | 14.66 | −3.5 | −51.310 | 214.9156 | 12.25 |
| 77 | 17.68 | 42.3 | 1.42 | 1.3 | 1.846 | 2.0164 | 1.69 |
| 78 | 20.23 | 45.1 | 3.97 | 4.1 | 16.277 | 15.7609 | 16.81 |
| 79 | 22.61 | 39.5 | 6.35 | −1.5 | −9.525 | 40.3225 | 2.25 |
| 1880 | 19.75 | 39.0 | 3.49 | −2.0 | −6.980 | 12.1801 | 4.00 |
| 81 | 15.76 | 40.4 | −.50 | −.6 | .300 | .2500 | .36 |
| 82 | 21.33 | 41.4 | 5.07 | .4 | 2.028 | 25.7049 | .16 |
| 83 | 15.66 | 38.3 | −.60 | −2.7 | 1.620 | .3600 | 7.29 |
| 84 | 23.36 | 37.9 | 7.10 | −3.1 | −22.010 | 50.4100 | 9.61 |
| 85 | 13.08 | 40.1 | −3.18 | −.9 | 2.862 | 10.1124 | .81 |
| 86 | 13.26 | 41.1 | −3.00 | .1 | −.300 | 9.0000 | .01 |
| 87 | 16.33 | 38.8 | .07 | −2.2 | −.154 | .0049 | 4.84 |
| 88 | 16.51 | 38.5 | .25 | −2.5 | −.625 | .0625 | 6.25 |
| 89 | 11.03 | 42.7 | −5.23 | 1.7 | −8.891 | 27.3529 | 2.89 |
| 1890 | 15.75 | 40.8 | −.51 | −.2 | .102 | .2601 | .04 |
| 91 | 20.50 | 40.7 | 4.24 | −.3 | −1.272 | 17.9776 | .09 |
| 92 | 18.17 | 39.3 | 1.91 | −1.7 | −3.247 | 3.6481 | 2.89 |
| 93 | 13.74 | 38.1 | −2.52 | −2.9 | 7.308 | 6.3504 | 8.41 |
| 94 | 14.32 | 42.2 | −1.94 | 1.2 | −2.328 | 3.7636 | 1.44 |
| 95 | 16.92 | 39.8 | .66 | −1.2 | −.792 | .4356 | 1.44 |
| 96 | 16.64 | 38.9 | −.38 | −2.1 | −.798 | .1444 | 4.41 |
| 97 | 14.33 | 39.5 | −1.93 | −1.5 | 2.895 | 3.7249 | 2.25 |
| 98 | 13.67 | 41.3 | −2.59 | .3 | −.777 | 6.7081 | .09 |
| 99 | 15.47 | 38.6 | −.79 | −2.4 | 1.896 | .6241 | 5.76 |
| 1900 | 17.88 | 42.9 | 1.62 | 1.9 | 3.078 | 2.6244 | 3.61 |
| 01 | 15.59 | 43.0 | −.67 | 2.0 | −1.340 | .4489 | 4.00 |
| 02 | 15.95 | 40.4 | −.31 | −.6 | .186 | .0961 | .36 |
| 03 | 17.96 | 39.9 | 1.70 | −1.1 | −1.870 | 2.8900 | 1.21 |
| 04 | 14.17 | 39.1 | −2.09 | −1.9 | 3.971 | 4.3681 | 3.61 |
| 05 | 17.19 | 40.2 | .93 | −.8 | −.744 | .8649 | .64 |
| 06 | 18.22 | 40.9 | 1.96 | −.1 | −.196 | 3.8416 | .01 |
| 07 | 16.55 | 39.0 | .29 | −2.0 | −.058 | .0841 | 4.00 |
| 08 | 16.91 | 42.7 | .65 | 1.7 | 1.105 | .4225 | 2.89 |
| 09 | 18.55 | 40.1 | 2.29 | −.9 | −2.061 | 5.2441 | .81 |

exactly as above; the correlation coefficient was found to be 0.069 ± 0.112.

The correlations in none of these three cases is particularly high arithmetically even when the negative signs are disregarded. The significant feature is the negative sign of the first two cases: although neither are high, they indicate that with higher precipitation the temperature is likely to be lower. The correlation for large areas on the other hand is practically zero, and indicates that there is no correlation between annual precipitation and temperature over substantial

TABLE 11. CORRELATION OF ANNUAL TEMPERATURE AND PRECIPITATION, BISMARCK, N. DAK. — *Continued*

| YEAR | PRECIPITATION *Inches* | TEMPERATURE *Fahrenheit* | $X = P - M_p$ | $Y = T - M_t$ | $XY$ | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 1910 | 11.98 | 42.8 | −4.28 | 1.8 | −7.704 | 18.3184 | 3.24 |
| 11 | 15.22 | 39.9 | −1.04 | −1.1 | 1.144 | 1.0816 | 1.21 |
| 12 | 19.11 | 40.2 | 2.85 | −.8 | −2.280 | 8.1225 | .64 |
| 13 | 12.84 | 42.2 | −3.42 | 1.2 | −4.104 | 11.6964 | 1.44 |
| 14 | 22.98 | 42.1 | 6.72 | 1.1 | 7.392 | 45.1584 | 1.21 |
| 15 | 23.25 | 41.6 | 6.99 | .6 | 4.194 | 48.8601 | .36 |
| 16 | 17.10 | 38.3 | .84 | −2.7 | −2.268 | .7056 | 7.29 |
| 17 | 11.43 | 38.6 | −4.83 | −2.4 | 11.592 | 23.3289 | 5.76 |
| 18 | 13.50 | 42.2 | −2.76 | 1.2 | −3.312 | 7.6176 | 1.44 |
| 19 | 12.98 | 40.9 | −3.28 | −.1 | .328 | 10.7584 | .01 |
| 1920 | 11.15 | 42.0 | −5.11 | 1.0 | −5.110 | 26.1121 | 1.00 |
| 21 | 14.05 | 44.4 | −2.21 | 3.4 | −7.514 | 4.8841 | 11.56 |
| 22 | 17.16 | 41.8 | .90 | .8 | .720 | .8100 | .64 |
| 23 | 15.81 | 42.9 | −.45 | 1.9 | −.855 | .2025 | 3.61 |
| 24 | 16.67 | 40.7 | .41 | −.3 | −.123 | .1681 | .09 |
| 25 | 13.64 | 43.0 | −2.62 | 2.0 | −5.240 | 6.8644 | 4.00 |
| 26 | 12.37 | 42.8 | −3.89 | 1.8 | −7.002 | 15.1321 | 3.24 |
| 27 | 20.84 | 40.0 | 4.58 | −1.0 | −4.580 | 20.9764 | 1.00 |
| 28 | 15.02 | 43.2 | −1.24 | 2.2 | −2.728 | 1.5376 | 4.84 |
| 29 | 14.33 | 39.9 | −1.93 | −1.1 | 2.123 | 3.7249 | 1.21 |
| 1930 | 16.76 | 43.0 | .50 | 2.0 | 1.000 | .2500 | 4.00 |
| 31 | 15.82 | 46.3 | −.44 | 5.3 | −2.332 | .1936 | 28.09 |
| 32 | 14.41 | 41.8 | −1.85 | .8 | −1.480 | 3.4225 | .64 |
| 33 | 10.86 | 43.3 | −5.40 | 2.3 | −12.420 | 29.1600 | 5.29 |
| 34 | 7.74 | 45.6 | −8.52 | 4.6 | −39.192 | 72.5904 | 21.16 |
| 35 | 17.93 | 41.8 | 1.67 | .8 | 1.336 | 2.7889 | .64 |
| 36 | 5.97 | 41.0 | −10.29 | 0 | 0 | 105.8841 | 0 |
| 37 | 16.60 | 41.1 | .34 | .1 | .034 | .1156 | .01 |
| 38 | 13.42 | 44.0 | −2.84 | 3.0 | −8.520 | 8.0656 | 9.00 |
| 39 | 14.49 | 43.8 | −1.77 | 2.8 | −4.956 | 3.1326 | 7.84 |
| 1940 | 14.19 | 42.3 | −2.07 | 1.3 | −2.691 | 4.2849 | 1.69 |
| Totals | 1073.13 | 2708.8 | | | −228.734 | 1085.7106 | 281.82 |
| Means | 16.26 | 41.0 | | | | | |
| Square root | | | | | | 32.95 | 16.79 |

areas that are subjected to intermittent precipitation such as the state of Iowa receives.

*Primary Elements of Climate.* Climate is composed of many elements, a knowledge of which is very desirable as a background for interpreting and interpolating hydrological data. The two most important elements are temperature and moisture. It is primarily on these that climate has been analyzed and classified and the same basis may conveniently be used by the hydrologist or engineer when he is called upon to compare climates or use data from another region.

The use of data from a distant region raises the question of homogeneity of climate. Homogeneous climates may be defined (65) as those having similar types, values, and like variations in all important climatic elements. Temperatures and precipitation should be similar in value or amount, annual range, distribution, and in variation. That is, any extensive variations should be contemporaneous over the areas under comparison in order that the climates can be considered homogeneous. The homogeneity of a climate may be determined by methods of correlation or indicated by some classification of climate.

*The Koeppen Classification of Climate.* Koeppen's scheme of classifying climate provides five general classes dependent upon moisture and temperature, a set of appropriate seasonal classes pertinent to precipitation, and a third set of designations for specified limits of temperature (57, 111). These classes are identified by letters, and the climatic formula is made of three or four letters which describe the climate in accordance with the letter definitions. The following letter symbols are used to designate the main classes:

A: Tropical rainy climate, with the temperature of the coolest month above 64.4 F
B: Dry climate, with two subclasses

  S, steppe or subhumid
  W, desert

C: Humid mesothermal (middle temperature) climate
D: Humid microthermal (low temperature) climate
E: Polar climates, with two subclasses

  F, eternal frost, that is, always below freezing
  T, tundra climate

The seasonal and limiting temperatures and symbols are as follows:

a: Average temperature of the warmest month above 22 C (71.6 F)
b: Average temperature of the warmest month below 22 C (71.6 F)
c: Fewer than four months with temperature above 10 C (50 F)
f: No real dry season, moist or wet entire year
s: Summer dry
w: Winter dry

Examples of the climatic formulas are given on Figure 5.

*Thornthwaite's Classification of Climate.* Thornthwaite (182) devised a scheme to evaluate climate numerically on the basis of temperature, precipitation, and evaporation, with precipitation appearing to play

the dominant role. Thornthwaite based his classification upon a study of some 1100 stations in the United States, and established an empirical relationship called "precipitation-evaporation ratio." This ratio, which was based on the mean monthly values, was expressed by the equation

$$\frac{P}{E} = 11.5\left(\frac{P}{T - 10}\right)^{10/9}$$

in which $P$ is the precipitation; $E$, evaporation; and $T$, the temperature. The sum of the $P/E$ ratio for 12 months, multiplied by 10 to eliminate decimals, is called the $P/E$ index, $I$, and is expressed thus:

$$I = \sum_{n=0}^{n=12} 115\left(\frac{P}{T - 10}\right)^{10/9}.$$

The values $I$, of the $P/E$ index, range from zero upward and are divided into five humidity provinces as follows:

| SYMBOL | DESIGNATION *Name* | TYPE OF VEGETATION | VALUE |
|---|---|---|---|
| A | Wet | Rain forest | 128 and upward |
| B | Humid | Forest | 64–127 |
| C | Subhumid | Grassland | 32–63 |
| D | Semiarid | Steppe | 16–31 |
| E | Arid | Desert | Less than 16 |

In a similar study made of temperature which was classified to conform to the grades of the precipitation evaporation index, Thornthwaite developed a "thermal efficiency factor" that was expressed by the formula

$$i = \frac{T - 32}{4}$$

in which $i$ is the thermal efficiency factor and $T$ the mean monthly temperature. Five temperature categories were fixed as follows:

| SYMBOL | NAME | VALUE |
|---|---|---|
| A′ | Tropical | 128 and up |
| B′ | Mesothermal | 64–127 |
| C′ | Microthermal | 32–63 |
| D′ | Taigo | 16–31 |
| E′ | Tundra | 1–15 |
| F′ | Frost | 0 |

*Descriptive Graph of Climate.* In order to provide a graphic comparison of climate, the author (66) devised a grid drawn on rectangular coordinates on which are plotted the mean monthly temperatures

against the precipitation. The grid automatically places each separate monthly point in its class of precipitation and temperature, which are plotted on the abscissas and ordinates of the graph.

As can be seen on Figure 5 there are five classes of temperature. These class limits were selected to coincide as closely as possible with points used by writers on the subject of climatology as significant points of temperature.

The class limits for the precipitation were determined from Thornthwaite's scheme of climate classification by substituting the various class values given above in the formula to obtain the corresponding values of temperature. The temperatures thus derived were in turn substituted in the formula for the $P/E$ index together with the class values of the latter, and the equation solved to obtain the corresponding precipitation.

The point values to fix the classification grid are shown in Table 12.

TABLE 12. CLASSIFICATION GRID POINTS OF CLIMATIC GRAPH

| CLASS | TEMPERATURE | | PRECIPITATION | |
|---|---|---|---|---|
| | *Fahrenheit* | *Centigrade* | *Inches* | *Millimeters* |
| Arid | 32.4 | 0.2 | 0.32 | 8.13 |
| | 83.2 | 28.4 | 1.03 | 26.16 |
| Semiarid | 32.4 | 0.2 | 0.59 | 14.99 |
| | 83.2 | 28.4 | 1.93 | 49.02 |
| Subhumid | 32.4 | 0.2 | 1.10 | 27.94 |
| | 83.2 | 28.4 | 3.60 | 91.44 |
| Humid | 32.4 | 0.2 | 2.05 | 52.07 |
| | 83.2 | 28.4 | 6.73 | 170.94 |

Certain features of the graphs are important indications of types and characteristics of climate, and since the graph is based on mean monthly values the indications are reasonably stable. Location on the graph shows the general type of climate, hot, cold, wet, or dry. Hot climates are located at the top, cold at the bottom, while wet climates are at the right and dry at the left. For example, a hot, dry desert climate would be located at the extreme upper left corner. Other types of climate are indicated by their location and also by the shape and slope of the loop made by the twelve monthly points for any given station. A narrow horizontal loop indicates little variation in temperature with a big range in mean monthly precipitation. A narrow loop with a vertical axis indicates uniform precipitation with much variation in temperature. A loop sloping upward to the left indicates a climate with dry summers and wet winters, while a slope upward to the right indicates a wet summer and a relatively dry winter. Examples of a few types of climate are shown on Figure 5.

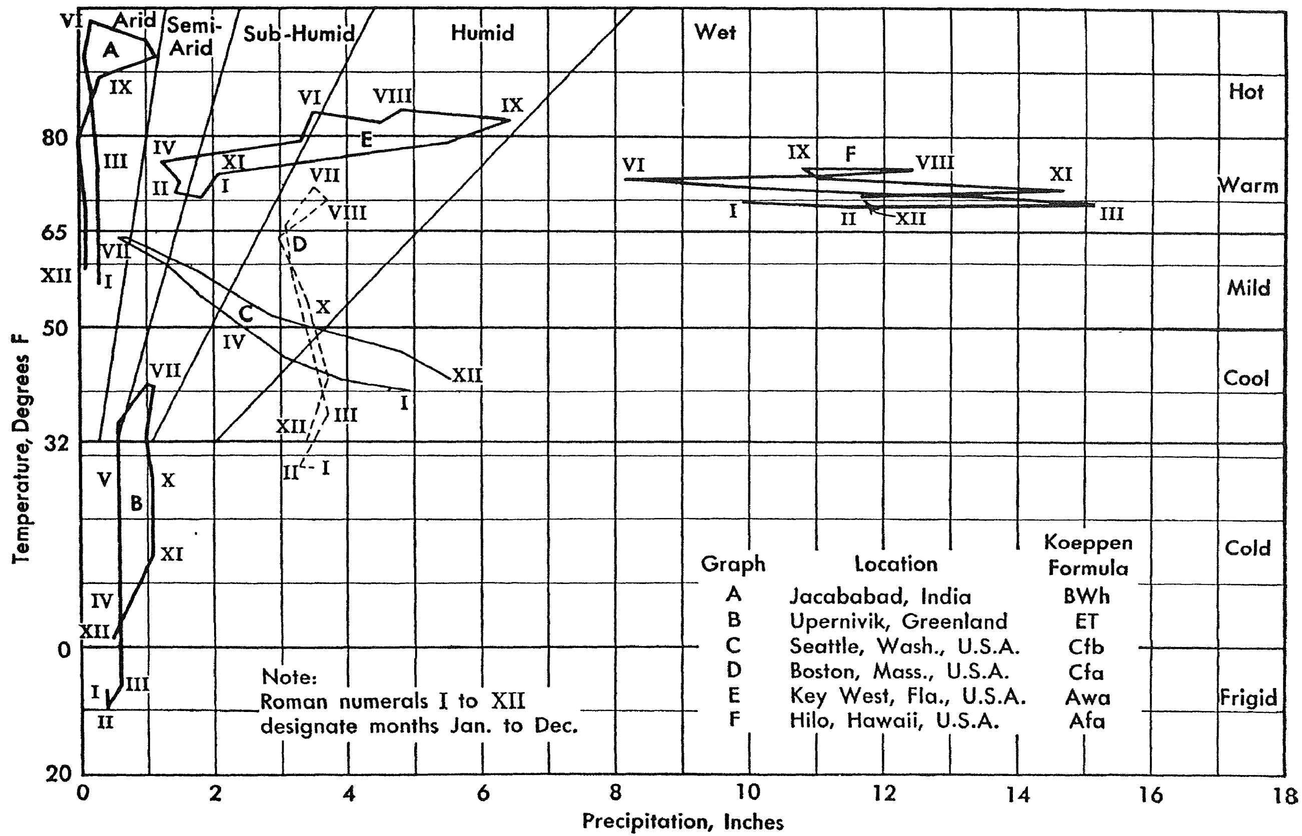



FIGURE 5. Climatic Graphs

# 3 AIR MASSES

*General Atmospheric Circulation.* In order to gain an adequate conception of air masses, it is first necessary to view the general circulation of the earth by considering it as a sphere that is heated in the equatorial latitudes and cooled in the polar regions. It then appears that the relatively thin envelope of gas on the earth's surface moves from the equator to the poles and upon the loss of its heat returns to the lower latitudes. The rotation of the earth and the disposition of the continents, together with other factors, affect this circulation so that it is far from being a simple convective action. Broadly speaking, it appears that great masses of the atmosphere in equatorial or subtropical regions become heated and for some reason not fully understood, move northward to merge with the general eastward drift in the middle latitudes and then enter the polar regions or return to the low latitudes. In a similar manner other masses of air become more or less stationary in the polar regions until some agency, probably like the warmer masses connected with the general planetary thermal convective circulation, compels it to move southward. The causes that initiate the movements are not entirely clear but they are not particularly important to the hydrologist. What is important to him is that the air masses acquire certain properties before moving that make their subsequent travels events of great moment.

*Terrestrial Winds.* Due to solar radiation and planetary movements of the earth, there is beneath all observed variations a consistent pattern of wind movements. These wind movements, which are known as "planetary" or "terrestrial" winds, are briefly described here to the extent necessary to appreciate their effect on air masses and precipitation. The fundamental factor promoting this general circulation is the unequal heating of the atmosphere by the sun, which is most effective over the tropical regions. The tropics and adjacent regions contain perhaps the best known planetary wind system, called the "trade

winds." In the northern hemisphere the trade winds lie between 3 and 35 degrees north latitude, approximately, and blow from the northeast; in the southern hemisphere they lie between 0 and 28 degrees south latitude and blow from the southeast. Between the two belts of trade winds there is a narrow band of latitude called the "doldrums," in which the winds are light and variable or the air is calm. In the latitudes both north and south of the trade winds is another belt of light variable winds, known as the "horse latitudes." Still farther northward and southward there is in the middle latitudes of each hemisphere a band in which the general winds are westerly and are referred to as the "prevailing westerlies." This wind system is not constant or steady in direction since these latitudes are also the region of secondary circulation. In addition to the trade winds and prevailing westerlies, there are in different parts of the world winds known as "monsoons" which are due to seasonal heating and cooling of the continents. These winds blow on shore in summer and off shore in winter. The monsoons of southern Asia are the best known and most effectively developed. There are also in many parts of the world other steady or characteristic winds of local importance, but which do not require discussion here.

*Secondary Circulation.* The secondary circulation consists of the winds of various types of storms. These winds are in general imposed upon the general wind circulation by causes less extensive than those producing the terrestrial winds. One portion of the secondary circulation is closely related to the movements of air masses. Air-mass movements are accompanied by considerable differences in barometric pressure; over one area the pressure is high and the winds, known as the anticyclonic circulation, are moving away from the center. On other areas, usually near the margins or points of convergence of the air masses, the barometric pressure is relatively low; these areas of low barometric pressure are known as "extratropical cyclones," and the winds moving inwardly around them as the "cyclonic circulation."

*Definition of Air Mass.* The term "air mass" is applied to the portion of the atmosphere that has remained stationary, or nearly so, over an extensive area until it has acquired an approximate horizontal homogeneity with definite properties. This definition is elaborated upon by Dr. H. C. Willett (193) thus: "The formation of an air mass in this sense takes place on the earth's surface wherever the atmosphere remains at rest over an extensive area of uniform surface properties for a sufficiently long time so that the properties of the atmosphere (vertical distribution of temperature and moisture) reach equilibrium with respect to the surface beneath. Such a region on the earth's surface is referred to as a source region of air masses."

*Origin of Air-Mass Analysis.* The original development of the theory of air masses is credited by Dr. Willett to the Norwegian meteorologist, T. Bergeron. Other Europeans, O. Moese and G. Schinze, have made notable contributions to the analysis and theory of air masses on that continent. To their names can be added those of Dr. H. C. Willett, C. G. Rossby, Dr. H. R. Byers, Jerome Namias, and others who have contributed to the study of air masses on this continent. The development of air-mass analysis has reached its present status within the past 20 years by utilizing the airplane and radio to obtain the necessary atmospheric data of the higher altitudes.

*Observational Data of Air Masses.* Observed data upon which analysis of air masses is made consist of temperature, relative humidity, and barometric pressure of the atmosphere taken at elevations up to several kilometers. These data are obtained principally by means of airplanes and sounding balloons which carry instruments to record or transmit the data sought to the ground observers by radio. The characteristic properties of air masses are derived from these observations. The elevation of the observations is obtained from the barometric pressure which is therefore of prime importance since the characteristic properties of the upper atmosphere with respect to altitude are indispensable for the identification and analysis of the air mass. Meteorological data used in air-mass analysis are expressed in the metric system of measurement, which is a custom that is followed in this chapter. See Figure 6.

*Characteristic Properties.* The principal characteristic properties are derived from observations of temperature and relative humidity. These properties and other supplementary features are given by Willett (193), Namias (143), and Byers (27), in the *Bulletin of the American Meteorological Society.* They are briefly described in the following paragraphs.

*Temperature.* The temperature of a particle of air at a given elevation is obtained by direct observation. The most conservative thermal property, that is, the one least subject to change, is a function of the observed temperature and is designated "equivalent-potential temperature." The equivalent-potential temperature of a particle of air is defined as the temperature it would have if all its moisture were condensed and removed, and the latent heat of condensation were added to the dry air and then it be brought adiabatically to a pressure of 1000 millibars. (A pressure of 1000 millibars is 760 millimeters of mercury; one millibar is slightly less than one one-thousand of the atmospheric pressure.) Equivalent-potential temperature is expressed in degrees centigrade above absolute zero.

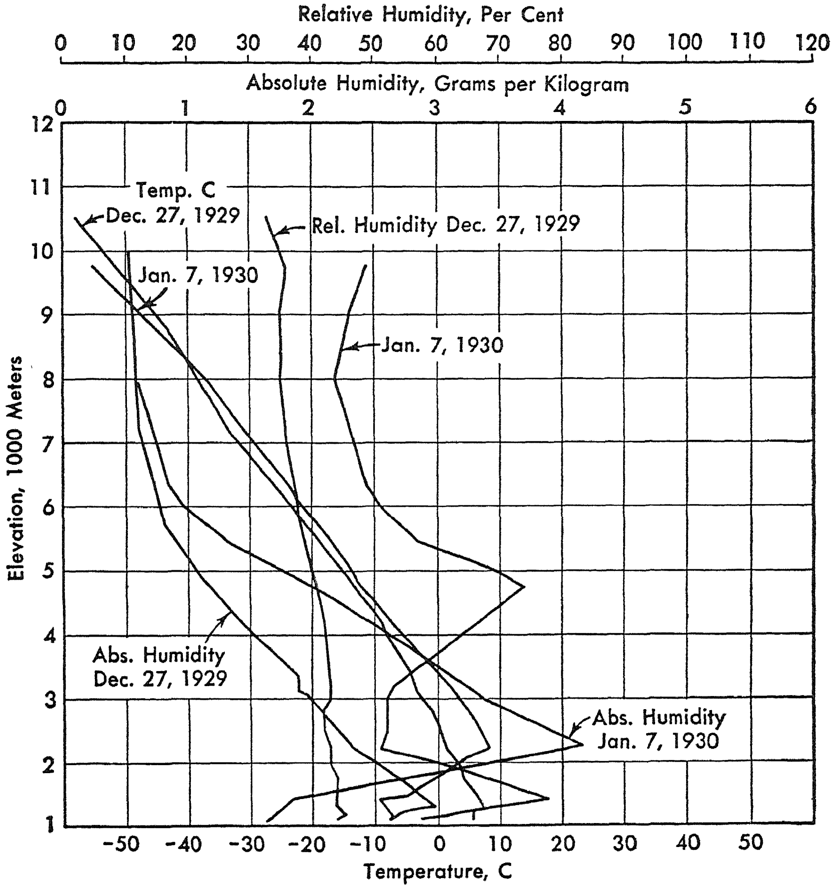



FIGURE 6. Sounding Observations, Amarillo, Texas

*Specific Humidity.* The atmospheric moisture is observed as relative humidity but the conservative characteristic property of air masses is the specific humidity. In practice it is taken as

$$q = 622e/p$$

where $e$ is the vapor pressure present and is obtained from psychrometric tables for the observed relative humidity; $p$ is the total barometric pressure; $q$ is the specific humidity, in grams per kilogram of air.

*Lapse Rate.* The term "lapse rate" is defined as the change of one property with respect to another. In meteorology the lapse rate is the change of temperature per unit increase in altitude. The units commonly used are degrees centigrade per 100 meters of altitude. There are two important lapse rates, one being that of dry air rising adiabatically and the other that of saturated air. The first rate is approximately

1 degree centigrade per 100 meters of altitude, while the second has no fixed rate since it is dependent upon the actual air temperature and amount of water vapor present.

*Condensation Forms.* The type of cloud formation depends largely upon the vertical distribution of temperature, that is, the lapse rate, and the specific humidity. It follows therefore that air masses with definite properties of temperature are likely to have accompanying them more or less definite forms of clouds.

*Visibility.* Good visibility is usually associated with cold air masses moving over a warm surface. In this case there is a steep lapse rate and rising atmosphere in which dust and smoke become diluted so that the air is clearer. In a stable, or small, lapse rate, dust and smoke tend to remain near the ground and thus tend to reduce visibility.

*Wind Direction and Velocity.* Wind direction and velocity are indications of the movements of the air mass but they are not constant features of a given type. For example, a polar air mass may have a wind with a southerly velocity component which would not assist in distinguishing it from, say, a Gulf air mass.

*Stability of the Atmosphere.* The term "stability" as employed in referring to movements of the atmosphere is used in the physical sense although the movements are caused primarily by differences or changes in heat. A particle of atmosphere is said to be stable if it resists displacement from its position or tends to return if moved to a different elevation or position. Stability or instability is a factor of importance in several respects. Instability of the atmosphere exists when the lapse rate is greater than the adiabatic rate for dry air; if the lapse rate is less than the rate for saturated air it is stable under all conditions; if the lapse rate is greater than the rate for saturated air but less than the adiabatic rate for dry air it is stable until the air becomes saturated. Stability or some aspect of it is a distinctive feature of certain air masses.

*Classification of Air Masses.* Air masses are classified according to their source area, that is, in accordance with the region in which they originated. This is the logical means of classification because the area of origin determines the properties of the air mass. The classification and nomenclature that was used in this country by the Weather Bureau until 1939 was that devised by Dr. H. C. Willett; in that year the Weather Bureau adopted the international system of Bergeron, but Willett's system is much used in this country, hence it will be used here. The following paragraphs are substantially as given by Willett (193).

The sources of air masses of North America fall into two general groups, namely, tropical or subtropical, and polar, or subpolar. The

reason for these two sources is the existence in the tropics and circumpolar regions of large areas of approximately uniform surface conditions and limited atmospheric activity. The interjacent region, that is, the middle latitudes, is the region of the prevailing westerly winds and the paths of the extratropical cyclones of the northern hemisphere; for that reason, the atmosphere in the middle region is not quiescent long enough to acquire characteristic properties of the surface there. However, air masses do move into the middle region from either side, and by so doing have their properties modified from the original. Hence there are really three general classes of air masses which are designated as follows:

| | |
|---|---|
| Polar | (P) |
| Tropical | (T) |
| Transitional | (N) |

The transitional air masses are those which originate in either tropical or polar regions but have been modified in their characteristic properties since leaving their source region. Both polar and tropical air masses are further classified in accordance with their source region which is indicated by a subscript. The complete classification with symbols is as follows:

1. Polar Continental, (Pc), from the northern continental area
2. Polar Pacific, (Pp), from the North Pacific
3. Polar Atlantic, (Pa), from the North Atlantic region
4. Tropical Gulf, (Tg), from the Gulf of Mexico and Caribbean Sea
5. Tropical Atlantic, (Ta), from the regions of the Sargasso Sea
6. Tropical Pacific, (Pa), from the middle Pacific Ocean
7. Tropical Continental, (Tc), from the southwestern continental area

Transitional types are classified only by their source region which is designated by a two-letter subscript. Then for each of the above types there is a transitional air mass which is designated thus: "Npc" for transitional polar continental; "Npp" for transitional polar Pacific. Usually transitional tropical air masses originating over water surfaces are not distinguished by source area because the characteristic properties of all tropical maritime air masses are very similar and their effect on land is virtually identical. They are the only transitional tropical air masses of any importance and are designated simply "Ntm," that is, transitional tropical maritime.*

* While the term "maritime" is commonly used to designate these air masses, the name "marine" would convey more accurately the idea of their origin over the ocean far from shore, but in order to abide by custom the term "maritime" is retained.

*Bergeron's Classification.* Another classification of air masses was given by Bergeron and it has been adopted by the U. S. Weather Bureau. The principal distinguishing feature of this classification is the inclusion in the symbol of each type of air mass a letter to designate whether the air mass is warm (w) or cold (k) with respect to the earth's surface. Thus a polar continental air mass in its source region would be designated "cP" without the letter to indicate the relative temperature of the air. By adding the notation just given the air mass would become "cPw" to indicate that the air was warmer than the earth in winter, and "cPk" in summer to show that it was colder. All three of these would be designated "Pc" in Willett's nomenclature until sufficiently modified to be labeled "Npc." The Bergeron nomenclature has the advantage of indicating the relative lapse rate which is a feature of importance in meteorological work but of less moment to the hydrologist. For the purposes of the latter a geographical classification is simpler and sufficient for all purposes, and is therefore utilized throughout this work, except that both systems are given in Table 13.

The various air masses are summarized with season of occurrence in Table 13.

TABLE 13. CLASSIFICATION OF AIR MASSES

| LATITUDE | NATURE | LOCAL SOURCE REGION | AIR MASS SYMBOL | | SEASON OF OCCURRENCE |
|---|---|---|---|---|---|
| | | | *Willett* | *Bergeron* | |
| Polar | Continental | Alaska, Canada and Arctic | Pc | cP<br>cPw<br>cPk | Entire Year<br>Winter<br>Summer |
| " | " | Modified in United States | Npc | cPw | Entire Year |
| " | Maritime | North Pacific Ocean | Pp | mP<br>mPk<br>mPw | Entire Year<br>Winter<br>Summer |
| " | " | Modified in United States | Npp | cPw<br>cPk | Entire Year |
| " | " | Colder Portion of North Atlantic Ocean | Pa | mPk | Entire Year |
| " | " | Modified Over Warmer Portion of Ocean | Npa | mPk | Spring and Summer |
| Tropical | Continental | Southwest United States and Northern Mexico | Tc<br>Ntc | cTk | Warm Half of Year<br>Negligible |

TABLE 13. CLASSIFICATION OF AIR MASSES—*Continued*

| LATITUDE | NATURE | LOCAL SOURCE REGION | AIR MASS SYMBOL | | SEASON OF OCCURRENCE |
|---|---|---|---|---|---|
| | | | *Willett* | *Bergeron* | |
| Tropical | Maritime | Gulf of Mexico and Caribbean | Tg | mTw mTk | Entire Year |
| " | " | Modified in United States and North Atlantic | Ntm(Ntg) | mTw | Entire Year |
| " | " | Sargasso Sea | Ta | mTw mTk | Entire Year |
| " | " | Modified in United States | Ntm(Nta) | mTw | Entire Year |
| " | " | Tropical Pacific | Tp | mTw | Winter |
| Upper Air | Subsidence | Upper Levels of Atmosphere | Ts or S | | Entire Year |

*Description of Polar Continental Air Masses.* Polar continental air masses are differentiated into two varieties, winter and summer, which have somewhat diverse properties. The primary source area includes the subpolar parts of the North American continent, namely, northern Canada, Alaska, and the adjacent arctic regions. This area is protected by the western mountain ranges from invasion of warm air from the Pacific Ocean. In winter practically the entire region is covered by snow and ice. Because there is little atmospheric activity in those regions, they being outside of the latitudes of planetary circulation, the air remains quiescent for several days at a time, and the bottom layers are cooled to low temperature by contact with the snow-covered surface. The middle elevations are farther away from the cold ground surface and have higher temperatures; then at still higher elevations its temperature again decreases. This condition, known as a temperature inversion, is shown by the following observations:

TABLE 14. OBSERVATIONS OF TEMPERATURE INVERSION

| ELEVATION *Km* | TEMPERATURE *C* |
|---|---|
| Surface | −26.1 |
| 1 | −25.3 |
| 2 | −20.1 |
| 3 | −21.5 |
| 4 | −25.4 |

Because of these low temperatures there is little evaporation of moisture from the surface. The western mountains cut off moisture from the Pacific, and the eastward movement of the atmosphere in the

middle latitudes eliminates the Atlantic Ocean as a source of moisture. Consequently, the moisture content of the polar continental air masses is very low in winter; the normal moisture is about 0.5 gram per kilogram.

In summer these air masses are somewhat different. During that season there is no snow on much of the area, so that the lower strata of the air are warmed on the bottom instead of cooled. Nevertheless, the temperatures at the middle elevations remain moderately low and the Pc air mass is characteristically cool as compared with masses of southern origin. Moisture is appreciably higher in summer and is usually 5 or 6 grams per kilogram but its relative humidity remains low. Due to this dryness Pc air masses are typically cloudless in summer as well as in winter.

*Movements of Polar Continental Air Masses.* Movements of air masses from their source region occur as a result of vaguely determinant causes. The first movement is southeastward or southward into the United States. It then spreads eastward to cover the eastern portion of the United States. Some movement occurs westward but movement in that direction is hindered by the mountain ranges and the westerly planetary winds.

*Modification of the Characteristic Properties.* Modifications of air masses take place as a result of movement from the source origin. However, during winter modification is not great while the movement continues over snow-covered ground. Since the snow cover extends well southward into the United States, the Pc air masses move in winter with little modification into the central portion of the country. After the spring melting of snow cover, or in summer, modification proceeds rapidly after the air masses leave the source region. The temperature is raised by contact with and radiation from the ground surface. The moisture content is augmented by evaporation from the same surface. A general subsidence of the air mass occurs which intensifies the temperature at low levels and effects a general warming of the upper strata. Turbulence of the atmosphere is caused as air masses pass over the rough land surface, and there is developed a thoroughly mixed lower stratum. Modification of air masses is likewise rapid over open water though probably affecting only the lowest stratum, but is noticeable only over large bodies of water. The effect of the Great Lakes is manifested by fogginess, clouds, and snow flurries, which disappear as the air mass passes eastward over the Appalachian Mountains.

*Polar Pacific Air Masses.* The principal source region of the polar Pacific air masses is the North Pacific Ocean. It is probable that the air

originates in the polar regions over Alaska and breaks out to the Pacific Ocean as the continental air over the land areas to the east and moves southward into the United States. These polar Pacific air masses have been traced by Byers (27) from the Asiatic shore to North America but they acquire their characteristic properties over the North Pacific Ocean. In this case the air masses acquire their characteristic properties not so much through remaining stationary over the area as by their long travel over a water surface. Air masses with these properties may properly be said to have their source region in the Pacific Ocean, and previous travel is of little importance in developing their characteristic values.

The cold, dry, and thermally stable polar continental air which comes to the ocean from Siberia, Western Alaska, or the frozen Bering Sea, is converted to polar Pacific air in its passage over the ocean by acquiring heat and moisture in the lower strata. When it arrives on the western coast of North America it is moderately cool and moist in the lower portions; the relative humidity is high. The vertical distribution of moisture at Seattle, Wash., based on twenty-six samples, is as follows:

At surface, 3 to 5 grams per kilogram
At 1 kilometer, 1 to 3 grams per kilogram
At 2 kilometers, 0 to 2 grams per kilogram

By traveling farther southward, say to San Diego, more moisture is acquired as indicated as follows:

At surface, 3 to 7 grams per kilogram
At 1 kilometer, 2 to 7 grams per kilogram
At 2 kilometers, 1 to 2 grams per kilogram

*Polar Atlantic Air Masses.* The source region of polar Atlantic air masses is the area of the North Atlantic Ocean off the Maine coast northeastward and north of the Gulf Stream. In this region the temperature is abnormally low throughout the year for ocean surface in comparable latitudes. These air masses are brought to New England and the Canadian Maritime Provinces by return winds from a stationary offshore anticyclone. The path of this air over the water surface is short, hence Pa air masses may be expected to be cold and dry as compared with other air masses from an ocean source region. The moisture content is small; therefore precipitation is small, consisting of light snow flurries or showers. The importance of these air masses lies in the sudden changes in weather that they cause. However, their influence does not extend west of the Appalachian Mountains. Polar

Atlantic air masses are not important in winter because the eastward movement of air is too strong to permit much westward travel of air masses.

In summer the source region of the polar Atlantic air masses becomes a source of cold air because the North Atlantic Ocean remains relatively cold throughout the year. It will be recalled that in summer the continental source region of Pc air is free of snow and is subject to considerable heating. For this reason Pc air moves onto the adjacent North Atlantic Ocean with temperature above that of the Pa source region. Because of its low temperature polar Atlantic air contains little moisture, which results in some cloud formation with small amounts of rain.

*Tropical Pacific Air Masses.* The source region of tropical Pacific air masses is the tropical portion of the Pacific Ocean. The properties with which these air masses reach the coast of North America have been changed by their travel. As they move eastward and northward from their source region they pass from a warm to a relatively colder surface thus becoming cooler and more stable in the lower strata. They are convectively unstable upon reaching the colder areas only above an altitude of one kilometer. Their moisture content shows a decrease as they move eastward and northward. At Pearl Harbor, Hawaiian Islands, it is as follows:

At the surface, 14 to 15 grams per kilogram
At 1 kilometer, 4 to 9 grams per kilogram
At 2 kilometers, 1½ to 3 grams per kilogram

Above San Diego, Calif., it is

At the surface, 9 to 12 grams per kilogram
At 1 kilometer, 6 to 8 grams per kilogram
At 2 kilometers, 3 to 6 grams per kilogram

Above Seattle, Wash., the general properties are similar to those at San Diego, Calif., except that it is cooler, but the difference in temperature is not great for the difference in latitude.

Tropical Pacific air has been observed occasionally at high altitudes as far inland as Ellendale in southeastern North Dakota, but it has not been observed at stations farther inland. The temperature of this air is lower at Ellendale than on the west coast. The water content is likewise lower at the lower levels but above two kilometers it is nearly the same as at Seattle, Washington, at corresponding elevations.

*Tropical Continental Air Masses.* The source region of tropical continental air masses is in the southwestern portion of the United

States and northern Mexico, an area that is relatively small when compared with the source region of other air masses.

The greater portion of the region is elevated so that its climate in summer is relatively cool; in winter it is quite cold. In winter this area is usually occupied by old Pc or Pp air which enters with low temperature. During the winter it receives neither heat nor moisture from the Tc source region, hence Tc air in winter is characteristically cool and dry.

In contrast to the cool winter conditions, the source region of Tc air in summer is strongly heated by the sun, which causes the distinctive feature of summer Tc air. The summer air of this region is characterized by high temperature and low specific and relative humidity. These characteristics are most extreme over the lower plains east of the Rocky Mountains. However, as the air from the plateau moves eastward, the dynamic heating in its descent brings it to the same degree of warmth and low humidity as the air on the low central plains, so that difference of temperature is not felt. The heated surfaces of the low plains cause marked instability, but convection seldom reaches condensation levels, hence condensation and precipitation rarely occur. The presence of Tc air is marked by dryness, clear sky, large diurnal range in temperature, high surface temperature, and convective instability. In these characteristics it is in contrast with high humidity and low diurnal range of temperature of the tropical Gulf air masses which are also found on the southern plains.

Tropical continental air is brought northeastward when low barometric pressure prevails over the northern plains. However, it is usually not found at the aerological stations at Broken Arrow in northeast Oklahoma, or Ellendale, North Dakota. It is seldom found at the ground at Groesbeck, Texas, but is found at elevations of one to two kilometers. Tc air is seldom found at the ground east of the Mississippi River, and does not get far from its source region before it is displaced by either Tg or Pc air. It is not, therefore, of great importance away from its source region and adjacent areas.

*Tropical Gulf and Tropical Atlantic Air Masses.* These two air masses are of prime importance in the eastern and central parts of the United States. Their source regions and consequently characteristic properties are so similar that they are not distinguishable in the interior of the United States. Their symbols, "Tg" and "Ta" respectively, are useful mainly to indicate the life history of the air masses as they approach the coast. The symbol "Tm" is used to designate a tropical maritime air mass from either source region.

Tropical Atlantic air masses advance into the United States only

with a general movement of air westward or northwestward from the South Atlantic region to the eastern coast. Such a movement is likely to occur when the western portion of the middle Atlantic high pressure area becomes particularly well developed so that anticyclonic circulation extends over the southeastern United States. This anticyclonic circulation occurs usually three or four days after a major outbreak of Polar air southward over the Atlantic Ocean. Consequently the properties of Ta air are probably in a condition of equilibrium with respect to the underlying ocean surface.

Tropical Gulf air masses have usually moved northwestward from the Caribbean Sea or West Indies before moving over the Gulf. These air masses, therefore, represent real equatorial air from the trade wind circulation, and in contrast with the short travel of Ta air masses, have probably had an equatorial sojourn of several weeks. Nevertheless, in spite of different life histories the two tropical air masses are so similar that they can scarcely be distinguished from each other. Both are characterized by much warmth and high specific humidity at the ground, and the same low specific and relative humidity at high levels. For this reason the two types are usually given a common designation, "Tropical Maritime" ("Tm").

The temperature of Tm air at its source is very close to that of the ocean surface, hence in winter it ranges usually from 20 C to 26 C (68 F to 79 F). The specific humidity of winter Tm air masses is high near the surface but has a marked decrease between the elevations of one and three kilometers. This condition is found in tropical air masses over the United States and it seems to be characteristic of all such air masses in all regions. Stability is great enough so that while the air mass is moving over the ocean, convection is noticeably absent. Even with insolational heating over land surface, a considerable ascent of air is needed at the coast line or an active cold front to produce convection. It is an exception rather than the rule in winter when the water is relatively warm that the low Gulf and South Atlantic coast line can effect enough vertical displacement of Tm air mass to initiate convective showers, though when sufficient lifting does occur showers are likely to be heavy. On the other hand, the active overrunning of a cold continental air mass by the Tm air sets off the potential thermal instability of the Tm air masses and causes intense rainfall.

Representative observations of Tg properties in winter are available at Groesbeck, Texas. At this point it has left its source region a comparatively short time so that its properties have changed but little, if any. Typical data for that station and Royal Center, in the northwest section of Indiana, are given in Table 15.

TABLE 15. PROPERTIES OF TG AIR IN WINTER

| | GROESBECK, TEXAS | | | ROYAL CENTER, IND. | | |
|---|---|---|---|---|---|---|
| *Elevation Km* | *Temp C* | *Sp Hum G per Kg* | *R.H. Per Cent* | *Temp C* | *Sp Hum G per Kg* | *R.H. Per Cent* |
| Surface | 18.8 | 12.6 | 90 | 17.5 | 11.3 | 90 |
| 1 | 14.0 | 10.4 | 95 | 12.5 | 9.6 | |
| 2 | 13.0 | 4.1 | 40 | 8.0 | 4.5 | |
| 3 | 7.5 | 1.2 | 15 | ... | ... | |

Only two observations are included in Table 15 for Groesbeck for the elevation of three kilometers, so that the figures given for that altitude are not typical. The value of fifteen per cent for relative humidity is lower than similar values observed in Tg air. What is especially important in the above tabulation is the close agreement in the properties at two such distant points as Groesbeck and Royal Center.

As Tm air masses progress northward in winter they move over ground that is colder than the lower strata, and are therefore subject to some cooling. This loss of heat tends to increase their thermal stability so that active convection is noticeably absent.

In winter Tg air currents usually do not reach a very high latitude before they are forced to ascend over cold continental air masses which they meet sometime in their northward advance. This ascent of the warm air frequently occurs before it reaches the north coast of the Gulf of Mexico. On the other hand, the southern shores of the Great Lakes mark approximately the extreme northern limit reached by Tg warm fronts on the ground in winter.

Except when the continent is covered by ice and snow there is little modification in the properties of Tg air masses as they move northward or eastward. This is shown by the small difference between values of Groesbeck and Royal Center in Table 15. Conservation of their properties in the absence of an underlying surface of ice and snow characterizes Tm air masses and is in sharp contrast to the rapid modification of Pc air masses when moving over similar surfaces. It is a matter of considerable importance to the hydrology of the interior since it provides a source of moisture for heavy precipitation, though with decreasing frequency in the higher latitudes.

Tropical maritime air comes to the North Atlantic states by three courses. Tg air moves northward on the ground over the Central States between the Gulf of Mexico and the Appalachian Mountains or goes northeastward over the South Atlantic coastal states. By the first course it reaches New England and is observed at higher levels over Boston, and by the second course it may move along the ground from the southwest. This air is modified by its passage over the land areas. In the third course Tm air comes to New England from the Sargasso

Sea region. This event occurs when a slowly moving trough of low pressure exists over the Great Lakes and the Ohio and Mississippi River Valleys, with an area of high pressure over the vicinity of the Bermuda Islands; at such a time New England may remain for several days in a rapidly moving current of Ta air mass. This occurred several times in 1932 and again in March 1936; at the latter time it was the primary cause of disastrous floods. This Ta air comes to New England with lower temperatures and humidity than the Tg air that comes overland through the interior. This is probably due to the fact that it comes from the northern edge of the source region and travels over cold water surface, which is much more effective for cooling than a land surface not covered by ice or snow. Nevertheless, this Ta air has a moderately high moisture content and high relative humidity above the elevation of 1 kilometer, as shown in Table 16.

Tropical maritime air in New England has a marked thermal stability due to cooling of the lower strata, but because of high relative humidity in the higher elevations, only a small uplift is necessary to cause condensation and precipitation. The frontal precipitation associated with Tm air on the North Atlantic coast is not found to be quite as extreme in local amounts but is comparable in average depth and more uniform than what is observed inland. Table 16 gives the average observed properties of Tm air at Boston in winter.

TABLE 16. PROPERTIES OF WINTER TM AIR, BOSTON, MASS.

| ELEVATION *Km* | TEMP *C* | SP HUM *Grams per Kg* | R.H. *Per Cent* |
|---|---|---|---|
| Surface | 14.0 | 8.8 | 82 |
| 1 | 13.7 | 6.5 | 60 |
| 2 | 8.7 | 6.2 | 86 |
| 3 | 2.0 | 4.6 | 84 |
| 4 | 3.7 | 2.9 | 85 |

*Summer Tropical Maritime Air.* Summer is much more favorable to the development and movements of Ta and Tg air masses. Two factors combine at that season to aid their movements; first there is the development of a low pressure area over the interior of North America; second there is the development of an area of high pressure over the western Atlantic Ocean. These two factors combine to produce a pressure gradient that sends Tm air over the United States and eastern Canada. Tm air, therefore, is found over the continent for a greater part of the time and over a much greater area in summer. Another change is found in the shift of the principal zone of frontal activity from the normal winter position in the Gulf of Mexico to the southern part of the Great Lakes region in summer.

In general the vertical structure of the summer Tm air masses is

similar to that of the winter air masses, but somewhat warmer and moister, the higher moisture content being the most important difference. In summer the temperature of the air at its source is almost identical with that of the water surface. The lapse rate is moderate, being only 0.6 of the adiabatic rate, which condition indicates stability for unsaturated air. The relative humidity is high at the ground and up to levels of three kilometers, which indicates potential instability that will become actual as soon as the air becomes saturated by cooling or otherwise. The high relative humidity aloft also indicates that only a little vertical displacement is needed to initiate condensation and precipitation.

As Tm air moves inland it travels over a heated land surface and is subject to heating so that it becomes warmer as it moves northward until it reaches the Great Lakes. The insolation heating frequently initiates convection that sets off the potential instability and causes thunderstorms with heavy rainfall.

Representative values of the characteristic properties of summer Tm air masses are given in Table 17.

TABLE 17. PROPERTIES OF SUMMER TM AIR

| | GROESBECK | | | PENSACOLA | | | ELLENDALE | | | ROYAL CENTER | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Elevation Km* | *Temp C* | *Sp Hum Gm Per Kg* | *R.H. Per Cent* | *Temp C* | *Sp Hum Gm Per Kg* | *R.H. Per Cent* | *Temp C* | *Sp Hum Gm Per Kg* | *R.H. Per Cent* | *Temp C* | *Sp Hum Gm Per Kg* | *R.H. Per Cent* |
| Surface | 28.0 | 17 5 | 72 | 28.5 | 20.8 | 85 | 28.5 | 16.5 | 66 | 29.0 | 15.9 | 61 |
| 1 | 20 8 | 13 0 | | 23 2 | 15.5 | | 27.0 | 13.3 | | 25 2 | 13.9 | |
| 2 | 15 8 | 6.4 | | 17.0 | 12.5 | | 21.8 | 8.7 | | 18.5 | 11.5 | |
| 3 | 9.0 | 4.9 | | 10.8 | 9.7 | | 12.8 | 5.7 | | 10.8 | 8.6 | |

It will be noted from the above table that the temperatures as far north as Royal Center, Ind., and even Ellendale, N. Dak., are as high as those at Pensacola, Fla. There has been some loss of moisture by rainfall, roughly one-fourth, but there still remains sufficient to produce heavy precipitation.

*Fronts of Air Masses.* By the term "front" is meant the boundary of an air mass in contact with another mass with different values of characteristic properties. These fronts are zones of rapid transition or change of the characteristic properties of one air mass to those of another, and may be accompanied by various manifestations of unsettled weather. The discontinuities which mark these fronts are not simply phenomena observed at the surface of the earth but extend to the upper levels of the air masses. Fronts fall naturally into two classes, namely, cold fronts when a cold air mass is moving into or under a warmer one, and warm fronts when warm air is advancing into or over a cold air mass.

In the case of a warm front, the air moves upward over the cold front since warm air has a lower density than cold. The lifting and con-

comitant cooling necessarily cause condensation since the warm air is nearly always relatively moist; the only important exception is the lifting of Tc (Ts or S) air which is unusual on the ground. In the case of Tg or Ta air masses, the condensation may result in heavy precipitation. The ascent of the warm moist air may set off its potential instability in convection and cause thunderstorms or occasionally tornadoes. The rate of rising, storm activity, and consequent precipitation depend upon many variable factors in the two conflicting air masses. These factors include: moisture content, which in turn depends upon the source region and life history of the air mass since its origin; slope of the front, which depends upon the difference in temperatures of the two air masses and their relative velocities; the location and topography of the surface over which the front passes. The land surface modifies the properties of the air masses and may provide upward gradients that initiate convection.

A cold front is formed when a cold air mass advances into a warm air mass. Rather, it is more accurate to say that it advances under the warm air mass, for in this case the cold air mass, having a greater density than the warm air, flows under the warmer air. As the cold air mass moves forward the warm air is forced upward. The resulting cooling, if sufficient moisture be present as it usually is, causes condensation, but as in the case of warm fronts the amount depends upon many variable circumstances of each particular front. Consequently variations in atmospheric activity, storm magnitude, and precipitation are largely a result of the differences in the characteristic properties of the two air masses.

Contiguous warm and cold fronts are usually found at points of convergence of two or more dissimilar air masses. These regions, since they occupy considerable area, are shown on the synoptic weather maps by the isobars of lower barometric pressure. These regions are known as "cyclones" or "extratropical cyclones" of the secondary circulation noted near the beginning of this chapter.

*Variations in the Properties of Air Masses.* It has been pointed out that moisture content and temperature of air masses are variable properties. Similar variations characterize all aspects of air masses because after leaving their respective source regions air masses are subject to continual change. All these deviations have their effect on the precipitation in any given locality. Although all precipitation is caused by the operation of definite physical causes, the amount and occurrence are clearly the result of chance variation and chance combination of those causes.

# 4 STORMS WHICH PRODUCE PRECIPITATION

*Precipitation and Storms.* Though there is always some water vapor in the air, it is common experience that rain or snow is not always falling. There is manifestly something needed in addition to moist air to cause rain or snow. This prerequisite, as was indicated in the previous chapter, is an atmospheric disturbance or storm with convergence in the air mass and a lifting of large volumes of warm moist air to higher altitudes where it is cooled to a temperature below dew point. While the lifting is the decisive factor, strong convergence is also necessary to produce heavy precipitation. A horizontal wind may be a storm but it produces no precipitation unless it blows up a slope of land surface or cold front. Storminess is therefore a characteristic of much importance to meteorologists, hydrologists, and climatologists; hence a rather detailed study is made of the kind and character of various storms and their resulting precipitation.

*Classification of Precipitation.* On the basis of causative storms precipitation may be divided into three types, namely, convectional, orographic, and cyclonic. This classification is based upon the meteorological phenomena which cause and accompany the precipitation. The fundamental factors in all types of storms are the accumulation of the moisture-laden air, and the lifting of it to higher altitudes for cooling with resulting condensation of the vapor, and subsequent precipitation.

*Convectional Precipitation.* In the convectional type of precipitation the actuating force is, as the name implies, thermal convection of the moisture-laden air. Two factors are thus necessary to produce convectional rainfall, heat to expand and raise the lower layers of atmosphere, and ample water vapor in the air to give it a high relative humidity. Solar radiation is the principal and very nearly the only source of heat sufficient to cause convection. It may operate directly on the air or may first heat the earth which then radiates the heat to the air to cause

convection. The only other sources of considerable heat are great conflagrations such as forest fires or volcanoes which are negligible as producers of precipitation. Convectional precipitation, being a consequence of warm weather, may be accompanied by other meteorological phenomena such as thunder, lightning, and local winds. This type of precipitation consists entirely of rain with occasional hail.

Convectional precipitation occurs in the tropics and in the temperate zones under favorable conditions. It is found in the equatorial region in the belt of calms called the doldrums between the north and south trade winds. Concerning this precipitation Davis (46) says: "The calms of the doldrums have already been described as occupying the belt of low pressure around the equator. They are supplied from either side by the inflowing trade winds which loiter here on the weakest gradients (of barometric pressure), allowing the air to reach a high temperature and humidity and to expand upward, causing an overflow aloft, and thus establishing the upper currents that run obliquely towards the poles. In the process of expansion, there is also a diurnal convectional movement, caused on the equatorial lands by warming of the lower air, but on oceans presumably due in greater part to the upward expansion and diffusion of the plentiful vapor there taken from the water surface. In this belt of warm damp air, the noonday witnesses the production of clouds, followed in the afternoon or evening by the occurrence of lively showers of rain, which frequently reach the activity of violent thunder storms; late in the night the clouds dissolve away, and in the morning the sky is generally clear. The belt of doldrums is therefore also known as the equatorial cloud belt and as the belt of equatorial rains, standing in strong contrast with the comparatively dry trade wind belts on either side. The equatorial rainfall is estimated at about 100 inches. Its large amount is due not only to the activity of the convectional processes on which it depends, but also and largely to the rapid decrease of the capacity for water vapor when air cools at high temperatures prevailing around the equator." This last statement is in general applicable to convectional precipitation wherever it may be experienced.

*Convectional Precipitation in the Temperate Zone.* Convectional rainfall in the temperate zones usually occurs with thunderstorms which develop with varying frequency wherever the two necessary factors, high temperature and moisture, are found. Thunder and lightning do not necessarily occur with convective precipitation, but since the factors which cause the rain also produce thunder and lightning which are the distinguishing features of the thunderstorm, it is advisable to consider that type of storm in lieu of convective precipitation in the temperate zones. Practically, the thunderstorm is the only con-

veniently observable manifestation of convective rainfall so that a study of those storms must be substituted for an analysis of such precipitation.

However, not all thunderstorms are caused solely by heating with resulting convection, although the convectional processes are essential in the genesis of all such storms. Other types owe their origin, at least in part, to frontal activity of moving air masses. However, since no distinction with respect to origin is made by the Weather Bureau in reporting thunderstorms, the several types are discussed.

*Thunderstorms.* Several types of thunderstorms have been recognized by meteorologists who have based the distinction on the actions of the air masses in which the storms occur. The classification given here is that used by Humphreys (96) except that one more class is included. The classes are as follows:

1. The heat or local convectional type
2. Those associated with a warm front
3. Those associated with a cold front
4. Those occurring within a cold air mass

Namias (143) distinguishes also thunderstorms occurring along an occluded front, those in horizontally converging air currents, and those produced by orographic effects. However, these three types seem to be variants of the foregoing enumerated classes.

*Heat Thunderstorms.* A heat thunderstorm is one produced essentially by a column of warm moist air rising by convection in a relatively slowly moving cumulonimbus cloud in which the vapor is cooled and precipitated with the accompaniment of thunder and lightning. As with other convectional rainfall, the genesis of heat thunderstorms depends upon heat and moisture. They are experienced most frequently in summer and in the humid regions. They occur occasionally in spring and fall but rarely in winter in the temperate climates. On account of deficient moisture they seldom occur in desert regions and because of the low temperatures, never in the polar regions. These thunderstorms are generated in all types of air masses in summer in the interior of the continent, but much more frequently in the Tg air as may be expected because of the characteristically plentiful humidity, high temperature, and high lapse rate. They commonly occur singly but may develop in groups if favorable conditions are widespread; Byers (25) reported a case in which 11 developed in one day over an area approximately 90 miles wide.

*Thunderstorms on a Warm Front.* Thunderstorms occur along a warm front as the warm air advances over a cold air mass; in this case the

thunderstorms are initiated by mechanical lifting of the warmer air, which upsets the potential instability of the air. The typical cloud of the thunderstorm is merged into the clouds of the normal warm front type. The rainfall of these storms constitutes a portion of the regular precipitation of the frontal action of the air masses, from which it is indistinguishable.

*Thunderstorms on a Cold Front.* Thunderstorms develop in the front of a cold air mass more readily than on a warm front. Ordinarily the slope of a cold front is steeper than that of a warm front on account of the friction on the ground surface with which the cold air comes in contact. The cold air mass, by acting as a blunt wedge, forces up the warm air until its potentially instable energy manifested by the high humidity and steep lapse rate is released and convection of the warm air is started. Thunderstorms are common phenomena on cold fronts when local heating and high humidity favor their development, and tend to become more violent and persistent when so developed than under other conditions. The genesis of these storms is not limited to the centers of extratropical storms but may occur within or at the boundaries of narrow areas of low barometric pressure known as "troughs."

*Thunderstorms in a Thermodynamically Cold Air Mass.* The passage of a cold air mass over a warm surface produces conditions which promote the development of thunderstorms. In this situation the air becomes heated in the lower strata; the lapse rate is increased as in local heating of a stationary air mass due to insolation and if thermal instability is sufficient to start convection, a thunderstorm results.

Even the comparatively simple meteorological action of a thunderstorm is seen to be the result of several causes which may operate more or less independently or may combine to produce the given results.

*Precipitation from Thunderstorms.* During the seasons of warm weather a large proportion of the precipitation is derived from heat or convectional thunderstorms in regions where conditions favorable to their formation prevail. Speaking of thunderstorms in Georgia, Scott (161) says: "Apart from the tropical disturbance, the main rainfall producer during summer months in Georgia is the thunderstorm. Convective action is at its height during the warm season and the cyclonic movement is weak." He states further: "The thunderstorm, being the result often of purely local convective action, affects only a limited area, and we have therefore at times single, isolated heavy downpours and at others a series of locally heavy showers. Anything that induces strong vertical convection of moisture-laden air causes heavy rains."

"Convective rains are frequent in southern Georgia during the summer months, July and August, especially. At Blakely, Quitman, and Brunswick in southern Georgia the July normal rainfall amounts are 7.22, 7.28, and 7.16 inches, respectively. Some of this precipitation undoubtedly is due to tropical storms, but the rains occasioned by thundershowers contribute most to the great July normals at these stations."

The same or closely similar conditions prevail in the entire southern portion of the United States as far west as Texas. In the central Gulf section a considerable proportion of the summer rainfall is produced by thunderstorms.

Great variability characterizes the formation, development, and existence of thunderstorms. Ward (189) says of them: "Thunderstorms are not all alike. There are some which are almost tornadic in their violence, last for hours, and cross several states, covering a distance as great as that from the Mississippi Valley to the Atlantic. There are some so small and so mild that they are limited to a county or two in a single state, and they bring but a few peals of thunder in a gentle shower of rain." Great variation is therefore found in the amount, distribution, and occurrence of rainfall from thunderstorms.

*Distribution of Thunderstorms.* Since thunderstorms are dependent upon definite thermal and moisture conditions of the atmosphere, they occur on all parts of the earth where proper conditions exist, including most of the habitable regions in the temperate and tropical zones. These storms are more frequent in humid continental areas subject to heating from insolation as is seen from further study of their distribution in North America.

*Thunderstorms in North America.* Thunderstorms occur in every portion of the United States and southern Canada during the warm seasons of the year at least, and as may be expected, many more occur in the southern latitudes than in the northern. In a study of the geographical distribution of thunderstorms for the 30-year period 1904 to 1933 Alexander (3) shows that there exist two primary centers of thunderstorm activity, one over Tampa, Florida, with 91 storms per year and the other over Santa Fe, New Mexico, with approximately 72 storms per year. Tampa has a humid subtropical climate with copious summer rainfall which is favorable to the development of thunderstorms. From that center the number per year decreases rather rapidly to the northern border of the State and thence more slowly to the isoceraunic line (a line of equal number of thunderstorms) of 40 storms per year; that line encloses a large part of the central United

States, including the Santa Fe center. In the Santa Fe area apparently the activity of thunderstorms is markedly increased by the orographic effects of the southern Rocky Mountains. No portion of the United States appears to be free from thunderstorms but they diminish rapidly in Canada above the international boundary.

Turning now to the distribution through the year, one finds another situation. In January the center of thunderstorm activity is over Louisiana; in February, March, and April it spreads out and moves northward into southern Arkansas; in May this peak flattens out and all but disappears, while the two centers appear over Tampa and Santa Fe. These two peaks grow through June, reach a maximum in July, begin to wane in August, and have disappeared by November, at which time the Louisiana center takes a definite form in Arkansas. During the colder months of the year there are few or no thunderstorms in Canada or the central northwest of the United States and only a small number throughout the western and northwestern portions of this country near the coast. This is to be expected since low temperatures prevail in that region. On the other hand, during the warm seasons the number is fairly uniform except for the two centers, Tampa and Santa Fe, and a smaller number experienced along the Pacific Coast.

*Frequency of Thunderstorms.* There is manifestly a great variation in the two primary causes of thunderstorms, namely, heat and water vapor, as well as the variable contributing effects of air mass fronts. The two primary factors are necessarily coexistent but variations in each are affected differently by the movements and activities of the air masses and are consequently more or less independent. Frontal and orographic effects are variable to the extent that one or the other may be the principal element in initiating action, or both may be entirely absent so that convection is the sole cause. From these circumstances it is evident that chance plays an important part in the development and frequency of thunderstorms. Just how extensively chance controls the frequency will be seen by studying a number of records of thunderstorms as they have been observed by the Weather Bureau.

The first example is taken from the records at Eastport, Maine. The observed frequency of thunderstorms during the summer months and for the year is compared with what may be predicted by the use of Poisson's equation. Poisson's equation is

$$P_{(n)} = \frac{e^{-m}m^n}{n!}.$$

The data are listed in Table 18.

TABLE 18. RECORDED THUNDERSTORMS AT EASTPORT, MAINE

| YEAR | MAY | JUNE | JULY | AUGUST | SEPTEMBER | FOR YEAR |
|---|---|---|---|---|---|---|
| 1893 | 1 | 1 | 5 | 1 | 0 | 10 |
| 4 | 1 | 3 | 5 | 0 | 0 | 14 |
| 5 | 1 | 1 | 1 | 1 | 3 | 9 |
| 6 | 2 | 4 | 2 | 2 | 1 | 14 |
| 7 | 1 | 5 | 3 | 4 | 4 | 18 |
| 8 | 2 | 2 | 1 | 1 | 1 | 8 |
| 9 | 0 | 6 | 3 | 0 | 1 | 11 |
| 1900 | 2 | 5 | 2 | 1 | 1 | 11 |
| 1 | 1 | 6 | 4 | 2 | 1 | 14 |
| 2 | 2 | 3 | 3 | 3 | 2 | 13 |
| 3 | 2 | 0 | 4 | 2 | 3 | 12 |
| 4 | 0 | 2 | 3 | 2 | 4 | 12 |
| 5 | 3 | 1 | 5 | 0 | 1 | 10 |
| 6 | 3 | 2 | 4 | 4 | 0 | 13 |
| 7 | 0 | 5 | 7 | 2 | 0 | 16 |
| 8 | 4 | 2 | 4 | 5 | 1 | 20 |
| 9 | 0 | 4 | 5 | 3 | 0 | 15 |
| 1910 | 4 | 8 | 7 | 1 | 0 | 16 |
| 1 | 2 | 8 | 4 | 3 | 0 | 17 |
| 2 | 2 | 7 | 4 | 10 | 1 | 23 |
| 3 | 3 | 1 | 4 | 3 | 3 | 15 |
| 4 | 2 | 4 | 4 | 1 | 4 | 18 |
| 5 | 1 | 5 | 3 | 3 | 1 | 15 |
| 6 | 0 | 1 | 6 | 5 | 0 | 14 |
| 7 | 0 | 3 | 8 | 4 | 0 | 16 |
| 8 | 3 | 2 | 6 | 2 | 2 | 20 |
| 9 | 0 | 2 | 8 | 1 | 1 | 12 |
| 1920 | 1 | 3 | 2 | 3 | 3 | 15 |
| 1 | 2 | 3 | 5 | 1 | 1 | 14 |
| 2 | 3 | 3 | 2 | 6 | 2 | 21 |
| 3 | 0 | 4 | 1 | 5 | 1 | 14 |
| 4 | 0 | 3 | 5 | 4 | 2 | 15 |
| 5 | 0 | 6 | 3 | 3 | 4 | 17 |
| 6 | 2 | 3 | 5 | 1 | 0 | 13 |
| 7 | 0 | 2 | 5 | 4 | 2 | 15 |
| 8 | 1 | 2 | 2 | 1 | 3 | 13 |
| 9 | 5 | 2 | 1 | 2 | 0 | 14 |
| 1930 | 2 | 5 | 7 | 4 | 4 | 23 |
| 1 | 2 | 2 | 5 | 3 | 7 | 22 |
| 2 | 1 | 2 | 7 | 3 | 0 | 13 |
| 3 | 3 | 4 | 3 | 3 | 1 | 17 |
| 1934 | 2 | 5 | 7 | 3 | 0 | 18 |
| 42 | 66 | 142 | 175 | 112 | 65 | 630 |
| Means | 1.57 | 3.38 | 4.17 | 2.56 | 1.55 | 15.0 |

In Tables 19, 20, and 21, the probable number of months in which the given number $n$ of thunderstorms per month will occur is computed by means of Poisson's equation and the computed number is then compared with the observations. Finally the chi-squared test is applied to determine goodness-of-fit of the computed values with the observa-

TABLE 19. CLASSIFICATION OF THUNDERSTORMS, EASTPORT, MAINE

| NUMBER: $(n)$ | NUMBER OF YEARS WITH $(n)$ THUNDERSTORMS OBSERVED IN MONTHS *May* | *June* | *July* | *August* | *September* | *Year* |
|---|---|---|---|---|---|---|
| 0 | 11 | 1 | 0 | 3 | 13 | |
| 1 | 9 | 5 | 4 | 10 | 13 | |
| 2 | 13 | 11 | 5 | 7 | 5 | |
| 3 | 6 | 8 | 7 | 11 | 5 | |
| 4 | 2 | 5 | 8 | 6 | 5 | |
| 5 | 1 | 6 | 9 | 3 | 0 | |
| 6 | 0 | 3 | 2 | 1 | 0 | |
| 7 | | 1 | 5 | 0 | 1 | 0 |
| 8 | | 2 | 2 | 0 | 0 | 1 |
| 9 | | 0 | 0 | 0 | | 1 |
| 10 | | | | 1 | | 2 |
| 11 | | | | 0 | | 2 |
| 12 | | | | | | 3 |
| 13 | | | | | | 5 |
| 14 | | | | | | 7 |
| 15 | | | | | | 6 |
| 16 | | | | | | 3 |
| 17 | | | | | | 3 |
| 18 | | | | | | 3 |
| 19 | | | | | | 0 |
| 20 | | | | | | 2 |
| 21 | | | | | | 1 |
| 22 | | | | | | 1 |
| 23 | | | | | | 2 |
| 24 | | | | | | 0 |
| Totals | 66 | 142 | 175 | 112 | 65 | 630 |

tions. The values for $P$ used in this test are taken from Karl Pearson's *Tables for Statisticians and Biometricians.* Since the computations for each month are identical, one sample is given and a summary of the computations for the remaining months and the year is shown.

TABLE 20. THE EXPECTED NUMBER OF THUNDERSTORMS FOR MAY, EASTPORT, MAINE

| NUMBER $(n)$ | MONTHS WITH $(n)$ STORMS | PROBABILITY $(P)$ | EXPECTED NUMBER | DEVIATION | DIVERGENCE |
|---|---|---|---|---|---|
| 0 | 11 | 0.2084 | 8.7 | −2.3 | 0.61 |
| 1 | 9 | 0.3265 | 13.8 | 4.7 | 1.61 |
| 2 | 13 | 0.2564 | 10.8 | −2.2 | 0.45 |
| 3 | 6 | 0.1341 | 5.6 | −0.4 | 0.03 |
| 4 | 2 | 0.0536 | 2.3 | 0.3 | 0.04 |
| 5 | 1 | 0.0166 | 0.7 | −0.3 | 0.13 |
| 6 | 0 | 0.0034 | 0.1 | 0.1 | 0.10 |
| | | | | | $\chi^2 = 2.97$ |

For $n'$ (number of divergences in the last column in the above table) $= 7$, and $\chi^2 = 2.97$, $P'$, which may be called the goodness or probability of fit, $=0.81$. This high value of $P'$ indicates that Poisson's equation

is an excellent fit for the frequency distribution of observed numbers of thunderstorms.

TABLE 21. SUMMARY OF COMPUTED DISTRIBUTIONS, THUNDERSTORMS AT EASTPORT, MAINE

| MONTH | AVERAGE NUMBER OBSERVED | $n'$ | $\chi^2$ | GOODNESS-OF-FIT $P'$ |
|---|---|---|---|---|
| May | 1.57 | 7 | 2.97 | 0.81 |
| June | 3.38 | 8 | 2.46 | 0.92 |
| July | 4.17 | 11 | 6.46 | 0.78 |
| August | 2.56 | 8 | 2.31 | 0.97 |
| September | 1.55 | 5 | 8.19 | 0.09 |
| Year | 15.0 | 18 | 6.66 | 0.99 |

It should be noted that $n'$ does not always equal the number of classes under $n$ as in Table 20. This difference, if any, is caused by grouping together under one deviation a number of classes with small expected numbers of events on either end of the computation. This practise is usually necessary if the expected number is less than 1.0 although it was avoided in Table 20.

*Thunderstorms at Montgomery, Alabama.* Another example of the distribution of thunderstorms is taken from the records of the Weather Bureau at Montgomery, Ala. Due to the warmer climate there is a larger number of thunderstorms observed at Montgomery than at Eastport, Maine.

The record begins in 1873 and is shown graphically in Figure 7. The record, however, does not appear to be consistent throughout the period. There is a steady increase in the number of such storms each year from the beginning of the record until the turn of the century. It is probable that this increase is due to increasing accuracy in observing and reporting them rather than to a real increase in the number of storms. Humphrey (96) states that there was a change in the method of reporting them that would necessarily increase the number. Prior to 1904, only those thunderstorms that accompanied rain at the observing station were reported whereas after that year all thunderstorms that were heard were reported as occurrences. This would tend to make an appreciable increase in storms reported in regions where they are relatively frequent although the difference would probably be negligible in other regions where the total number is small, as at Eastport, Maine. Because of the noticeable change in the data resulting from this change of plan of reporting, only the record from 1904 to 1934 inclusive is used for computing the frequency at Montgomery. The length of record is thus 31 years.

The data of thunderstorms at Montgomery demonstrates the need for careful examination of hydrological data before attempting any

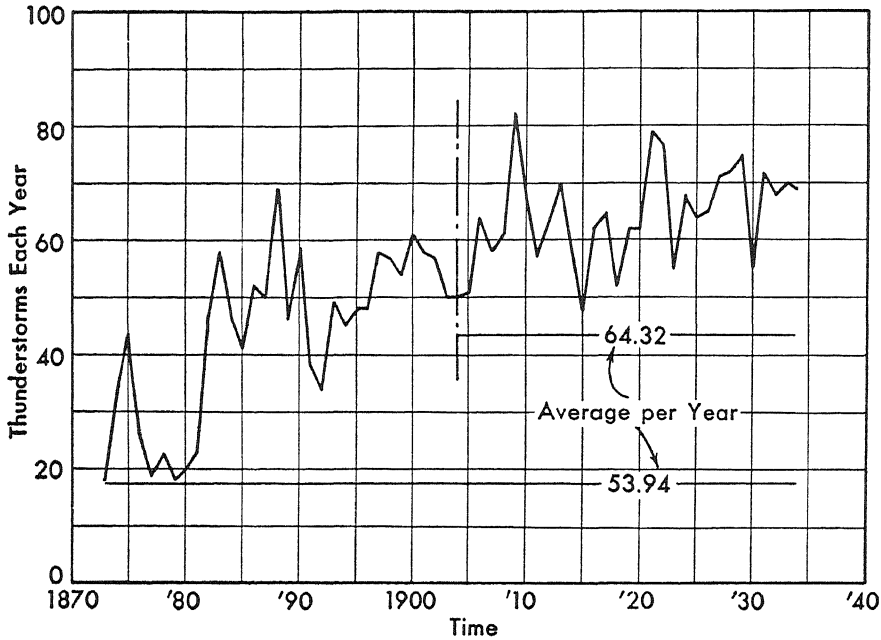



FIGURE 7. Annual Thunderstorms, Montgomery, Ala.

form of analysis. The methods of statistics, like any other method of analysis, do not remove or correct defects inherent in an array of data.

TABLE 22. CLASSIFICATION OF THUNDERSTORMS, MONTGOMERY, ALA.

| NUMBER | JAN. | FEB. | MAR. | APR. | MAY | JUNE | JULY | AUG. | SEPT. | OCT. | NOV. | DEC. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8 | 2 | 1 | 1 | | | | | 0 | 9 | 11 | 11 |
| 1 | 9 | 8 | 2 | 1 | 0 | | | | 2 | 6 | 8 | 9 |
| 2 | 7 | 3 | 5 | 2 | 2 | | | | 3 | 11 | 7 | 4 |
| 3 | 6 | 10 | 7 | 5 | 3 | 0 | | 0 | 5 | 2 | 4 | 5 |
| 4 | 0 | 3 | 4 | 3 | 6 | 1 | | 1 | 5 | 2 | 0 | 1 |
| 5 | 1 | 3 | 3 | 10 | 3 | 1 | | 0 | 3 | 0 | 0 | 0 |
| 6 | 0 | 1 | 1 | 3 | 5 | 3 | 0 | 1 | 1 | 0 | 1 | 1 |
| 7 | | 1 | 1 | 2 | 1 | 0 | 3 | 1 | 0 | 1 | 0 | 0 |
| 8 | | 0 | 5 | 3 | 3 | 3 | 1 | 4 | 7 | 0 | | |
| 9 | | | 2 | 1 | 1 | 0 | 1 | 3 | 1 | | | |
| 10 | | | 0 | 0 | 3 | 4 | 4 | 4 | 1 | | | |
| 11 | | | | | 2 | 3 | 6 | 3 | 1 | | | |
| 12 | | | | | 1 | 8 | 1 | 3 | 2 | | | |
| 13 | | | | | 0 | 3 | 4 | 3 | 0 | | | |
| 14 | | | | | 0 | 1 | 0 | 2 | | | | |
| 15 | | | | | 0 | 0 | 6 | 1 | | | | |
| 16 | | | | | 0 | 1 | 3 | 2 | | | | |
| 17 | | | | | 1 | 2 | 1 | 3 | | | | |
| 18 | | | | | 0 | 0 | 1 | 0 | | | | |
| 19 | | | | | | 0 | 0 | | | | | |
| 20 | | | | | | 1 | | | | | | |
| 21 | | | | | | 0 | | | | | | |
| Total | 46 | 84 | 135 | 147 | 203 | 343 | 381 | 350 | 174 | 49 | 40 | 42 |
| Means | 1.48 | 2.71 | 4.35 | 4.74 | 6.55 | 11.06 | 12.29 | 11.29 | 5.61 | 1.58 | 1.29 | 1.35 |

The summary of observed and computed frequency and goodness-of-fit is given in Table 23.

TABLE 23. SUMMARY OF FREQUENCY OF THUNDERSTORMS, MONTGOMERY, ALA.

| MONTH | AVERAGE | $n'$ | $\chi^2$ | GOODNESS-OF-FIT, $P'$ |
|---|---|---|---|---|
| January | 1.48 | 6 | 1.69 | 0.89 |
| February | 2.71 | 8 | 6.19 | 0.52 |
| March | 4.35 | 10 | 18.89 | 0.026 |
| April | 4.74 | 10 | 9.29 | 0.32 |
| May | 6.55 | 13 | 10.93 | 0.54 |
| June | 11.06 | 13 | 18.25 | 0.12 |
| July | 12.29 | 14 | 18.55 | 0.14 |
| August | 11.29 | 13 | 4.20 | 0.98 |
| September | 5.61 | 9 | 21.21 | 0.007 |
| October | 1.58 | 6 | 5.30 | 0.38 |
| November | 1.29 | 5 | 1.98 | 0.74 |
| December | 1.35 | 5 | 3.91 | 0.42 |

The average goodness-of-fit of all months is 0.42. This is a fair fit although definitely not so good as that for the thunderstorms at Eastport, Maine. This may be caused in part by the relatively short period but it is also probable that a part of the cause lies in the application of Poisson's equation to a series in which the variations are relatively too great in proportion to the total number of events in the series. According to the theory of Poisson's equation, the values of $n$ should be small when compared to $m$. This condition was not well satisfied in the case of the thunderstorms at Montgomery in which $m$ was less than three times as great as $n$ in several instances. Nevertheless, these examples illustrate the use of the function. A frequency function consisting of series derived from Poisson's equation would very likely provide a better fit but it is not attempted here.

These two samples which are considered typical, indicate that the theory of probability, or law of chance, is applicable to the data of thunderstorms. It may reasonably be inferred that the same theory could be applied to other phenomena, which like the data used above, are from the results produced by multiple causes. By inference also, it may be expected that data of rainfall may be amenable to the same laws.

*Orographic Precipitation.* Orographic precipitation is that which results from cooling of moisture-laden air masses which are lifted by contact with elevated land masses. This type is most pronounced on the windward sides of mountain ranges lying athwart the prevailing terrestrial or continental winds where those winds pass from the relatively warmer ocean to the land. When these winds strike the high land, the air is forced upward and thereby cooled and the moisture

condensed and precipitated as rain or snow. Regions in which these conditions exist in a conspicuous manner are: the northwestern coast of the United States and the western coast of Canada where the prevailing westerly winds of the north latitudes strike the coast ranges of North America; the west coast of Chile where the prevailing westerly winds of the south latitudes strike the Andes Mountains; India where the monsoon winds reach the Himalaya Mountains; the coast of Mexico and central America where the mountains and highlands lie across the trade winds. These regions are not the only places where prevailing winds produce orographic rainfall but they are conspicuous examples.

Orographic precipitation occurs also far inland wherever mountain ranges, highlands, or ridges rise above the surrounding country in the path of the moisture-bearing air masses. Essentially the same conditions must exist inland as on the coasts, namely, moist air masses are forced upward from the relative warmth of the lowlands to the greater altitudes of the highlands where the low temperatures cause condensation and precipitation. On inland ranges precipitation of this type is intermingled with other types which may be prevalent in the region. Orographic precipitation on inland ranges is especially likely to be irregular in occurrence and quantity on account of interference by the atmospheric disturbances resulting from cyclonic storms. Orographic precipitation occurs usually as either rain or snow.

Inland orographic precipitation is found on the southwesterly slope of the Appalachian Mountains to which the moisture-laden air is brought by tropical maritime air masses from the Gulf of Mexico and the South Atlantic Ocean. It is found on the west slopes of the Rocky Mountains to which moisture is brought by the Pacific Maritime air masses. This type also occurs less conspicuously along the southern coast of the United States where a very flat ridge is sufficient to precipitate the moisture brought by the maritime air of the Gulf of Mexico and produce a notable increase in annual rainfall.

*An Example of Concentrated Orographic Effect.* A case of extremely high rainfall caused by the trade winds blowing across the peculiar topography of the Hawaiian Islands is recorded by Nakamura (142). The average annual rainfall on Mount Waialeale, Island of Kauai, for the 12 years in the period 1911 to 1933 was 456 inches, which makes this spot one of the wettest in the world, ranking with the Khasi Hills in Assam, India.

He says: "The summit of Mount Waialeale rises to a height of well over 5,000 feet above sea level, and it is situated near the geographic center of the Island of Kauai. At the top the country is generally rolling in all directions except to the east where it breaks down

abruptly from the rain gage and drops about 4,000 feet to form the east scarp.

"A topographic map of the region discloses a long ridge that extends northeastward from the summit. East and northeast winds blowing over the plains to the south sweep into the gorge bounded by this ridge and another shorter ridge farther south and, after reaching the head of the gorge, ascend the precipitous east scarp of Mount Waialeale. In ascending the winds are cooled and the moisture precipitated. The falling rain is then blown up to the top by the upward draft."

Apparently from this description the two ridges act as a funnel to concentrate the moisture-laden trade winds to the scarp where they are cooled and carry upward in their ascent the falling rain which dropped over the edge of the scarp.

*Rain Shadow.* Just as an obstruction to the sunlight causes a shadow on the obstructed side, so does a mountain range or highland obstruct the moisture bearing winds and leave a relatively dry area on the lee side, which is known as the "rain shadow." The portions of the states of Oregon and Washington east of the Cascade Mountains are in such a rain shadow as are the intermontane valleys of the western coastal area which are sheltered from the rainy westerly winds. There are numerous rain shadows in the Rocky Mountain region, for example, the Big Horn Basin in Wyoming, which is sheltered by the Big Horn Mountains from the moisture-bearing air masses from the Gulf and from the maritime air of the Pacific Ocean by the Absaroka and other ranges of the Rocky Mountains. The North, Middle, and South Parks in Colorado are similarly situated in the rain shadows of the Rocky Mountain Ranges. Jefferson (100) points out that the interior valleys of Chile are in the rain shadows of coastal ranges of that country. The lee side of the mountains of the Hawaiian Islands are in the rain shadows, sheltered from the trade winds. There are many other extensive regions and small areas that may be found in rain shadows. The cause of these rain shadows may be easily discovered; the moist air has been forced up the windward side and precipitated its moisture, and upon passing the peak of the range, no further orographic or other lifting occurs so that the rainfall is the residual of previous condensation or the result of other and more uncertain lifting. If the obstructing mountain ranges are sufficiently massive and the low regions on the leeward are sufficiently extensive, the wind may descend on the lee side, thereby undergoing compression and heating and becoming still more unfavorable for precipitation.

*Precipitation in Black Hills.* A good example of inland orographic precipitation is found in the Black Hills, South Dakota. These hills,

located within the Great Plains, are the result of an isolated batholithic uplift which formed the now maturely dissected dome, the peaks of which rise some 4,000 feet above the subjacent plain on the east. The mean annual rainfall on the surrounding plains ranges from 14.66 to 19.70 inches. Within the Black Hills the average annual precipitation ranges up to 25.90 inches, based on normals of the Weather Bureau to 1940. Tables 24 and 25 show the comparative depths of normal precipitation to 1940, and for the period from 1910 to 1922 inclusive, and elevations of the stations of record.

TABLE 24. PRECIPITATION WITHIN THE BLACK HILLS, S. DAK.

| STATION | ELEVATION, FEET *Mean Sea Level* | ANNUAL PRECIPITATION, INCHES *Mean to 1940* | *Mean 1910–22* |
|---|---|---|---|
| Deadwood, S. Dak. | 4535 | .... | 28.32 |
| Deerfield, S. Dak. | 6000 | 19.07 | 17.83 |
| Dumont, S. Dak. | 6192 | 22.58 | 23.50 |
| Elk Mountain, S. Dak. | 4700 | .... | 15.50 |
| Greenmount, S. Dak. | 6340 | .... | 25.73 |
| Hardy, R.S., S. Dak. | 6600 | 20.73 | 21.86 |
| Harvey's Ranch, S. Dak. | 6284 | 25.90 | 24.13 |
| Hot Springs, S. Dak. | 3426 | 19.23 | 19.28 |
| Lead, S. Dak. | 5152 | 24.39 | 24.64 |
| Rochford, S. Dak. | 5224 | 21.04 | 20.03 |
| Spearfish, S. Dak. | 3647 | 20.43 | 18.94 |
| Water's Ranch, S. Dak. | 4000 | 20.26 | 20.50 |

TABLE 25. PRECIPITATION ON THE SUBJACENT GREAT PLAINS AROUND THE BLACK HILLS

| STATION | ELEVATION, FEET *Mean Sea Level* | ANNUAL PRECIPITATION, INCHES *Mean to 1940* | *Mean 1910–22* |
|---|---|---|---|
| Belle Fourche, S. Dak. | 3019 | 14.66 | 13.99 |
| Dowling, S. Dak. | 2250 | .... | 19.76 |
| Hermosa, S. Dak. | 3297 | .... | 16.37 |
| Knowles, Wyo. | 4500 | .... | 20.18 |
| Oelrichs, S. Dak. | 3341 | 19.70 | 19.50 |
| Orman, S. Dak. | 2920 | 15.12 | 14.91 |
| Rapid City, S. Dak. | 3259 | 18.00 | 18.19 |
| Vale, S. Dak. | 2773 | 16.64 | 16.39 |

Although there is considerable diversity in annual precipitation at comparable elevations, it is apparent that greater precipitation is obtained at the higher levels. For the four stations at or above 6000 feet having records to 1940, the mean precipitation is 22.07 inches; and for the same four stations, the mean precipitation 1910–22 was 21.83 inches. In contrast to those means, the five stations on the Great Plains (Table 25) below elevation 3,500 feet have a mean to 1940 of 16.82 inches and for the period 1910–22, a mean of 16.60 inches. Since the Black Hills form a rugged mountainous area, it is not possible to

explain the diversity among the individual stations, but the data show clearly an increase in precipitation with an increase in altitude.

*Orographic Effect in Individual Storms.* In the storms of March 1936 in New England, there can be observed the orographic effect in the course of individual storms. The rainfall received on the given dates and periods through those storms is given in Table 26.

TABLE 26. RAINFALL IN NEW HAMPSHIRE, MARCH 1936

| STATION | ELEV. FT. | DAY OF MONTH OR PERIOD | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *11* | *12* | *13* | *11–13* | *17* | *18* | *19* | *17–19* |
| Berlin | 1,110 | .02 | 1.68 | 1.01 | 2.71 | .89 | .52 | 2.20 | 3.61 |
| Concord | 350 | 1.39 | 1.82 | .06 | 3.27 | .03 | 2.04 | .61 | 2.68 |
| Durham | 83 | .10 | 2.18 | .06 | 2.38 | | .46 | 1.35 | 1.81 |
| Mt. Washington | 6,288 | .62 | 2.67 | .72 | 4.01 | .87 | 2.10 | 2.54 | 5.51 |
| Pinkham Notch | 2,000 | .69 | 6.46 | .63 | 7.78 | .44 | 6.27 | 4.05 | 10.76 |
| Plymouth | 500 | .46 | 3.46 | .33 | 4.25 | .31 | 2.12 | 1.82 | 4.25 |

Another point is brought out by a study of the data of the rainfall on Mt. Washington, N. H., in the storms of March 1936. While the mean annual precipitation (and perhaps the mean for shorter periods) is greater at the higher altitudes, the maximum reported rainfall of both storms was observed at Pinkham Notch at an elevation of slightly less than 2,000 feet while on Mt. Washington, at an elevation of 6,288 feet, the rainfall was approximately 51 per cent of the precipitation at Pinkham Notch on the two peak days. It may be inferred from this that convection was not predominantly active and that heavy condensation and precipitation occurred at moderate elevations. This situation could be expected since it is recalled that tropical marine air becomes more stable because of its passage over the cold northern waters. The stable air then apparently flows around the high isolated peaks when possible instead of being lifted up the slope and forced over the top.

These observations support the statements of Dr. M. McEwen (121): "If the air current is stable (density decreases at a greater than average rate as the height increases), as it is forced up a slope the surrounding air is lighter and thus ascent is resisted. The air will tend to flow out around the obstacles rather than continue upward. This explains why oftentimes the heaviest precipitation does not occur at the top of a mountain but at some intermediate elevation."

*Some Effects of Low Topography.* The effects of relatively low topography on precipitation have been noted in Texas. Lowry (117) states the situation there as follows: "Lines of equal rainfall run in a north and south direction in most cases with some deviations. The latter may disappear as additional years of record are accumulated.

However, some deviations are caused by geological or physical conditions which will prevent any future straightening. This is particularly true of the area west of San Antonio. Here the lines of equal rainfall swing far to the West. It may be noted that this area is just below the Balcones Escarpment. There is no doubt that this fault is the cause of the increased rainfall."

The data of the foregoing tables show plainly that even relatively small differences in elevations have an effect on the precipitation obtained in a locality. It is probable that local topography is the cause of many irregularities in data which cannot be accounted for by chance variations in the precipitation.

*Precipitation of Rotational Storms.* The term "rotational storms" designates all those storms in which the moving air of some portion of the storm appears to rotate about a vertical axis. Three general types of rotational storms are recognized, namely, the tornado, the hurricane or typhoon, and the extratropical cyclone, which are named in the order of the magnitude of the area covered by the respective paths and in inverse order of violence. The cause of these storms is not of primary concern except in so far as the cause may effect precipitation. The main consideration is the fact that the passage of any one of these storms is accompanied by precipitation in varying amounts and intensities.

*The Tornado.* Although the tornado covers the smallest area of the three types of storms, it is undoubtedly the most violent in proportion to its size. It has been described by Davis (46) as "a progressive, limited, local, violent whirlwind, characterized by a funnel-like cloud which hangs suspended from an intensely black mass of storm clouds; the apex of the funnel sweeps over the earth surface, sometimes touching it, and sometimes receding from it, to come down again to the ground farther on in the course of the cloud as it moves forward." Tornadoes are always accompanied by heavy thundershowers. This precipitation constitutes the reason for considering it here, for the destructive effects, which are great, are outside the scope of this study.

The tornado occurs under meteorological conditions favorable to the development of violent thunderstorms, which conditions are a warm moist air-mass under strong local heating, and the advent of a cold front. All these factors produce rapid convection of unstable air so that intense precipitation is also the accompaniment of all tornadoes.

The geographical distribution of tornadoes is quite extensive although not as great as that of thunderstorms. Central and southeastern United States are the best-known regions. They may be expected to occur wherever strong local heating of warm moist air masses invade regions subject to the passage of cold fronts.

The frequency of tornadoes is fortunately not so great as that of thunderstorms. Sufficient data are not available to study the observed frequency with that computed from theoretical methods, but because of the similarity of conditions required for development it may reasonably be assumed that the frequency of tornadoes can be computed by the same methods as those used for thunderstorms. For this reason no further study need be given to tornadoes here.

*The Tropical Cyclone.* The tropical cyclone is the great rotating storm of the tropical seas and adjacent lands. In different parts of the world it is known by various names: in the West Indies and western part of the Atlantic Ocean it is called the "hurricane"; in the East Indies and adjacent seas of the east Asian continent, it is the "typhoon"; over the Philippines, it is known as the "baguio"; in Australia, it is sometimes named the "willy-willy" (177). The name "hurricane" is used hereinafter for the tropical cyclone since that name is most commonly used in the United States, and the following discussion is in general confined to the West Indian storms.

The hurricane is a large and violent storm and is a producer of copious rainfall. The area covered is exceeded only by that of the extratropical cyclone which will be discussed later. The velocity of its winds is exceeded only by the winds of the tornado and winds at very high altitudes.

Certain features of action and form as well as location of origin distinguish the hurricane. The most prominent of these characteristic features is the "eye," or the quiet rainless center varying from 6 to 30 miles in diameter, about which the in-blowing winds circulate. The isobars around the center are nearly circular and the atmospheric pressure begins to drop at the periphery of the winds. The distribution of temperature around the central eye is nearly uniform since all incoming air has been subject to the same tropical heating. Whether or not cold continental air masses enter into the genesis of these storms is still a moot point, but if they do they have been profoundly modified by heating and absorption of moisture on their paths to the tropical seas. Precipitation is not distributed uniformly throughout the hurricane, but in storms moving over the Gulf Coast at least, it is concentrated in the right front quadrant, as has been shown by Cline (33). However, Cline also states that in three cyclones that had ceased to advance, the heaviest rainfall took place in the rear of the storms. The occurrence of precipitation at any one point is not steady but is irregular in amount and may be intermittent. The occurrence of the hurricane is distinctly seasonal, late summer being the season of greatest frequency.

Although the places of origin of the hurricane are the tropical seas, they frequently leave those regions and travel long distances into the temperate zones. This is characteristic of storms of hurricane violence and also of those storms of weaker action which originate in the same region and follow similar paths.

In the western hemisphere hurricanes have their origin in the southern part of the North Atlantic Ocean and Caribbean Sea. The usual path is then westward with a gradual shifting northward. This change of direction increases as the storm leaves the tropics, after which it moves northward, joins the prevailing westerly winds and extratropical cyclones, and travels northeastward. When the hurricane departs from the tropics it leaves the source of thermal energy which produces and sustains it, and although its violence continues to be great for long distances over water, it gradually subsides and becomes in effect only another extratropical cyclone. When these storms originate far enough west, as many do, they encounter the islands of the West Indies and beyond them the continent of North America. The friction of the rough land surface and the cutting off of the warm moist air operate to diminish rapidly the violence of the storm. Rarely a northward-moving hurricane may encounter a moving front which supplies it with an additional source of energy and augments its life to a relatively small extent.

For the land areas along the Gulf of Mexico and the South Atlantic Ocean hurricanes are important producers of rainfall, causing especially the heavy rains that result in floods. Not only are those storms of hurricane violence important in this respect but also are those storms of lesser violence of similar origin. The importance of storms of this type as moisture carriers is evidenced by the seasonal precipitation received in northeastern Mexico and southwestern Texas during the months of August and September, as shown by the hyetograph for Corpus Christi, Texas, on Figure 19.

The effects and importance of hurricanes in extratropical regions are illustrated by the histories of a few well-known storms which have traversed the eastern portions of the United States.

*The Hurricane of September 1–12, 1900.* This storm was first reported southwest of Haiti on September 1, 1900. During the next two days it traveled westward, then veered northwest, then north, and passed over Cuba during the morning of September 5. On the sixth it turned westward again when near the Florida coast. It crossed the Gulf of Mexico and struck the coast near Galveston, Texas, which sustained heavy losses of life and property. The storm continued

northward, as far as Nebraska, losing however, much of its violence, then turned eastward and passed down the St. Lawrence Valley. Although it retained its shape and high winds, it became in effect another extratropical cyclone. However, the path of this hurricane is not the typical course of these storms in that it moved farther westward than usual.

*The Hurricane of June 29 to July 10, 1916.* Compared with the preceding storm the path of this hurricane was short. It apparently originated in the western Caribbean Sea, and moving in a northerly direction it struck the coast near Mobile, Ala., between July 5 and 6. It did not continue its northerly movement far but instead remained stationary over Mississippi and Alabama, where it gradually filled and died out. The precipitation of this storm was notable. The Miami Conservancy District (135) in its studies of this storm found an average 24-hour rainfall of 8.2 inches and an average 48-hour rainfall of 11.3 inches over 6,000 square miles. Markedly destructive floods were caused on the Coosa River in Alabama.

*The Hurricane of October 21–31, 1921.* This storm illustrates another unusual path because it recurved to the eastward rather than to the northeast. It originated in the western part of the Caribbean Sea and moved northwesterly through the Yucatan Straits, past Cuba, and then turned due east. It hit the coast of Florida and crossed the peninsula on a northeasterly course. Heavy precipitation occurred over the State; 11.73 inches in 24 hours were reported at St. Leo, Fla.

*The Hurricane of September 16–22, 1938.* The storm was first definitely located, according to Tannehill (178), about 500 miles northeast of the Leeward Islands, September 16. It traveled northwestward along a path north and east of the Windward Islands and the Bahamas, and then turned northward and moved along the Atlantic Seaboard until it struck the south coast of New England, near New Haven, Conn. According to Pierce (151) the hurricane on its journey northward entered a trough of low pressure in front of a cold front moving slowly eastward, for which the center of low pressure was located over Ontario, Canada, while tropical air was over New England. Upper air which was moving northward drew the hurricane away from the usual northeast path and across New England into Canada, where it faded out. Although the course of this hurricane over New England is not unprecedented, such storms are infrequent, so that the storm of September 1938 was wholly unexpected in New England.

*Seasonal Distribution of Hurricanes.* As stated above, the occurrence of hurricanes is limited to a definite season of the year, principally June

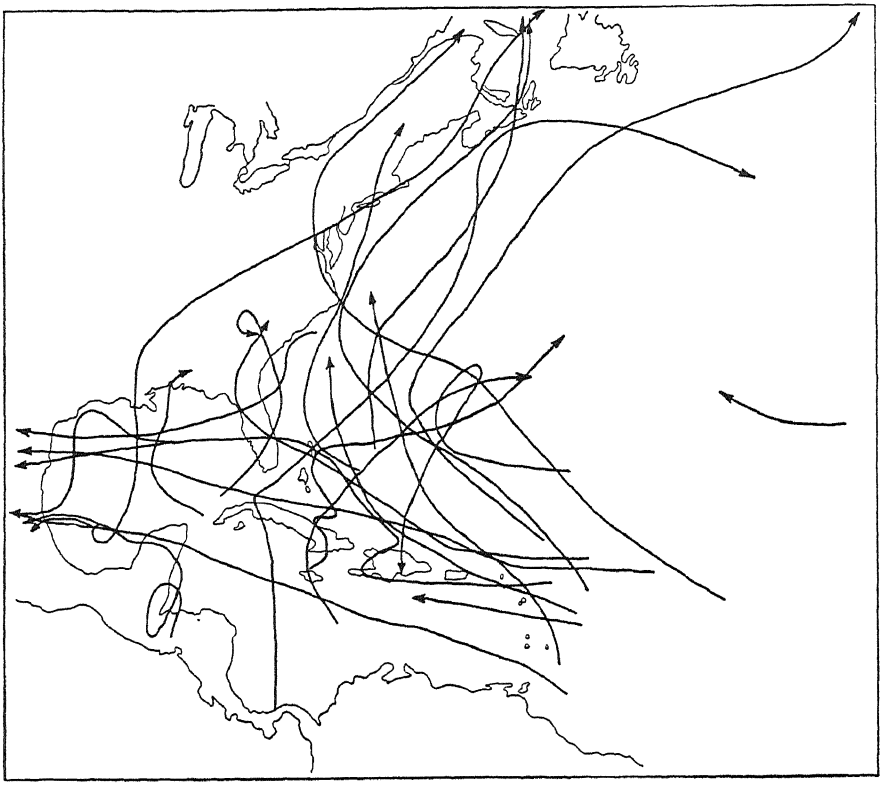


FIGURE 8. Paths of Tropical Hurricanes, North America, 1933–1934

to October with a peak in September. The following tabulation gives the monthly number of hurricanes which occurred in the various months from 1887 to 1939.

| | MAY | JUNE | JULY | AUG. | SEPT. | OCT. | NOV. | DEC. | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| Number of Storms | 4 | 25 | 26 | 77 | 119 | 95 | 26 | 2 | 374 |

The data for the years from 1887 to 1932 are from Tannehill (177) and the remainder were obtained from the *Monthly Weather Review*. They include all storms reported to have been of hurricane or lesser violence.

*Variation in Paths of Hurricanes.* From the few illustrations given above it may be inferred that considerable variation exists in the paths of hurricanes. In Figure 8 is shown the paths of all hurricanes reported in the *Monthly Weather Review* for two years, 1933 and 1934. In all 15 paths of storms of hurricane force are shown, and 5 of lesser violence. These paths are typical of all and show in general the area covered, except that these storms do range somewhat farther west in the United

States. It is readily seen that many paths are decidedly erratic and apparently move in response to whatever external forces chance to act upon them.

Other series of paths show similar erratic courses. Tannehill (177) gives a series of figures depicting the paths of all hurricanes by months from 1874 to 1933, which show all the irregularities of those in Figure 8. Apparently the course of a hurricane is strongly guided within the broad zone of travel by whatever upper air currents act upon it or the air mass in which it exists.

*Distribution of Hurricanes on the Coasts of the United States.* The distribution of hurricanes along the coasts of the United States for the years 1879 to 1936 is given in Table 27 from Tannehill (177), and to those data are added the storms to include those of 1939.

TABLE 27. HURRICANES ON THE COASTS OF THE UNITED STATES

| STATE | NUMBER OF STORMS OF HURRICANE INTENSITY, 1879–1939 | LENGTH OF COAST LINE MILES | AVERAGE NUMBER PER 100 MILES OF COAST PER CENTURY |
|---|---|---|---|
| Texas | 31 | 367 | 13.9 |
| Louisiana | 17 | 397 | 7.1 |
| Mississippi | 10 | 65 | 25.2 |
| Alabama | 7 | 53 | 21.7 |
| Gulf coast of Florida | 34 | 798 | 7.0 |
| Atlantic coast of Florida | 13 | 399 | 5.4 |
| Both coasts of Florida | 47 | 1,197 | 6.4 |
| Georgia | 7 | 100 | 11.5 |
| South Carolina | 12 | 187 | 10.5 |
| North Carolina | 16 | 301 | 8.7 |
| All states listed above | 147 | 2,667 | 9.1 |

*Frequency of West Indian Hurricanes.* Like thunderstorms the frequency of hurricanes over the region of occurrence is a question of how many per annum rather than how many years between storms since there are usually a number experienced every year. The same methods used to calculate the number of thunderstorms can be used to compute the number of hurricanes to be expected in a given unit of time. As with thunderstorms the events under consideration form a series of numbers of discrete or integral quantities that in statistical terms constitutes an integral heterograde series. Poisson's formula is particularly applicable to such a series where the probable number of events is small, as is evident in this case, and therefore the frequency is computed by that formula and the results compared with the observations.

The data of the number of hurricanes observed annually were taken

TABLE 28. HURRICANES IN THE WEST INDIES, 1887-1940

| YEAR | WEST CARIB. | EAST CARIB. | TOTAL | YEAR | WEST CARIB. | EAST CARIB. | TOTAL |
|---|---|---|---|---|---|---|---|
| 1887 | 3 | 13 | 16 | 1913 | 2 | 2 | 4 |
| 8 | 3 | 7 | 10 | 4 | 1 | 1 | 2 |
| 9 | 3 | 5 | 8 | 5 | 1 | 4 | 5 |
| 1890 | 0 | 1 | 1 | 6 | 3 | 10 | 13 |
| 1 | 2 | 9 | 11 | 7 | 0 | 2 | 2 |
| 2 | 4 | 5 | 9 | 8 | 0 | 4 | 4 |
| 3 | 5 | 5 | 10 | 9 | 1 | 3 | 4 |
| 4 | 2 | 4 | 6 | 1920 | 2 | 2 | 4 |
| 5 | 4 | 2 | 6 | 1 | 3 | 2 | 5 |
| 6 | 3 | 3 | 6 | 2 | 4 | 1 | 5 |
| 7 | 2 | 3 | 5 | 3 | 1 | 4 | 5 |
| 8 | 2 | 5 | 7 | 4 | 6 | 2 | 8 |
| 9 | 2 | 3 | 5 | 5 | 2 | 1 | 3 |
| 1900 | 1 | 5 | 6 | 6 | 4 | 6 | 10 |
| 1 | 2 | 8 | 10 | 7 | 1 | 6 | 7 |
| 2 | 3 | 1 | 4 | 8 | 0 | 6 | 6 |
| 3 | 0 | 8 | 8 | 9 | 1 | 1 | 2 |
| 4 | 3 | 6 | 9 | 1930 | 0 | 2 | 2 |
| 5 | 2 | 1 | 3 | 1 | 3 | 5 | 8 |
| 6 | 4 | 5 | 9 | 2 | 5 | 6 | 11 |
| 7 | 1 | 3 | 4 | 3 | 9 | 12 | 21 |
| 8 | 2 | 4 | 6 | 4 | 4 | 7 | 11 |
| 9 | 5 | 7 | 12 | 5 | 2 | 3 | 5 |
| 1910 | 1 | 3 | 4 | 6 | 5 | 12 | 17 |
| 1 | 1 | 1 | 2 | 7 | 4 | 5 | 9 |
| 2 | 6 | 2 | 8 | 8 | 5 | 3 | 8 |
| | | | | 9 | 2 | 3 | 5 |
| | | | | 1940 | 6 | 2 | 8 |
| | | | | Totals | 143 | 236 | 379 |
| | | | | Means | 2.6481 | 4.3703 | 7.0185 |

from a paper by Ray (153) for the period 1887 to 1933; for the period 1934–1940 they were obtained from the *Monthly Weather Review*. The storms for the period 1934–1940 include all that are listed as hurricanes in the *Monthly Weather Review* regardless of whether or not they were of hurricane violence. This usage appeared to be in agreement with Mr. Ray's practise. The number of tropical storms that occurred in the region of the West Indies during the period 1887–1940 is divided into two classes according to the area of occurrence; first, those occurring in the eastern Caribbean region and eastward; second, those in the western Caribbean and Gulf of Mexico. The number of storms observed each year is given in Table 28.

In Table 29 the same data are arranged to show the number of years in which the given number of hurricanes occurred.

TABLE 29. CLASSIFICATION OF HURRICANES

| NUMBER OF HURRICANES | NUMBER OF YEARS IN WHICH NUMBER OF HURRICANES OCCURRED *Western Caribbean* | *Eastern Caribbean* | *Total* |
|---|---|---|---|
| 0 | 6 | 0 | 0 |
| 1 | 10 | 8 | 1 |
| 2 | 13 | 9 | 5 |
| 3 | 9 | 9 | 2 |
| 4 | 7 | 5 | 7 |
| 5 | 5 | 8 | 8 |
| 6 | 3 | 5 | 6 |
| 7 | 0 | 3 | 2 |
| 8 | 0 | 2 | 7 |
| 9 | 1 | 1 | 4 |
| 10 | 0 | 1 | 4 |
| 11 | | 0 | 3 |
| 12 | | 2 | 1 |
| 13 | | 1 | 1 |
| 14 | | 0 | 0 |
| 15 | | | 0 |
| 16 | | | 1 |
| 17 | | | 1 |
| 18 | | | 0 |
| 19 | | | 0 |
| 20 | | | 0 |
| 21 | | | 1 |
| 22 | | | 0 |

Except for the western Caribbean region, the distributions are rather poor, that is, the numbers of years for the given numbers of hurricanes are rather small and scattered, and it can be anticipated that efforts to fit frequency curves to those distributions are not likely to be wholly successful. Such proved to be the case when attempts were made to fit them to Poisson's function, for only the first distribution was satisfactorily fitted. Poisson's function was selected because this is an integral heterograde series, so that it becomes the logical function to use because of the type of data. However, the two widespread arrays in the right two columns would suggest that a Gram-Charlier series might be an appreciably better basis. Moreover, there are two modes in the second and third distribution which may be accidental groupings or may indicate a compound distribution, that is, a distribution resulting from more than one principal cause. This situation would require more extended analysis than given here to achieve a satisfactory solution.

The theoretical distribution of the number of years with the given observed number of hurricanes in the west Caribbean region was computed by means of Poisson's function, and the computation and results of the chi-squared test are given in Table 30.

TABLE 30. EXPECTED NUMBER OF HURRICANES IN WEST CARIBBEAN REGION

| NUMBER OF HURRICANES | NUMBER OF YEARS | PROBABILITY | EXPECTED NUMBER | DEVIATION FROM OBSERVED | | DIVERGENCE |
|---|---|---|---|---|---|---|
| 0 | 6 | 0.070740 | 3.82 | +2.18 | | 1.24 |
| 1 | 10 | .187283 | 10.13 | − .13 | | .00 |
| 2 | 13 | .248004 | 13.39 | − .39 | | .01 |
| 3 | 9 | .219020 | 11.83 | −1.83 | | .28 |
| 4 | 7 | .145118 | 7.84 | − .84 | | .09 |
| 5 | 5 | .076949 | 4.15 | + .85 | | .17 |
| 6 | 3 | .034015 | 1.84 | +1.16 | | .73 |
| 7 | 0 | .012892 | .70 | − .70 | | |
| 8 | 0 | .004278 | .23 | − .23 | −.01 | .00 |
| 9 | 1 | .001261 | .07 | + .93 | | |
| 10 | 0 | .000335 | .01 | − .01 | | |
| | | | | | Total | 2.52 |

For $n' = 8$ and $\chi^2 = 2.52$, $P' = 0.92$.

The probability of fit of 0.92 is excellent and from that it may safely be inferred that Poisson's function describes accurately the distribution of number of years with a given number of hurricanes, or in other words, it gives the number of hurricanes to be expected annually.

The frequency with which hurricanes strike a given region may also be computed by means of Poisson's function. An observed distribution consists of the data of the number of hurricanes that have struck the state of Florida during the years 1886 to 1943. The data for the years 1886 to 1930 are taken from a paper by Gray (74); the remainder are obtained from the annual summary of tropical storms published in the *Monthly Weather Review.* A total of 64 hurricanes was recorded over that state during that period, or an average of 1.10 per annum. The computed frequency and the chi-squared test are given in Table 31.

TABLE 31. FREQUENCY OF HURRICANES IN FLORIDA

| NUMBER OF HURRICANES | PROBABILITY OF NUMBER | EXPECTED NUMBER | OBSERVED NUMBER | DEVIATION | DIVERGENCE |
|---|---|---|---|---|---|
| 0 | 0.3329 | 21.3 | 20. | +1.3 | 0.79 |
| 1 | 0.3662 | 23.5 | 21. | +2.5 | .27 |
| 2 | 0.2014 | 12.9 | 9. | +3.9 | 1.18 |
| 3 | 0.0738 | 4.7 | 7. | −2.3 | 1.13 |
| 4 | 0.0203 | 1.3 | 1. | + .3 | .07 |
| 5 | 0.0045 | .3 | 0. | − .3 | 0.30 |
| | | | | Total | 3.74 |

For $n' = 6$ and $\chi^2 = 3.74$, $P' = 0.59$. This probability of the fit is reasonably good.

Of the four small distributions in Tables 29 and 31 of the annual

hurricanes, two have been compared with theoretical distributions computed from Poisson's function; one of the two had an excellent fit and the other had a reasonably good fit. The other two observed distributions were too scattered for the total number of storms to permit fair comparisons with a theoretical distribution. It may be fairly concluded, however, that the frequency of hurricanes could be reliably computed from the selected theoretical frequency functions.

*The Extratropical Cyclone.* An extratropical cyclone is an extensive storm of the middle latitudes in which air from two or more air masses moves in a counterclockwise direction about a center of low barometric pressure. The winds are only approximately rotational and may vary from a tangential to a nearly radially inward direction. The drop in barometric pressure is not so great, the winds not so violent, nor the precipitation, which may be rain or snow, so torrential as in the tropical cyclone. There may, in fact, be no precipitation.

The extratropical cyclone is further distinguished from its tropical prototype in the predominating interaction of different air masses and it is this action that makes the former the principal producer of precipitation in the latitudes of its occurrence, particularly in plains regions. The warm maritime and cold continental air masses approach the center in the general directions of their respective source regions so that precipitation occurs in the sector of the storm area nearest the source region of the warm moist air. This sector in the eastern part of the United States, for example, is the south or southeastern portion of the storm area. In the northwest it is the western portion of the storm area. A schematic portrayal of the interacting elements of an extratropical cyclone is shown in Figure 9. In Figures 10 and 11 are shown the weather maps of two storms, August 6, 1935, and March 17, 1936. These figures show only the barometric pressure and the related fronts with the air masses designated according to the Bergeron classification.

The areal extent of an extratropical cyclone is much greater than that of the tropical cyclone; the diameter of the former may vary from 1000 to 1500 miles. These dimensions vary greatly in different storms and they are likewise variable because of the indefiniteness of the boundaries of the storm. The foregoing distances cover the diameters of the significant isobars which, however, may be beyond the area of the actual atmospheric disturbance.

A number of theories has been advanced for explaining the generation of extratropical cyclones but none has been conclusively proved. Some of the older theories which ascribed the origin of extratropical cyclones to thermal convection have been definitely disproved by observed facts.

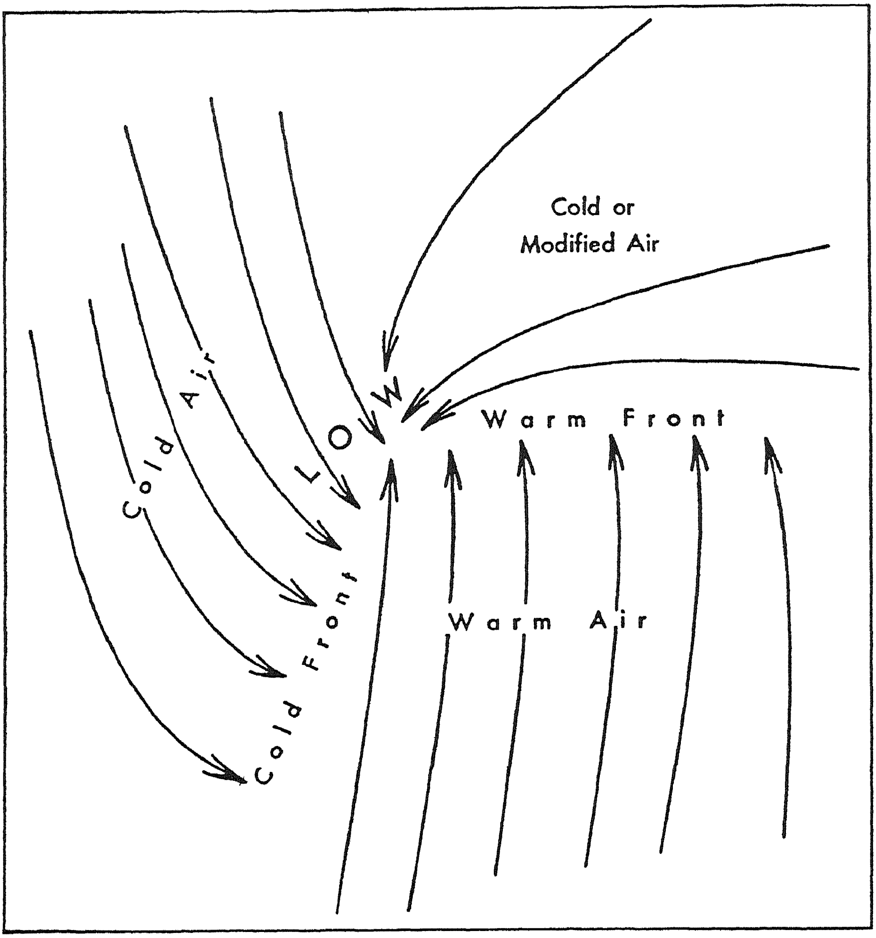



FIGURE 9. Schematic Outline of an Extratropical Cyclone

The latest hypothesis, one that has considerable supporting facts and theory, is the "wave theory" or "air mass" concept which was developed by the Norwegian meteorologists in connection with their study of air-mass analysis. This theory postulates that the extratropical cyclone with approximately rotational movement is generated from atmospheric waves, particularly shearing waves along the fronts or zones of discontinuity between air masses and the inertia waves in the air masses. This theory has not been fully accepted in all sections of meteorology nor have all details of the theoretical development been completely worked out.

The full acceptance of complete details of any one theory purporting to explain the genesis of extratropical cyclones is not necessary for the

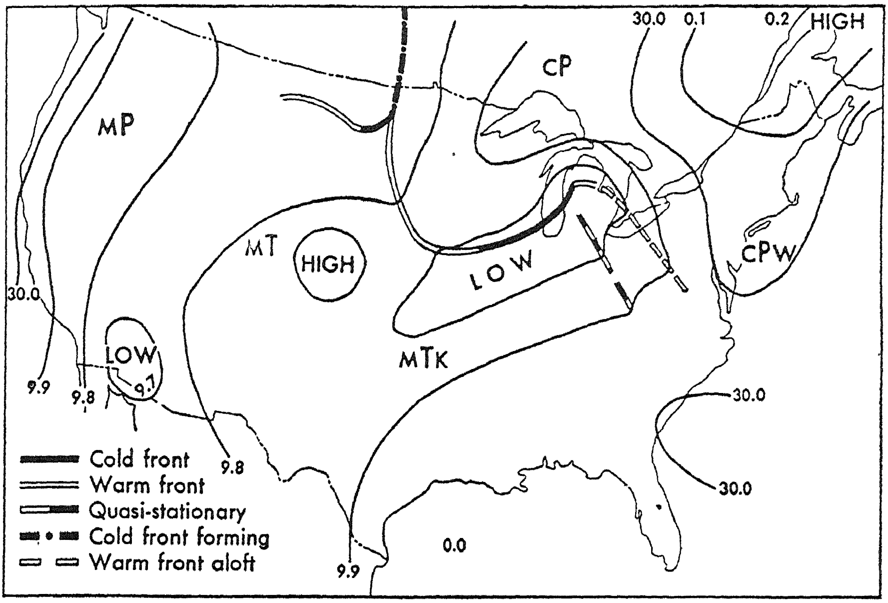



FIGURE 10. Weather Map, August 6, 1935, 8 P.M. (E.S.T.)

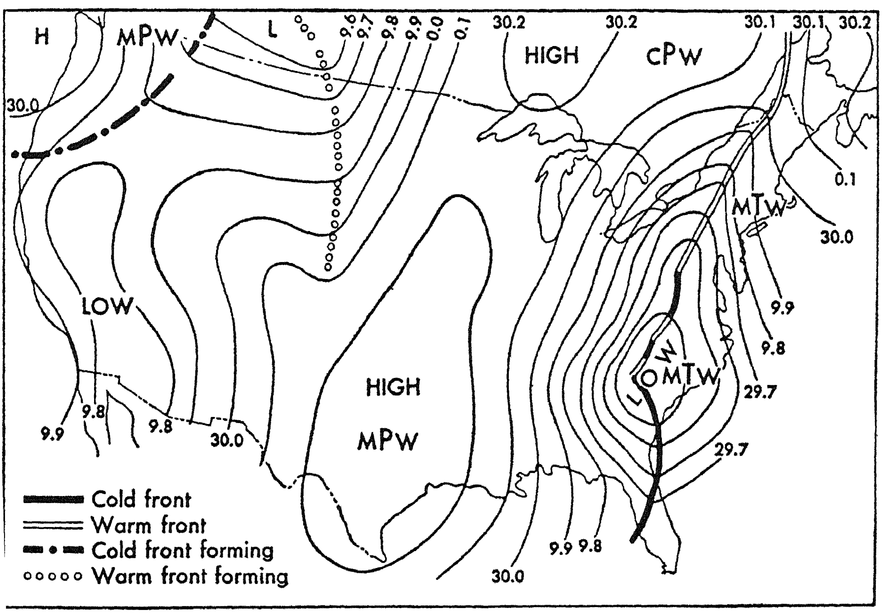



FIGURE 11. Weather Map, March 17, 1936, 8 A.M. (E.S.T.)

study of the effects of such storms. The principal effect to be considered in this discussion is the precipitation during those storms.

For present purposes extratropical cyclones are envisaged as gigantic atmospheric eddies in the areas of convergence of two or more air masses of diverse properties. The air moving out from the centers of high barometric pressure constitutes in itself a great irregular stream of atmosphere traveling in more or less variable directions. As two diverse streams of air meet, they tend to produce or at least accentuate low barometric pressures at the point of convergence and then tend to rotate about that point.

Regardless of the ultimate cause of the formation of extratropical cyclones, it is definitely known that they are areas of convergence of air masses of different properties, in which the warmer, moister, and consequently lighter air is lifted to higher elevations. The lifting causes cooling and condensation of the moisture, and if the moisture and cooling are sufficient, precipitation in varying amounts occurs.

*Regions of Cyclogenesis.* There exist regions in which cyclogenesis, or the generation of cyclones, occurs with greater frequency than is prevalent on the rest of the earth's surface. A continuous observation of the paths of lows in and across North America will reveal that many originate along the eastern slopes of the Rocky Mountains. As those mountains form a partial barrier to the westward movement of both polar continental and tropical Gulf air masses and thereby act as a guide wall forcing those masses to meet along the eastern slopes, it may reasonably be expected that cyclogenesis would occur frequently in that region. Dr. Sverre Petterssen (148) and others have pointed out that the region along the Middle Atlantic States east of the Appalachian Mountains is a region of cyclogenesis. A region of frontogenesis may become a region of cyclogenesis. Byers (27) has shown that the eastern Pacific where the polar continental air masses from Siberia meet those moving up the tropical regions is a region of active frontogenesis. On this evidence the region may be accepted as one of frequent cyclogenesis. Richardson (155) has shown that there exists a zone extending from the Aleutian Islands to Japan, in which cyclones are frequent. There may be many other regions where extratropical cyclones are formed, since they may be generated where air masses of diverse properties may meet.

*Paths of Extratropical Cyclones.* Considerable study has been given to the paths of extratropical cyclones in the North America where a well-developed network of observation stations exists from which ample data have been obtained to facilitate study. Despite much variation in the paths of those cyclones, many investigators have

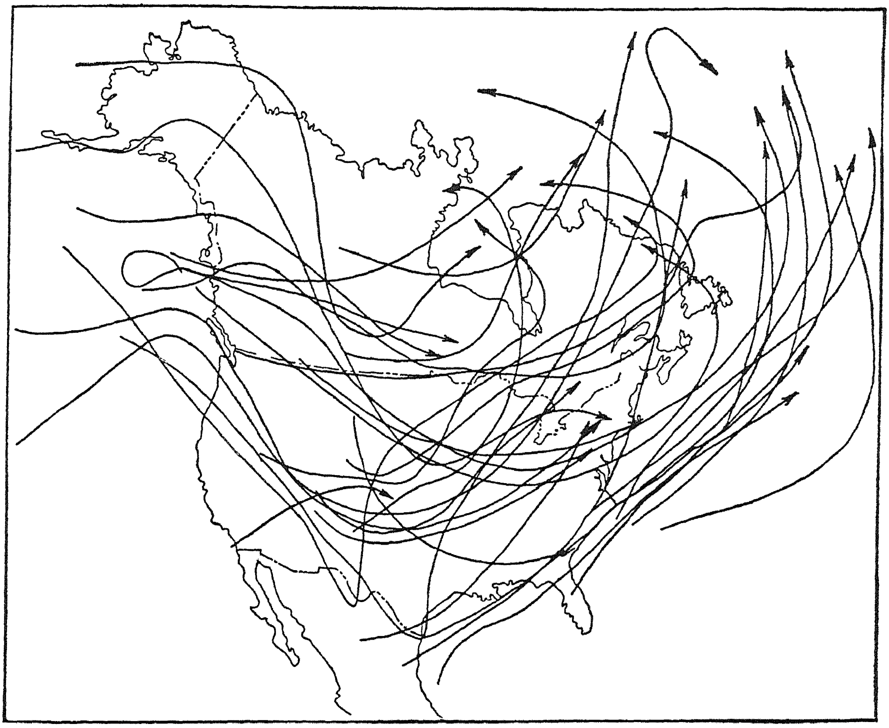


FIGURE 12. Paths of Extratropical Cyclones for January 1938, 1939, 1940

attempted to classify the routes. These classifications bring out distinctly a notable difference in the paths of the lows for various seasons of the year.

Ward (189) shows a system of paths that indicates clearly the seasonal changes in the different seasons. In summer there is one band of paths crossing the continent from British Columbia to Nova Scotia; extratropical cyclones originating along the Rocky Mountains are represented by a system of paths from southern Colorado to join with the transcontinental paths in Ontario. In winter there are three generalized transcontinental paths which enter in the region of Washington and British Columbia: one moves eastward across southern Canada; one turns southeasterly through the central states and thence northeasterly through New England; while the third loops down through the Gulf States and northeasterly along the Atlantic Coast. Two secondary paths represent lows originating in the Rocky Mountains area.

Bigelow arranged a highly generalized system of two main transcontinental paths, connected by crossovers, and a number of secondary paths. Russell (136) published in his *Meteorology* (1895) a system of

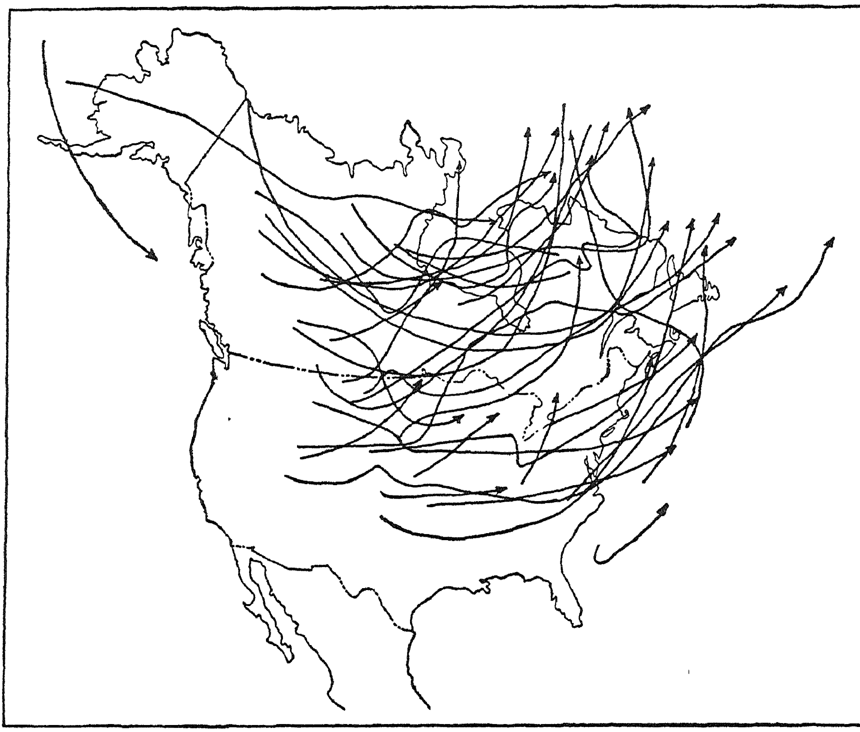


FIGURE 13. Paths of Extratropical Cyclones for July 1938, 1939, 1940

eleven paths covering the eastern two-thirds of the United States. These paths ranged from southern Canada to the north coast of the Gulf of Mexico. Van Cleef classified storm tracks into a system of 27 principal and subsidiary paths. This system also contained paths traveling routes from southern Canada to the Gulf Coast.

The variation in the courses of extratropical cyclones is shown best by a group of actual paths followed by the lows. A number of such paths published in the *Monthly Weather Review* is shown in Figures 12 and 13. In Figure 12 is shown the paths made in January for the years 1938 to 1940. It can be seen that the courses of these storms which enter the continent on the west coast or originate in the northwest vary from southern Canada to the Gulf of Mexico. The paths of the lows during July of the years 1938 to 1940 are shown in Figure 13. These paths, except those of the storms originating in the United States, are restricted to the latitude of southern Canada and the northern portion of the United States; they do not in July swing southward and thence move easterly along the Gulf of Mexico as they do in January.

*Types of Extratropical Storms in the Eastern United States.* These extratropical cyclones are some of the most, if not the most, important

rain producers in the temperate regions. From the foregoing discussion it is evident that the courses taken by successive disturbances are highly variable. There is also considerable variation in their action, so much that it has been found convenient to classify them with respect to their flood-producing capabilities. Precipitation varies greatly, being dependent upon the characteristics of the air masses involved; there may be no precipitation or there may be intense precipitation. Over the eastern half of the United States, storms which produce heavy rainfall may be classified into six types with respect to the meteorological conditions attending the occurrence. These types as distinguished by the U. S. Weather Bureau, are as follows:

1. Thunderstorms in a north-moving Tm air mass. This type may produce heavy precipitation over a small drainage basin.
2. Decadent tropical hurricanes that have entered from the Gulf of Mexico. These storms may cause heavy precipitation over large areas.
3. Quasi-stationary or slow-moving fronts along which horizontal waves may move.
4. Occlusion of an extended tongue of Tm air.
5. Occlusion of deep extratropical cyclones.
6. Rapidly moving extratropical cyclones.

Thunderstorms and hurricanes have already been discussed. The remainder are forms of the extratropical cyclone which have been considered above, but because they are important as flood producers further description is desirable.

*Quasi-stationary Fronts.* In this type of storm a stationary or slowly moving cold front may exist over which a warm moist air mass ascends. Horizontal waves of the air masses involved traverse the front; these waves are manifested at a specific point on the front by shifting wind as a wave passes. The storm is characterized by intermittent heavy and light rainfall which is distributed over a relatively long narrow area. Figure 11 shows the various fronts of a quasi-stationary wave-front storm in the portion over the Middle Atlantic States.

*Occlusion of an Extended Tongue of Moist Air.* This type of storm may occur when a cold air mass moves well southward through the central or west-central parts of the United States behind a low that is passing over eastern Canada. Under this condition a high pressure in the warmer regions to the south or southeast will force a long tongue of warm moist air northward which can be cut off and lifted, that is, occluded on the surface, by the eastward-moving cold air mass, so that heavy precipitation is probable. The entire movement may be materially aided by orographic lifting in mountainous regions.

*Occlusion of a Deep Extratropical Cyclone.* This type of storm is produced by the movement of a cold air mass southward along the east slope of the Rocky Mountains with cyclogenesis of its eastern front. Deep cyclonic currents of air move around the low which travels slowly, usually in a northerly direction. Currents of warm moist air are drawn into the center from the Gulf regions which are forced aloft as the low is occluded. This type of storm is favorable for causing rapid and extensive snowmelt in early spring. A form comparable to this storm is illustrated in Figure 10.

*Rapidly Moving Extratropical Cyclones.* When a mass of cold polar air pushes far south of its normal position and develops cyclonic circulation aloft, there is a pronounced tendency for surface cyclogenesis to occur at some distance east of the center of the cold cyclone aloft. If the cyclones which develop in this manner do not become too large they may move rapidly northward and produce heavy rain in the passage. This happened during the storm of November 1927 in New England. Cyclogenesis had occurred off the coast of North Carolina by 8:00 A.M., November 3, 1927. The storm moved northward, passing New Jersey at 8:00 P.M. on the same date, and had traversed New England by 8:00 A.M.; the total distance traveled in 24 hours was approximately 600 miles.

Storms of the extratropical cyclones are in fact well developed in the portion of North America east of the Rocky Mountains. There is no elevated land mass or mountain range to prevent movement of tropical air masses northward nor that of polar air southward. The region is indeed a veritable playground of air masses.

*Extratropical Storms in Western North America.* Turning now to the portion of the continent west of the Rocky Mountains, quite a different situation is found. Although the region is invaded by the same air masses and traversed by extratropical cyclones, the high plateaus and mountain ranges prevent freedom of movement of the former and effective development of the latter in so far as production of precipitation is concerned. Furthermore, in the middle latitudes the geographical location is such as to eliminate extensive movement of Gulf and Atlantic tropical air masses, and to increase the importance of Pacific air. In the southeastern portion of the United States and northern Mexico tropical air from the east and west invades the region under suitable conditions.

Because of the mountain ranges and elevated plateaus, orographic action is an important factor in the production of precipitation throughout this western region. It is effective in producing rain alike in the region of the prevailing westerlies and in the region southward of those

winds through the action of scattered showers and thunderstorms. The predominant effect of orographic action is indicated by the observation that isohyetal patterns of intense storms sometimes coincide quite closely that of the average annual isohyets. The same topographic features inhibit extensive action among air masses so that full development of extratropical storms does not obtain west of the Rocky Mountains as to the eastward, and therefore the same diversity of type is not found in that region.

*The Sacramento Storm.* The Hydrometeorological Section of the Weather Bureau has, however, distinguished one type of storm that is conspicuous on the West Coast. This type it calls the "Sacramento Storm." Since it is a producer of heavy precipitation, frequently causing disastrous floods, and is experienced over a large portion of the Pacific coast region, it merits some discussion.

The Sacramento Storm occurs in winter when there is a well developed anticyclone over the Pacific Ocean between the Hawaiian Islands and the California coast. Under proper conditions tropical maritime air moves northeasterly from this region of high barometric pressure and blows over the California coastline. This warm moist air is normally unstable, but by its passage over the cooler ocean surface it is cooled somewhat so that its lower strata become more stable until it reaches the mountain slope. There orographic action upsets its stability so that convection is added to the lifting forces and heavy precipitation results.

# 5 DISTRIBUTION OF PRECIPITATION

*Nature and Importance of Distribution of Precipitation.* There are three categories of distribution in which precipitation may be considered: first, there is dispersion over the earth surface which is hereinafter referred to as areal or geographical distribution, and second, the occurrence of precipitation as related to time or to distinct periods which is referred to as time distribution. The third category consists of frequencies which are statistical distributions of magnitude as well as time.

*Precipitation Normals.* In studies of variable data such as those of precipitation some standard is required to which the variations and fluctuations can be referred as a normal. For this standard or normal precipitation, the Weather Bureau has adopted the average over a long period of years. It may be questioned whether or not the average should properly be used if the concept of the normal is taken to indicate the amount of precipitation usually received. For the purpose of representing the amount most likely to be expected, or the usual amount, the median has been proposed by Mindling (138). The question was taken up in 1940 by the American Geophysical Union which established a committee to examine and report on it.

The principal objections to the use of the average as a normal is that it is too greatly influenced by extreme values and that it indicates for all units of time considered, the occurrence of phenomena that in reality may not be observed for the greater portion of the period. The two objections are closely related since an extreme value in one year can be averaged over many and thus show an annual value during years of no occurrence. The average does not in all cases express the usual state or the prevailing conditions. This objection is valid, however, only where an element occurs rarely or in scanty quantities.

Instead of the average, Mindling advocated the use of the median in order to show the prevailing or usual condition of a given element of weather, because it is not greatly influenced by extremes and shows the

value of a weather element at the point of 50 per cent of the time; the value is just as likely to be greater or smaller. The advantages and disadvantages of the use of the median were summarized by the committee of the American Geophysical Union (5) as follows:

*Disadvantages of the Median*

(1) The median, as the method of expressing the normal, is not always the figure which will be representative of the central tendency for all purposes. For certain purposes the arithmetical average is more useful.

(2) A large amount of work will be required to recompute the vast amount of data on record now available. This work would fall largely on the Weather Bureau which has amassed the greater part of the data now in use. The Weather Bureau does not have sufficient assistance to recompute this body of data at present.

(3) Comparatively few people outside the mathematical and engineering profession understand the exact meaning of the median whereas nearly everyone understands how the arithmetical average is computed. If the median was used it would be necessary to instruct those using the data as to the meaning of the median.

(4) When there is an extended number of observations, the arrangement of the data to determine the median is tedious and although no computation is necessary to determine this figure there is no machine on the market which will make the necessary arrangement. It is a simple and quick process to compute the arithmetical average because adding machines are almost universally available.

(5) The sum of the monthly medians for a year does not equal the annual median, whereas the sum of the monthly arithmetical averages equals the annual arithmetical average. The annual median would have to be computed from the annual amounts.

(6) Where more than half the figures in a series are zero, the median would convey the impression that there was a lack of measurable quantity.

*Advantages of the Median*

(1) The median, while not always the figure which will be representative of the central tendency for all purposes, is superior to the arithmetical average in many cases.

(2) The median can be determined by a simple arrangement of the series of observations and no computation is necessary. Where adding machines are not available, the determination of the average is far more tedious than the median.

(3) The median is unaffected by the abnormally large or small values of a series of observations. In the case of precipitation the abnormal values are always in excess of both the median and the arithmetical averages because of the limiting value of zero. The arithmetical average is "strongly influenced by extreme variants in a series of values."

(4) In a series of observations, if there is a greatly outlying value, either real or the result of an error, the median will be less affected than the arithmetical average.

(5) Negative departures of precipitation are of greater frequency than plus departures when the arithmetical average is used as a measure of central tendency. This would not be true when the median is employed.

(6) Those who would make active use of the median as the normal are mainly hydrologists, engineers, meteorologists, etc., who would not have to be instructed as to the meaning of the median. Those who do not know what the word median portrays could learn that as readily as the context of the arithmetical average, normal, or mean. On published data a definition of median could be inserted.

After careful consideration of the foregoing statements it is not convincing that the median should be used in preference to the mean to show normal conditions, although it may be desirable for certain purposes and certain types of data. From a statistical standpoint, however, the mean, being a first moment, possesses a distinct advantage over the median. The computation of the latter does not lead to any result which could be utilized to compute the important statistical parameters. Since "normal" signifies mere accordance with a rule, principle, or norm, not necessarily a "central tendency," the well-established means can very properly serve as a normal, and since the means have a long-established usage for that purpose they will be considered as normals hereinafter in conformity to the practise of the Weather Bureau.

*Annual Distribution.* By "annual distribution" is meant the distribution of precipitation through the yearly cycle. For most purposes the monthly mean totals of a given observation station are used to show the annual distribution. It could be shown more accurately by daily values but for most hydrological purposes the work required would be much too great for the additional accuracy obtained over that reached by the use of monthly values.

For a distribution over a single year, the observed monthly total precipitation is used as shown in Figure 14. This graph may be considered as a bar chart in which width of the bar represents a unit of time or as a form of line chart. For presenting the data of a single year any form of bar or single line chart is suitable. In the type presented, the area is proportional to the total precipitation of the year. Furthermore,, in the case of a graph showing an average annual distribution the horizontal top line of the individual months represents logically the mean monthly precipitation. For these reasons the graph shown in Figure 14 is preferred for mean annual distribution.

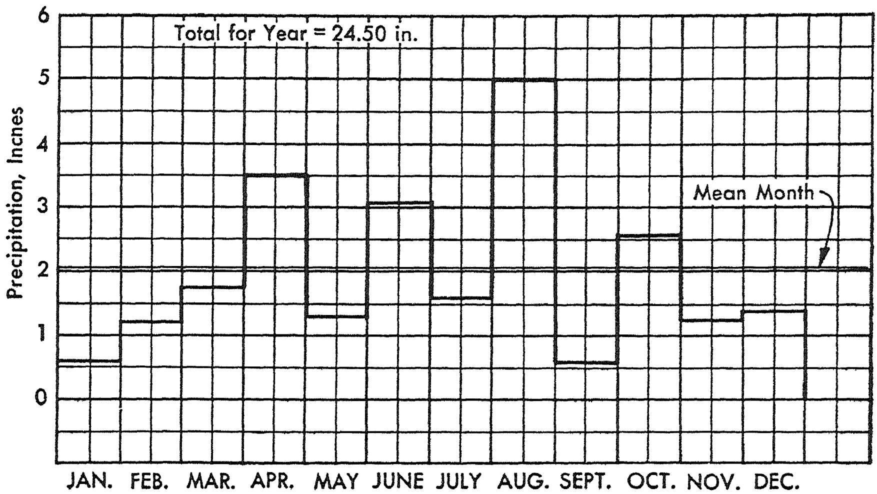



FIGURE 14. Monthly Precipitation, Omaha, Nebr., 1940

*Computation of Monthly Means.* Computation of monthly means involves consideration of variations of individual months from year to year, or statistically speaking, the mean implies variations which entail consideration of the magnitudes of the probable error and of dispersion.

Table 32 contains the data of monthly rainfall for June for the years 1871 to 1940 at Omaha, Nebr.; these data are used to illustrate the computation of means, probable error, and dispersion, or mean standard error.

The mean rainfall is obtained by the usual method of dividing the total by the number of years, thus $316.70/70 = 4.524$ inches. To obtain the deviations the mean, taken as 4.52, is subtracted from the observed monthly data. Better accuracy would be secured by using the third decimal place or making adjustment for it by statistical methods, but the additional labor involved is not considered justifiable for the added refinement. The dispersion, or standard error is computed by the formula

$$\sigma = \sqrt{\frac{\Sigma (x^2)}{N}}.$$

Upon substituting appropriate values of $x$ and $N$ from Table 32 in this formula,

$$\sigma = \sqrt{\frac{486.70}{70}} = 2.64 \pm \text{ inches.}$$

The probable error, $P.\ E.$, equals $0.6745(\pm\sigma)$; then $P.\ E. = 0.6745$ $(\pm 2.64) = \pm 1.78$ inches.

TABLE 32. JUNE RAINFALL, OMAHA, NEBR., 1871–1940

| YEAR | RAINFALL *Inches* | DEVIATION $x$ | |
|---|---|---|---|
| 1871 | 2.65 | −1.87 | 3.4969 |
| 1872 | 3.91 | −0.61 | .3721 |
| 1873 | 5.86 | +1.34 | 1.7956 |
| 1874 | 6.93 | +2.41 | 5.8081 |
| 1875 | 10.95 | +6.43 | 41.3449 |
| 1876 | 3.47 | −1.05 | 1.1025 |
| 1877 | 8.36 | +3.84 | 14.7456 |
| 1878 | 8.48 | +3.96 | 15.6816 |
| 1879 | 4.09 | −0.43 | .1849 |
| 1880 | 3.14 | −1.38 | 1.9044 |
| 1881 | 5.56 | +1.04 | 1.0816 |
| 1882 | 12.05 | +7.53 | 57.7009 |
| 1883 | 12.70 | +8.18 | 66.9124 |
| 1884 | 6.11 | +1.59 | 2.5281 |
| 1885 | 2.67 | −1.85 | 3.4225 |
| 1886 | 1.50 | −3.02 | 9.1204 |
| 1887 | 4.56 | +0.04 | 0.0016 |
| 1888 | 3.86 | −0.66 | 0.4356 |
| 1889 | 5.44 | +0.92 | 0.8464 |
| 1890 | 5.04 | +0.52 | 0.2704 |
| 1891 | 6.66 | +2.14 | 4.5796 |
| 1892 | 1.51 | −3.01 | 9.0901 |
| 1893 | 7.11 | +2.59 | 6.7081 |
| 1894 | 4.74 | +0.22 | 0.0484 |
| 1895 | 4.31 | −0.21 | 0.0441 |
| 1896 | 1.90 | −2.62 | 6.8644 |
| 1897 | 1.43 | −3.09 | 9.5481 |
| 1898 | 5.16 | +0.64 | 0.4096 |
| 1899 | 5.77 | +1.25 | 1.5625 |
| 1900 | 3.07 | −1.45 | 2.1025 |
| 1901 | 5.31 | +0.79 | 0.6241 |
| 1902 | 7.32 | +2.80 | 7.8400 |
| 1903 | 1.31 | −3.21 | 10.3041 |
| 1904 | 3.11 | −1.41 | 1.9881 |
| 1905 | 1.70 | −2.82 | 7.9524 |
| 1906 | 6.30 | +1.78 | 3.1684 |
| 1907 | 4.69 | +0.17 | 0.0289 |
| 1908 | 8.72 | +4.20 | 17.6400 |
| 1909 | 7.54 | +3.02 | 9.1204 |
| 1910 | 0.43 | −4.09 | 16.7281 |
| 1911 | 0.94 | −3.58 | 12.8164 |
| 1912 | 3.09 | −1.43 | 2.0449 |
| 1913 | 2.28 | −2.24 | 5.0176 |
| 1914 | 7.01 | +2.49 | 6.2001 |
| 1915 | 2.83 | −1.69 | 2.8561 |
| 1916 | 2.58 | −1.94 | 3.7636 |
| 1917 | 5.18 | +0.66 | 0.4356 |
| 1918 | 1.80 | −2.72 | 7.3984 |
| 1919 | 4.44 | −0.08 | 0.0064 |
| 1920 | 2.62 | −1.90 | 3.6100 |
| 1921 | 3.57 | −0.95 | 0.9025 |

TABLE 32. JUNE RAINFALL, OMAHA, NEBR., 1871-1940—*Continued*

| YEAR | RAINFALL *Inches* | DEVIATION $x$ | $x^2$ |
|---|---|---|---|
| 1922 | 2.68 | −1.84 | 3.3856 |
| 1923 | 6.00 | +1.48 | 2.1904 |
| 1924 | 9.08 | +4.56 | 20.7936 |
| 1925 | 7.42 | +2.90 | 8.4100 |
| 1926 | 2.01 | −2.51 | 6.3001 |
| 1927 | 1.27 | −3.25 | 10.5625 |
| 1928 | 5.20 | +0.68 | 0.4624 |
| 1929 | 2.94 | −1.58 | 2.4964 |
| 1930 | 1.56 | −2.96 | 8.7616 |
| 1931 | 4.96 | +0.44 | 0.1936 |
| 1932 | 4.79 | +0.27 | 0.0729 |
| 1933 | 0.25 | −4.27 | 18.2329 |
| 1934 | 2.97 | −1.55 | 3.4025 |
| 1935 | 5.25 | +0.73 | 0.5329 |
| 1936 | 3.28 | −1.24 | 1.5376 |
| 1937 | 4.77 | +0.25 | 0.0625 |
| 1938 | 2.03 | −2.49 | 6.2001 |
| 1939 | 5.42 | +0.90 | 0.8100 |
| 1940 | 3.06 | −1.46 | 2.1316 |
| Totals | 316.70 | | 486.6992 |
| Mean | 4.524 | | |

In Table 33 are data of distributions of monthly precipitation at Omaha, Nebr., for all months of the year. The means, standard, and probable deviations have been computed by the method given above or by the shorter method of grouping the data into classes and deriving

TABLE 33. DATA OF DISTRIBUTIONS OF MONTHLY PRECIPITATION OMAHA, NEBR., 1871-1940

| MONTH | MAXIMUM *In Inches* | MINIMUM *In Inches* | MEAN *In Inches* | STANDARD DEVIATION | PROBABLE ERROR |
|---|---|---|---|---|---|
| January | 2.80 | 0.01 | 0.71 | 0.54 | 0.36 |
| February | 3.09 | 0.03 | 0.86 | 0.61 | 0.41 |
| March | 4.91 | T | 1.32 | 0.98 | 0.66 |
| April | 6.34 | 0.23 | 2.46 | 1.45 | 0.98 |
| May | 11.29 | 0.55 | 3.60 | 2.18 | 1.47 |
| June | 12.70 | 0.25 | 4.52 | 2.64 | 1.78 |
| July | 10.35 | 0.45 | 3.69 | 2.61 | 1.76 |
| August | 12.50 | 0.18 | 3.21 | 1.65 | 1.11 |
| September | 9.32 | 0.24 | 3.06 | 2.01 | 1.36 |
| October | 5.86 | 0.07 | 2.15 | 1.43 | 0.97 |
| November | 6.24 | 0.03 | 1.23 | 1.24 | 0.84 |
| December | 3.33 | 0.07 | 0.86 | 0.71 | 0.48 |

the means and deviations by moments. The latter method, although shorter, requires greater familiarity with statistical methods. The two methods are equivalent for all practicable purposes although differences of a few hundredths usually occur.

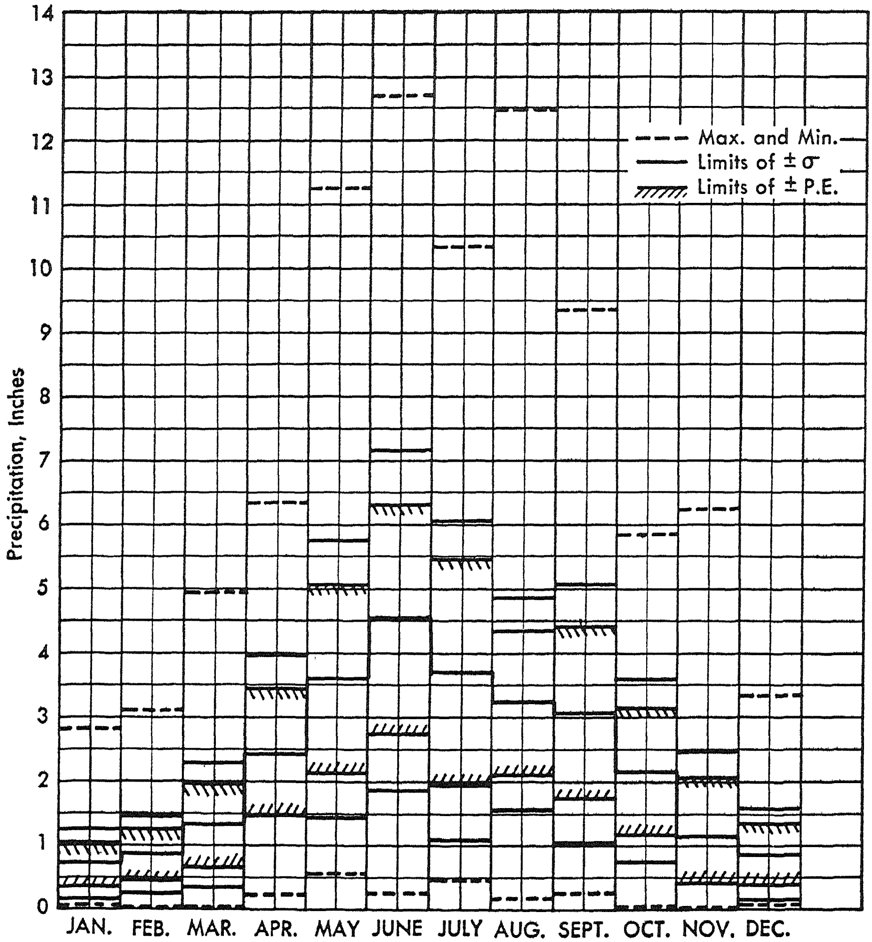



FIGURE 15. Deviations of Mean Monthly Precipitation, Omaha, Nebr.

In Table 33 the standard deviation varies from approximately 51 to 101 per cent of the mean and the probable deviation varies from 35 to 68 per cent. These may be considered representative of the area around Omaha. Regions of less variable precipitation would not show such great percentage variations. In view of the relatively large standard deviation, however, variations of a few hundredths in the mean are decidedly negligible.

In Figure 15, is shown the annual distribution of mean monthly precipitation at Omaha, Nebr. The band chart depicts the standard and probable deviations, the monthly means, maximums, and minimums. This particular chart illustrates the annual distributions of an inland station with a continental type of climate. The greater portion of the precipitation occurs as summer rains, which are a distinguishing

feature of the climate of interior regions sheltered from oceanic influences.

The standard deviations, given in Table 33, of the mean monthly precipitation at Omaha appear to be large in proportion to the means, and in view of the relatively large magnitude of the deviations some question may be raised as to the reliability of the means themselves. The standard deviations and probable errors, of course, apply to the samples used which are composed of the data from 1870 to 1940. The magnitudes of those statistics indicate primarily a large spread of the data from the mean of the sample. As they stand, they do not reflect directly how close the mean of the sample may be to the mean of the universe, which would be the number of years of rainfall possible at Omaha.

The relation of the mean of the sample to that of the universe is obtained from the formula

$$\sigma_m = \frac{\sigma_x}{\sqrt{N}}$$

where $\sigma_m$ is the standard deviation of the universe, and the other symbols are as given heretofore. Substituting the data for June in this formula,

$$\sigma_m = \frac{2.64}{\sqrt{70}} = \pm\, 0.316.$$

From the above result the corresponding probable error is $0.6745 \times \pm 0.316 = \pm 0.213$ inch. These values do not of course give the mean of the universe, but since the standard deviation of 0.316 and probable error of 0.213 are measured from that mean, it may be inferred that the mean of the 70-year sample (4.56 inches) is within those ranges from the mean of the universe. It can be expected that the mean of the universe is between the limits of 4.347 and 4.773 inches. Furthermore, since a range of plus 3 to minus 3 times the standard deviation includes 99.7 per cent of all deviations, it may be inferred that the mean of the universe is practically certain to be within 3.612 and 5.508 inches. The same principles apply to the means of the other months so that the probable limits can be set for the means.

The same reasoning can, of course, be used to determine the limits of the means of the universe of other types of data. Some good common-sense judgment should be exercised in interpreting the concept of universe of hydrological data. The science of geology teaches that the climate of the earth does change over very long periods. The universe of data cannot be extended through such a change as the recent Ice

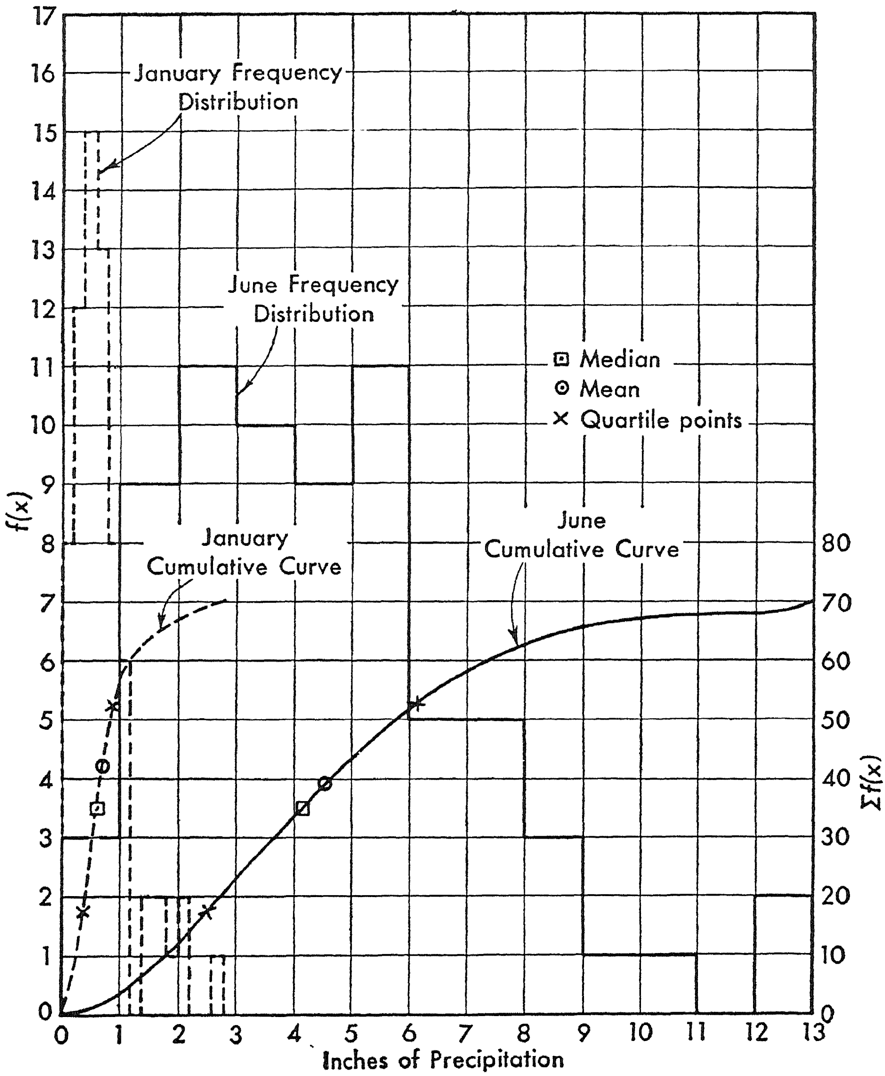



FIGURE 16. Frequency Distribution of Monthly Precipitation, 1871–1940

Age, for example. The term should be limited to such a period that a climate similar to the present one can reasonably be anticipated. This has probably prevailed for one, two, or three thousand years past, which indicates what may be expected in the future.

The standard and probable deviations of Table 33 and Figure 15 have been computed by methods based on the normal probability function. It is now pertinent to see whether or not the frequency distribution of data of monthly precipitation has any skew that would preclude its being considered normal. Frequency histograms have been

plotted in Figure 16 for the months of January and June, the months of maximum and minimum monthly precipitation for Omaha, Nebr. There is plainly some skew to the right, and the data therefore do not follow exactly the normal probability function. The same data have been plotted also in the integral form as the cumulative curves. From these curves the medians and the first and third quartile values have been picked and used to compute the skew by means of the formula

$$S_K = \frac{2(Q_3 + Q_1) - 4M}{Q_3 - Q_1}$$

where $S_K$ = skew; $Q_1$ = first quartile value; $Q_3$ = third quartile; and $M$ = Median.

From these computations the skew of the January distribution was found to be 0.32 and for June 0.19, which values would compare with zero for a condition of no skew. These values of skew are not considered great enough to invalidate the values of the deviations given in Table 33.

*Representative Annual Distribution of Precipitation.* Just as the interior continental regions have these characteristic annual distributions of precipitation, so other regions have theirs. This annual distribution is important in many ways and is a fundamental element of local climate. In Figures 17 to 21, are given some distributions of precipitation of well-known climates, which are described briefly in the following paragraphs.

*Seattle, Washington.* The climate of Seattle illustrates the west coast type of the middle latitudes in the prevailing westerly winds. Since most of the annual rainfall occurs in winter it is marked by wet cool winters and relatively dry summers. See Figure 17 and also Figure 5.

*Red Bluff, California.* Red Bluff represents the well-known Mediterranean climate, a type named for its prevalence around the Mediterranean Sea. It is characterized by hot, dry summers and mild winters. See Figure 17. Since most of the annual rainfall occurs in winter it is similar to the preceding type but differs by higher temperatures and less copious precipitation.

*Denver, Colorado.* The climate at Denver is representative of the subhumid or semiarid interior plains; it is characterized by dry winters and occurrence of small quantities of precipitation which fall principally in early summer. See Figure 17.

*Montgomery, Alabama.* The locality of Montgomery has a mild, humid climate, with abundant precipitation throughout the year. In Figure 18 the precipitation graph shows both oceanic and con-

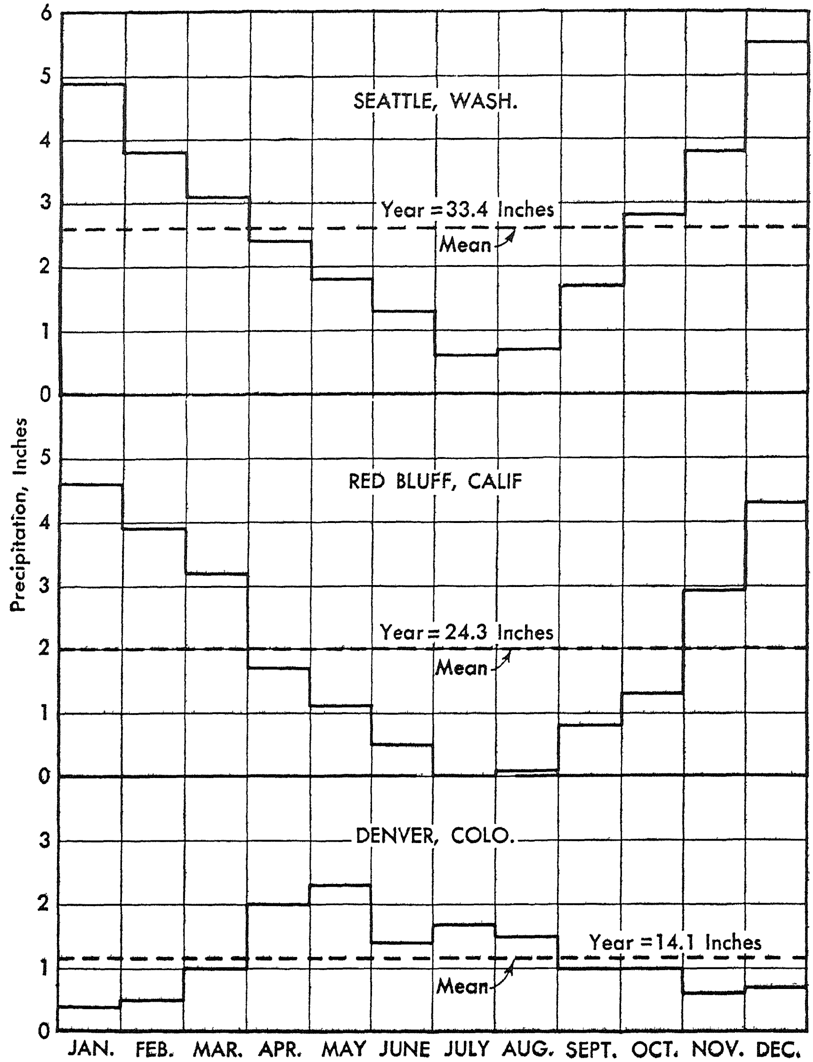



FIGURE 17. Annual Hyetographs of Precipitation

tinental influences, the former by the copious year-round rainfall and the latter by the summer peak. The primary peak, however, occurs in March and is probably due to the prevalence of frontal rains at the southern limit of travel of winter polar continental air masses at a season when tropical air masses become more active.

*Miami, Florida.* Miami has a subtropical climate, warm or mild temperature with abundant rainfall throughout the year. During the summer it is within the belt of the northeast trade winds with a resulting peak of summer precipitation in June; another and larger

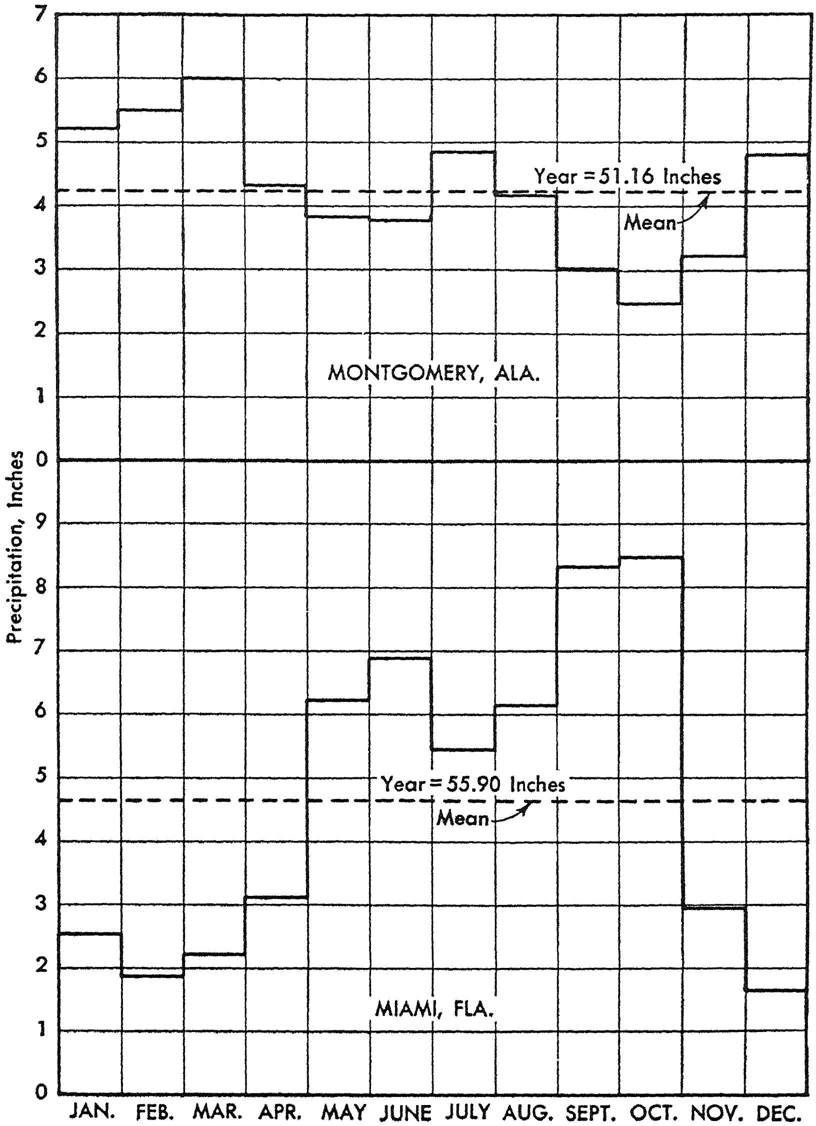



FIGURE 18. Annual Hyetographs of Precipitation

peak occurs in September and October, the months of hurricanes. See Figure 18.

*Boston, Massachusetts.* Boston illustrates an east-coast marine climate having a moderate range of temperature and unusually evenly distributed precipitation. See Figure 19 and also Figure 5.

*Corpus Christi, Texas.* Corpus Christi is another locality with a subtropical climate. Rainfall is moderate and fairly uniform except for a

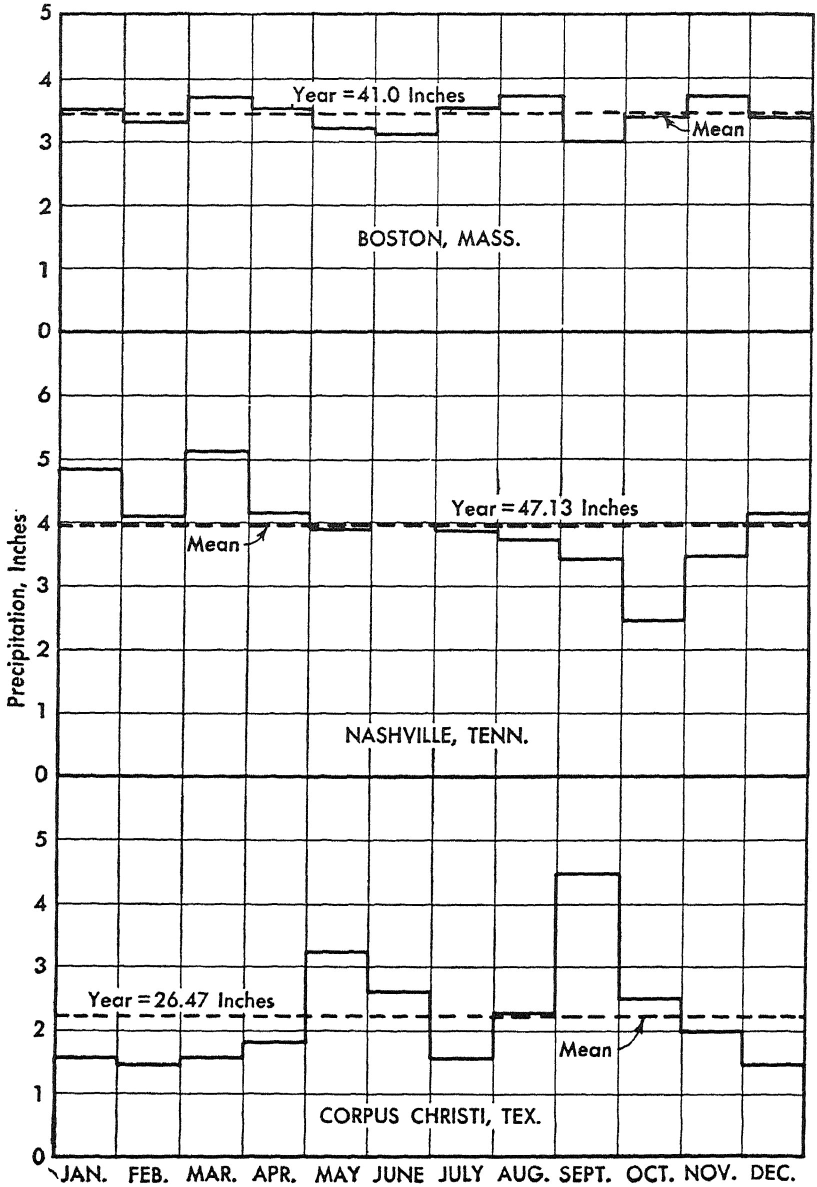



FIGURE 19. Annual Hyetographs of Precipitation

smaller peak in June and the larger one in September, which is the month of maximum hurricane frequency. See Figure 19.

*Nashville, Tennessee.* Nashville represents a moderate humid climate with rather uniform precipitation, although it is located well inland. See Figure 19.

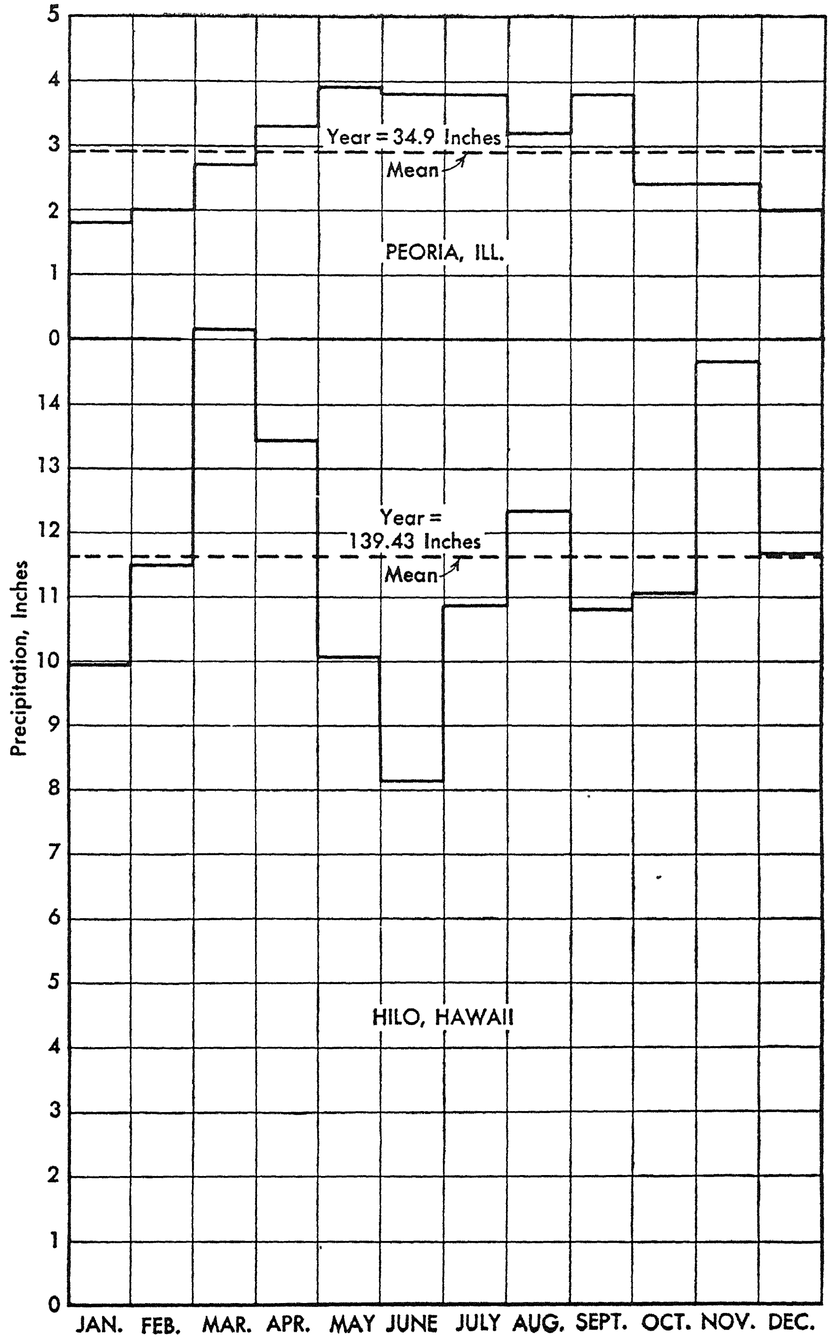



FIGURE 20. Annual Hyetographs of Precipitation

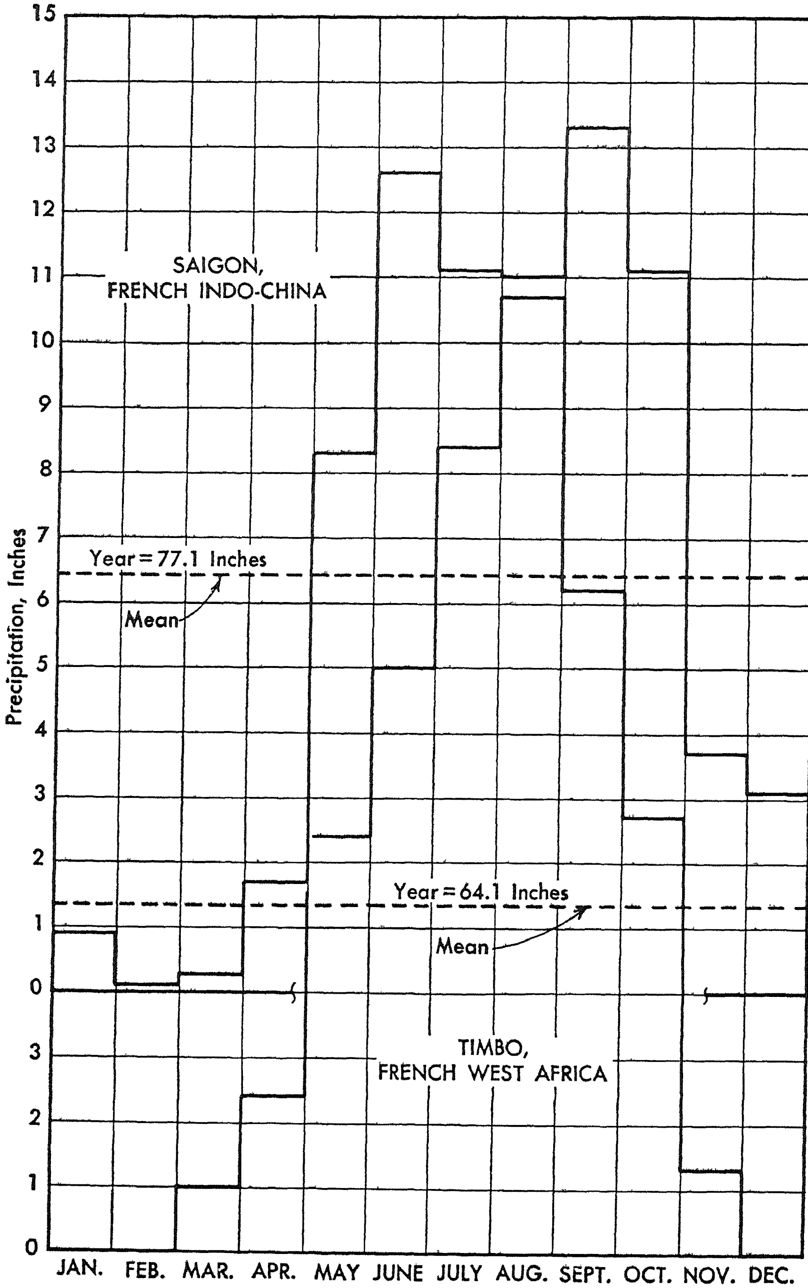



FIGURE 21. Annual Hyetographs of Precipitation

*Peoria, Illinois.* Another interior climate is illustrated by Peoria. It is located farther inland and exhibits (Figure 20) the typical continental characteristic of high summer rainfall.

*Hilo, Hawaii.* The effect of the northeast trade winds is depicted in Figure 20, showing the annual distribution of precipitation at Hilo, Hawaii. Precipitation is distinctly heavy throughout the year with peaks of distribution occurring in March and November. See Figure 5.

*Saigon, Cochin China.* Saigon is in the monsoon region of southeastern Asia and its annual distribution of precipitation (Figure 21) shows distinctly the influences of the monsoon winds. The rainfall is concentrated in the summer months and is heavy, whereas the winter months have comparatively little rain. Saigon, however, is not remarkable for total annual quantity.

*Timbo, French West Africa.* The savannah type of climate, represented here by Timbo, is similar to the monsoon type but the former has appreciably less annual precipitation occurring in a shorter rainy season. The winter, or dry season, is relatively longer and dryer. See Figure 21.

*Diurnal Distribution.* Diurnal distribution of precipitation is variation of precipitation through the day. This distribution over long periods of time in so far as the United States is concerned is fairly uniform. In regions where orographic action, extratropical, or tropical cyclones are the principal causes of rain it is manifest that there could be little average variation through the day, since such storms act irrespectively of the hour. Other storms, being dependent upon local surface heating, as purely convective thunderstorms, will readily affect diurnal distribution if a sufficiently large proportion of the rain is produced by such storms.

The diurnal variation of precipitation and the changes from month to month at Concord, N. H., are shown in Figures 22 to 24, in which the hourly percentage of daily total precipitation is plotted against the hour. The period of hourly records covers the years 1906 to 1932 inclusive. It will be noted that the graphs are irregular, a condition that is undoubtedly due to the fact that a 27-year record is not sufficient to smooth out the irregularities caused by occasional intense precipitation. For example, there is no other apparent reason to account for the heavy precipitation for the hour 11:00 to 12:00 P.M. in November. Likewise, the difference in the distributions for January and February are difficult to account for on other grounds than chance unless there is the possibility of an influx of different air masses in the two months. On the other hand, July, August, and September show a definite rise in the percentage of rainfall received in the afternoon, 1:00 to 8:00 P.M., which may logically be attributed to thunderstorms.

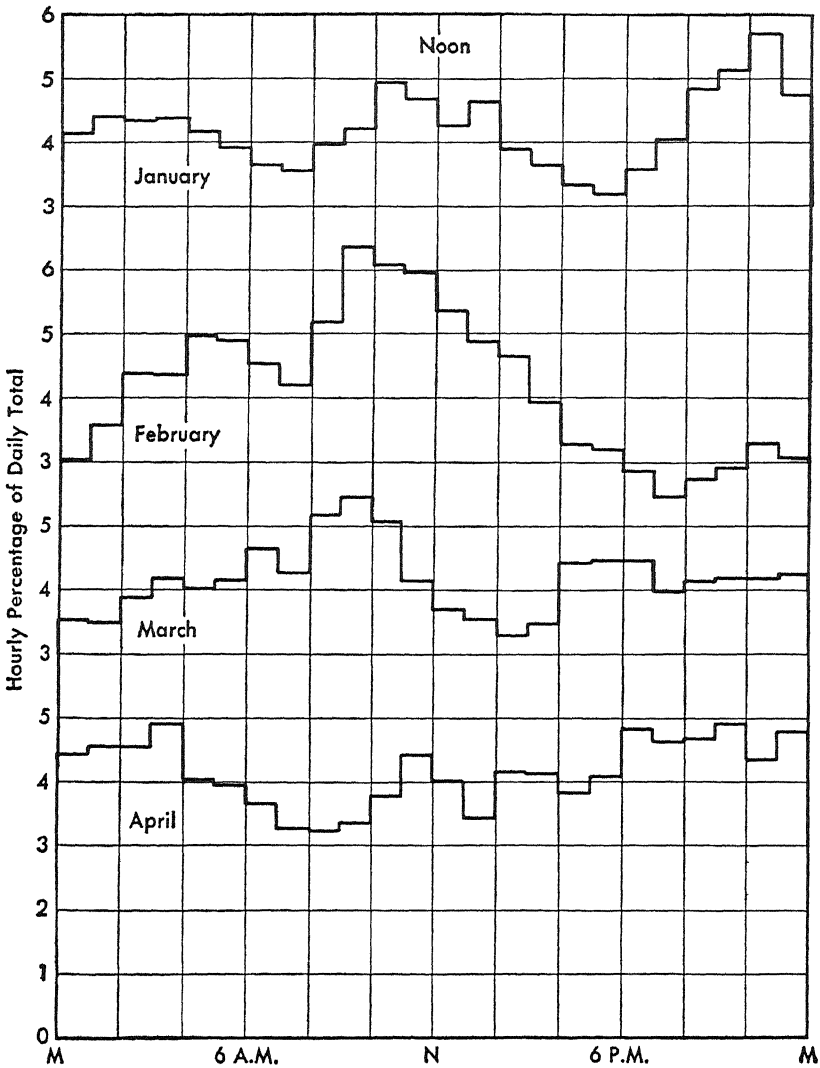



FIGURE 22. Diurnal Distribution of Rainfall by Months, Concord, N. H., 1906–1932

The diurnal distribution of precipitation varies in different regions just as any other rainfall distribution, being controlled apparently by prevailing meteorological conditions. A pronounced difference in winter and summer diurnal distributions exists at Mobile, Ala., as shown by the study of Armstrong (10). As shown by the accompanying graphs (Figure 25) depicting the hourly distribution for January and July, there is a decided peak of rainfall early in the afternoon through the later month whereas in January the rainfall is fairly evenly distributed with small fluctuations. The marked irregularities in the Concord

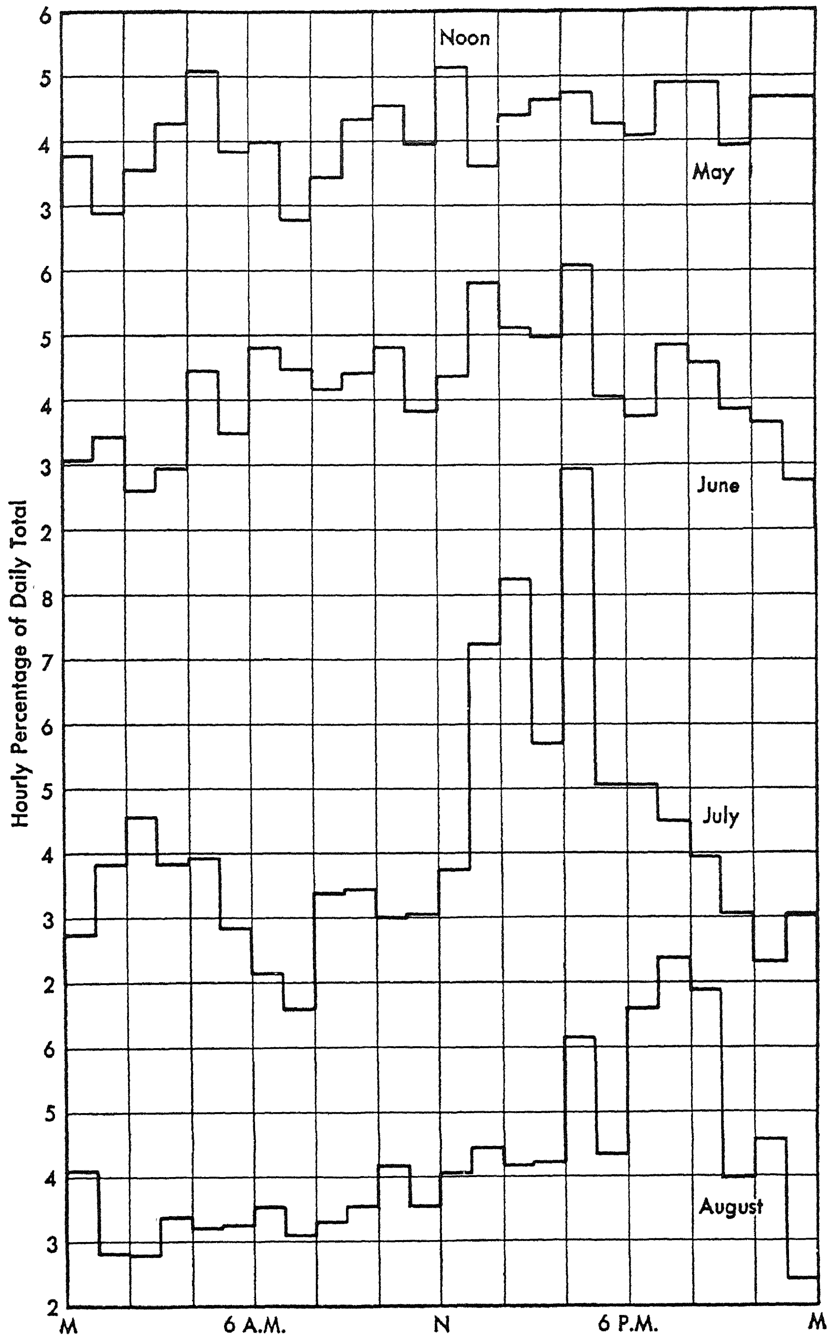


FIGURE 23. Diurnal Distribution of Rainfall by Months, Concord, N. H., 1906–1932

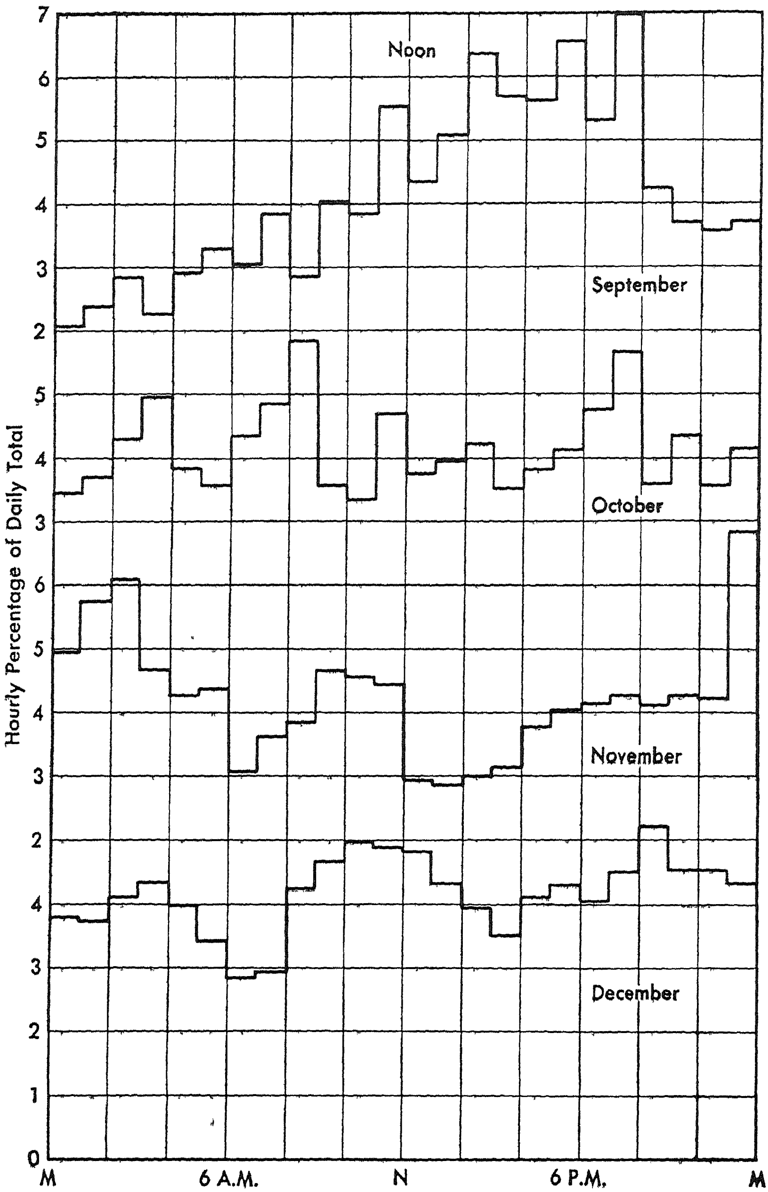



FIGURE 24. Diurnal Distribution of Rainfall by Months, Concord, N. H., 1906–1932

graphs have been eliminated from the data shown (Figure 25) by a method of artificial smoothing by which the value of the precipitation of the hour is doubled and added to the preceding and following hourly values and the sum divided by four. This process would smooth out sharp irregularities without materially affecting diurnal trends.

In the central portion of the United States (in so far as a large area

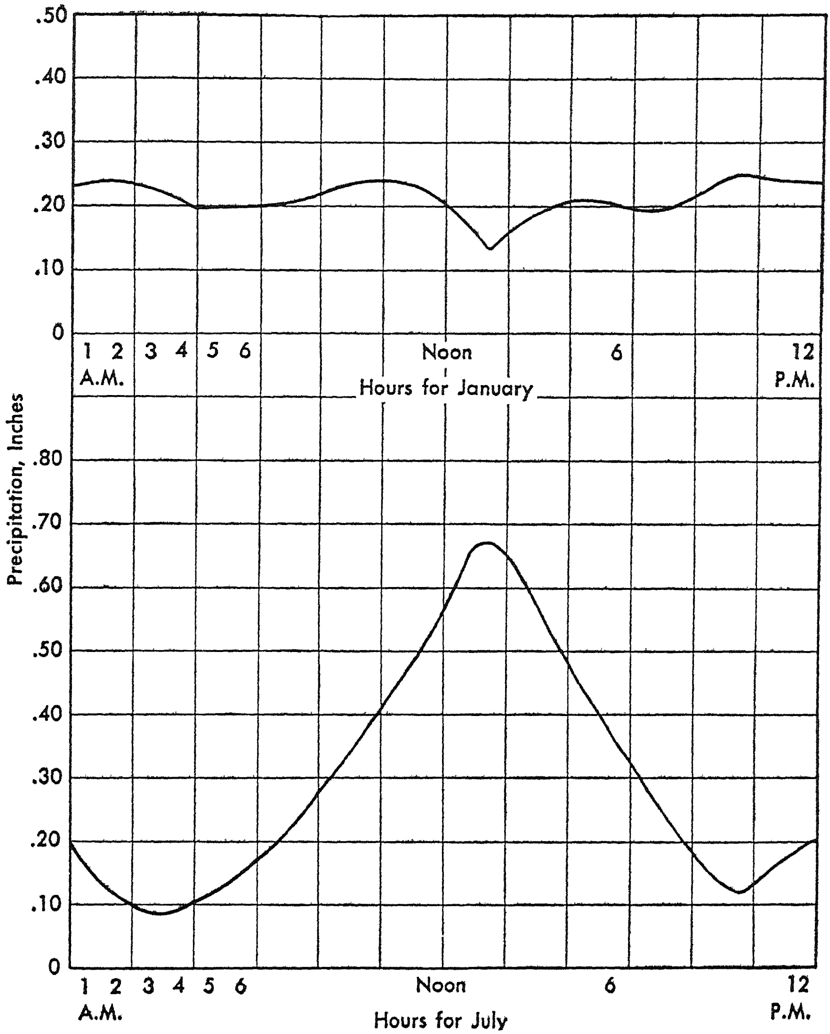



FIGURE 25. Average Diurnal Distribution of Precipitation, Mobile, Ala.

can be represented by two stations) a distinct difference is also shown in the diurnal distributions of winter and summer. During the winter the diurnal distributions observed at Oklahoma City, Okla., (2) and Kansas City, Mo., (79) are sensibly uniform. In comparison with irregularities found in the Concord, N. H., graphs, these two stations are without appreciable variation.

In St. Joseph, Mo., (17), Oklahoma City, and Kansas City, a larger portion of summer precipitation falls at night, being approximately 60, 59, and 59 per cent respectively. On the other hand, night precipitation at Denver, Colo., (38) constitutes only a relatively small proportion

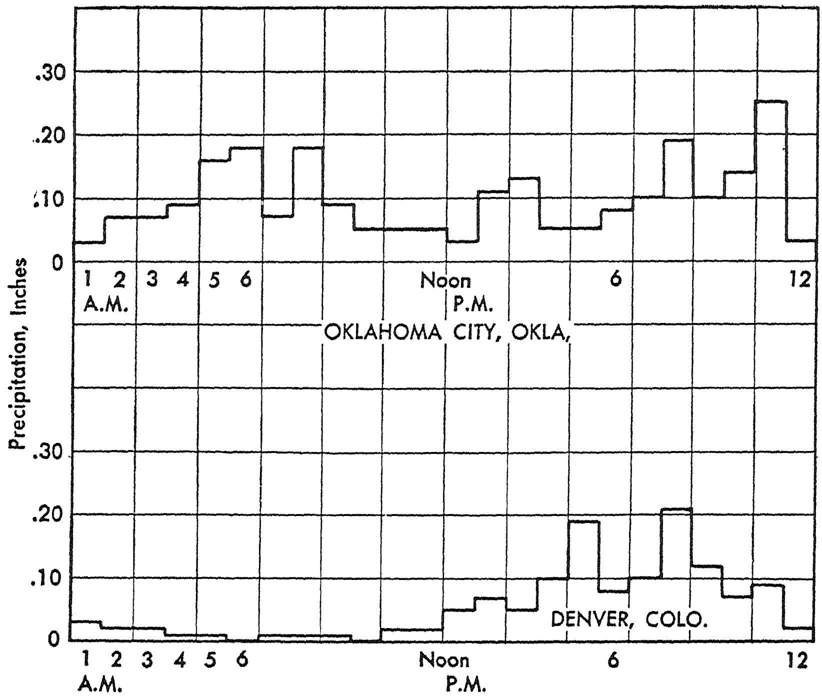
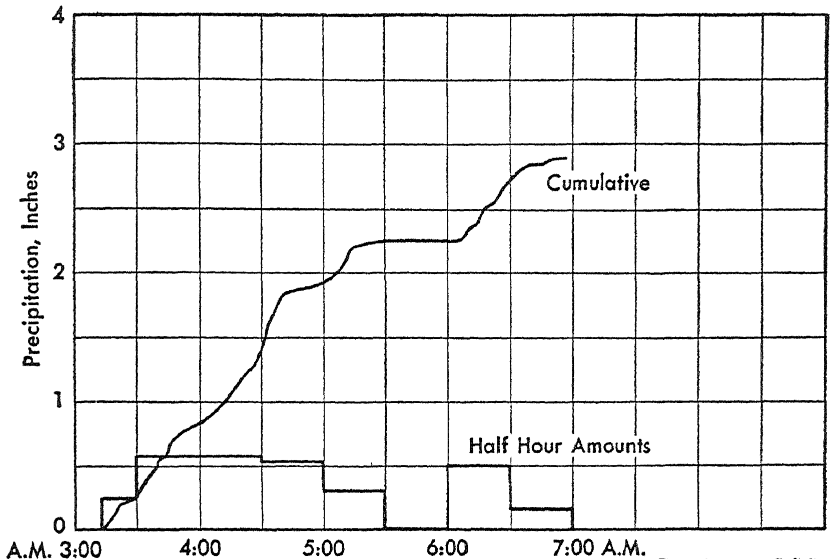



FIGURE 26. Average Diurnal Distribution of Precipitation for July

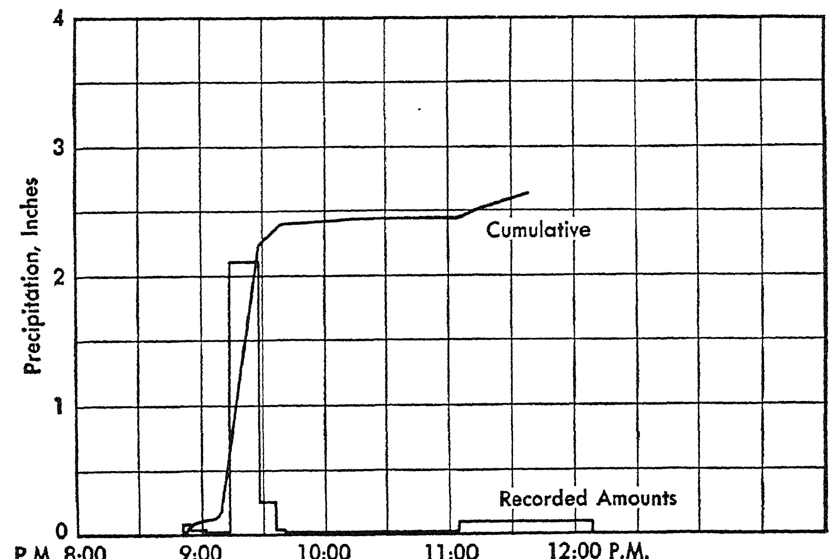
during summer, the larger proportion of rainfall occurring during afternoon and early evening. The daily distribution of rainfall at Denver appears to be closely related to the occurrence of thunderstorms which generally take place during the same hours. In Figure 26 is shown the diurnal distribution of precipitation for July to illustrate the hourly variation at Oklahoma City and Denver.

*Distribution Through Storms.* The distribution of rainfall during storms is a matter of importance in a number of types of investigations including drainage, sewerage design, and studies for design floods of reservoirs. Storm distribution may be studied by comparing the total amount of precipitation in a given time or by rates which express depths in a fixed unit of time; the latter is commonly expressed as inches per hour. Since many of the data obtained by recording gages are published as inches per hour the methods are combined in effect when the unit of time is the hour. The unit of inch and hour may be conveniently used for many purposes but for very small drainage areas it may be necessary to use units of time as small as five or ten minutes. In studies of this sort it is convenient to compare the rate per hour as well as the actual amount. As may be inferred from the number and variation of the factors causing precipitation, its unit distribution




*Data from the S.C.S.*

FIGURE 27. Storm Distribution, June 21, 1931, at Hays, Kan.




*Data from the S.C.S.*

FIGURE 28. Storm Distribution, June 8, 1932, at Hays, Kan.

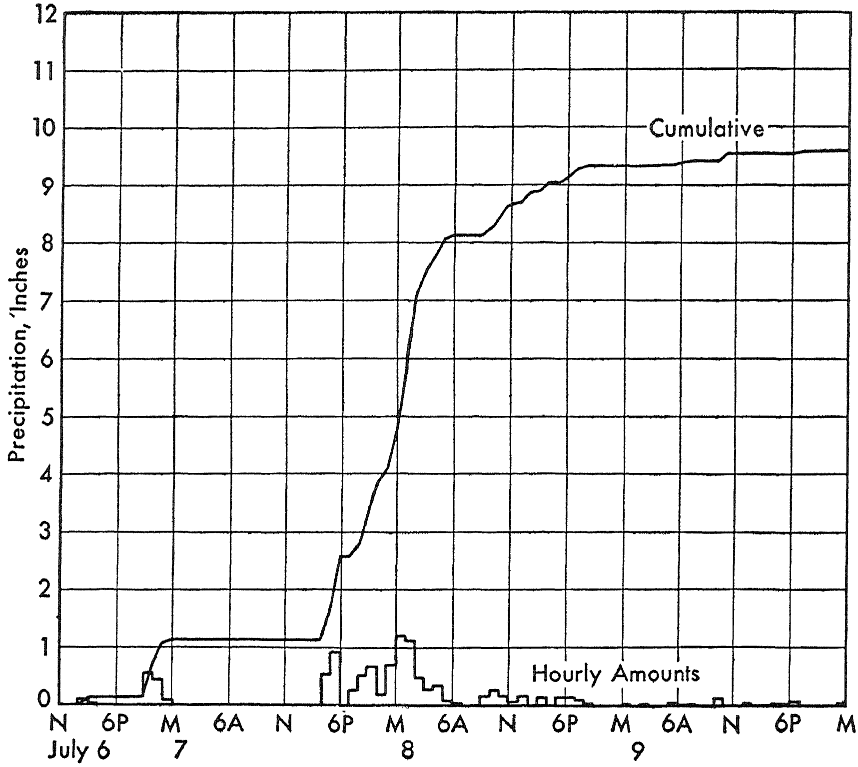



FIGURE 29. Storm Distribution, July 6–9, 1935, Ithaca, N. Y.

through a storm is highly variable. Although storms may be classified into recognizable groups, the distribution of precipitation cannot be readily grouped in similar corresponding classes. The distribution of storm precipitation can be studied best by a number of typical samples.

Figures 27 to 31 inclusive illustrate the distribution of precipitation by the horizontal line graphs through a number of storms selected more or less at random. The storms represented by Figure 27 and Figure 28 were typical summer downpours in Kansas of moderate and not unusual intensity, lacking much of being severe. Although producing nearly the same total rainfall they show considerable difference in hourly rates of precipitation. Figure 29 is constructed from data of the storm in New York State of July 6–9, 1935, which was caused by air-mass frontal action and resulted in a flood that caused extensive damage. The storm of March 17–19, 1936, in West Virginia, shown in Figure 30, illustrates a type of storm caused by an invasion of a warm air mass with extensive frontal action. The New England hurricane of September 21, 1938, and the storms preceding it furnished the data for Figure 31; at no time was the hourly rate of precipitation unusually great but the

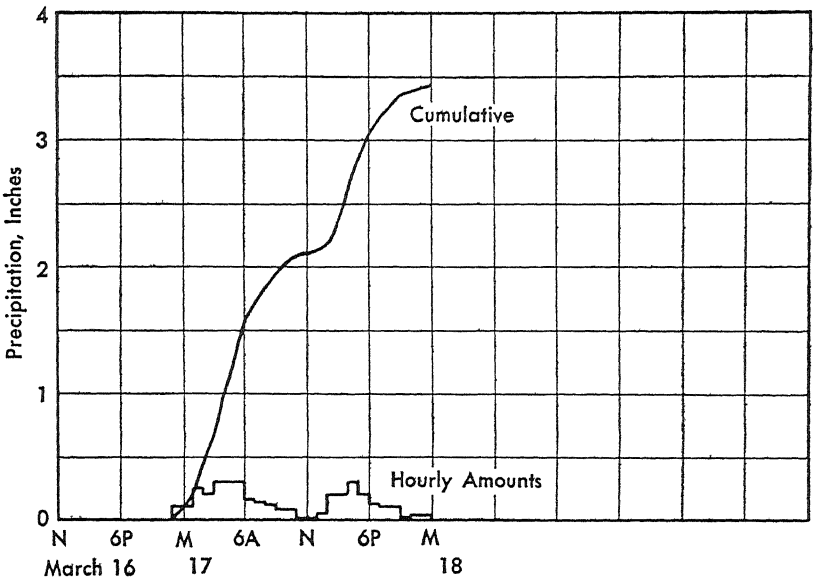



FIGURE 30. Storm Distribution, March 17–18, 1936, Elkins, W. Va.

total rainfall from September 17 to September 21, 1938, exceeded 13 inches and caused the second largest flood of record on many of the streams in that region.

The precipitation of the above storms on Figures 27 to 31 is shown also in the accumulative form. In these curves the rainfall for each period is added to the sum of the rainfall for the previous periods and the total plotted against the time for that period. Let $r$ be the rate of precipitation in any convenient increment of time, $R$ the total depth to the end any given period $T$, then

$$R = \int_{o}^{T} r \, dt.$$

The slope of the curve at any point is the rate at that time. The slope of a line connecting two points is the average rate for the time represented by the difference in abscissa between the two points. While the work is usually performed in distinct increments of time and precipitation, the accumulation is plotted in a smoothly curved line as indicated by the integral.

This method of representing storm precipitation is frequently used in comparing rainfall and runoff. It is especially useful in comparing precipitation distributions at adjacent stations without recording gages to obtain incremental depths. For this purpose the precipitation at a recording station is plotted in a cumulative graph, and the data of

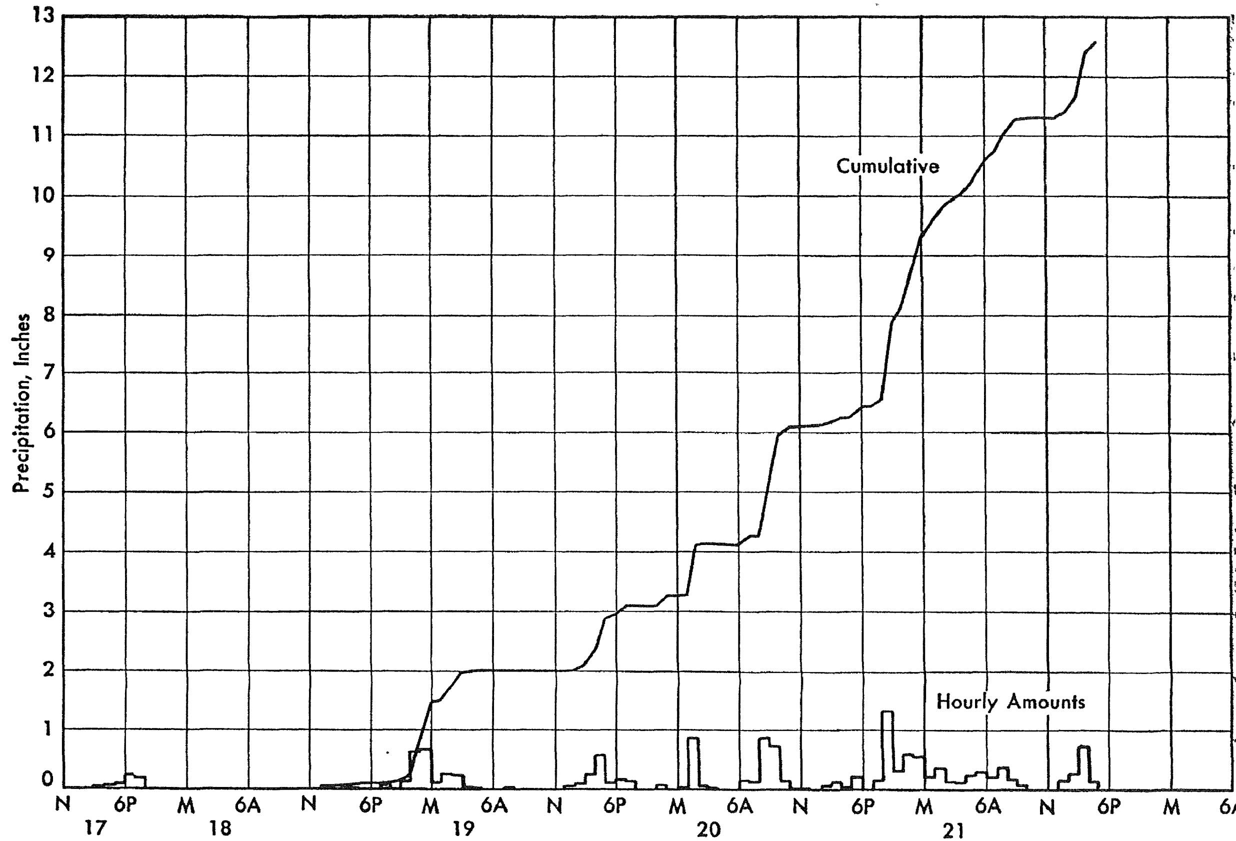



FIGURE 31. Storm Distribution, September 17–21, 1938, Hartford, Conn.

total depth from adjacent non-recording stations are spotted on the same sheet on the ordinate of time at which the observations were made. Assuming similar distribution at adjacent stations, the amounts recorded control the magnitude and the cumulative curves can be approximated for the non-recording station by comparison with the curve of the recording station.

The data plotted in Figures 27 to 31 demonstrate the variability of precipitation during the course of a storm. Such variability is of course to be expected in view of the manifold changes through which a storm may pass.

*Depth-Time Curves.* Analysis of the relationship between time and depth of precipitation is necessary for many uses of storm data. This analysis is achieved by means of the depth-time, or duration-depth, curves, which are constructed by plotting the depths of precipitation against time. In order to construct such a curve it is convenient to start plotting with the adopted unit of time having the largest amount of rainfall and extending the time by suitable intervals to each successive unit having the next greatest amount of precipitation until the entire storm is included. Any period of storm time from the beginning of the summation thus includes the greatest amount of precipitation for the given time.

In Figures 32 and 33 depth-time curves have been plotted from the precipitation data observed at one station for each of the storms of July 1935, March 1936 and September 1938. In the first figure the data were plotted on rectangular coordinates for the purpose of illustrating the relationship between time and depth and to show the effect of the changing phases of the storm. Logarithmic coordinates were used for Figure 33 in an effort to show the relationship curve as a straight line; this effort succeeded only for the third storm, the curve of which approximates a straight line.

In Figure 34 is shown the depth-time curves of the four most intense storms experienced at Kansas City, Mo., as reported by Jones (102). These curves on logarithmic coordinates show a distinctly straight segment in the lower left portions; curvature begins in the latter parts of the storm apparently as the supply of moist air diminishes. Also on Figure 34 is the curve of depth versus time of the excessive precipitation at the Kansas Weather Bureau stations during the period 1896 to 1929. This curve plots approximately a straight line.

In view of the tendency in the lower portion of the depth-time curves to form a straight line on logarithmic coordinates, a question arises as to how far this tendency would go if there were an abundant supply of moist air for an unlimited time. This question has received a sub-

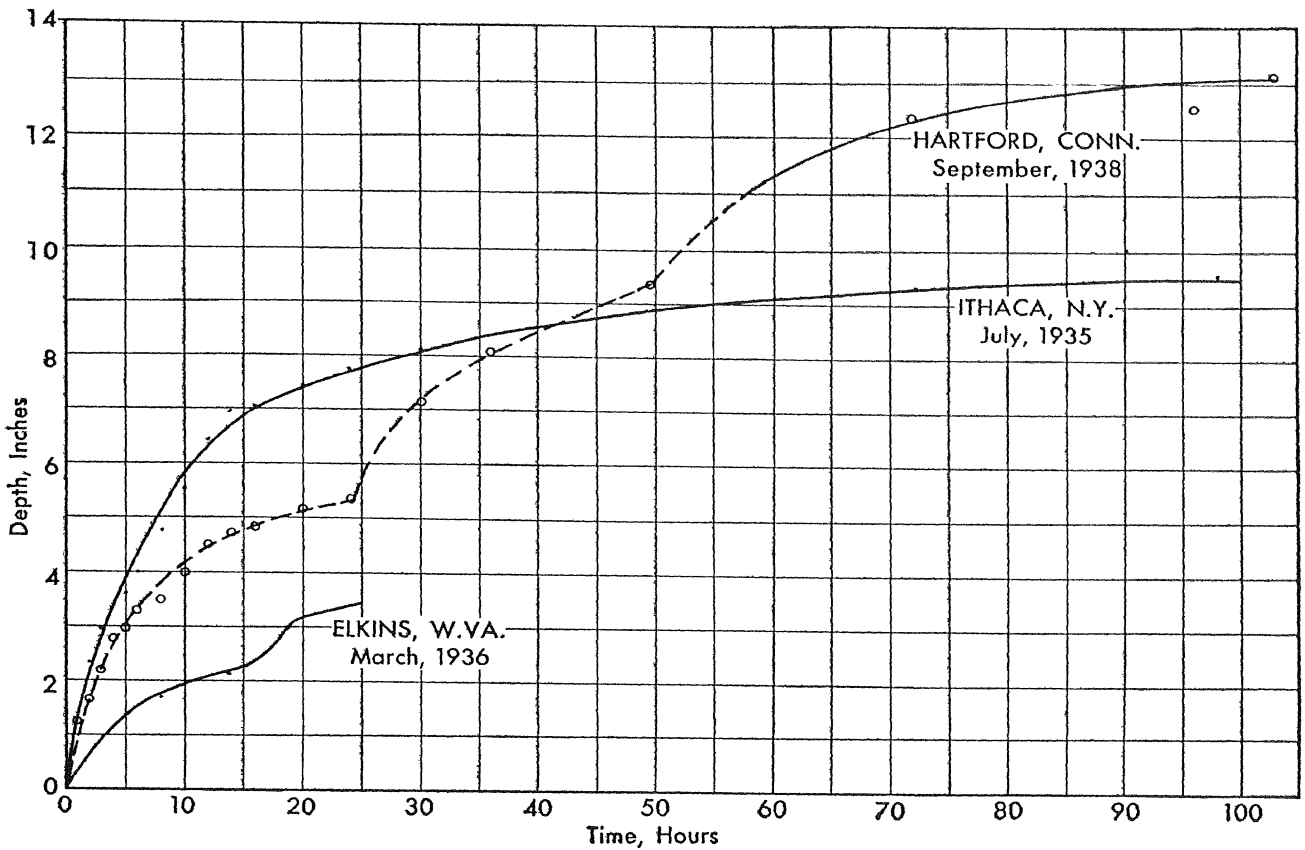



FIGURE 32. Depth versus Time for 3 Storms (Rectangular Coordinates)

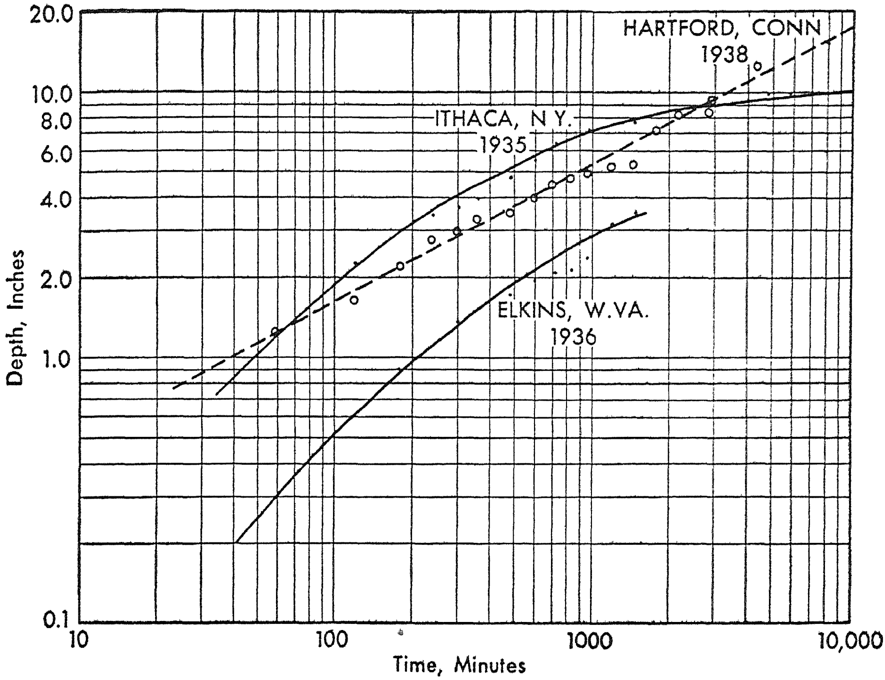
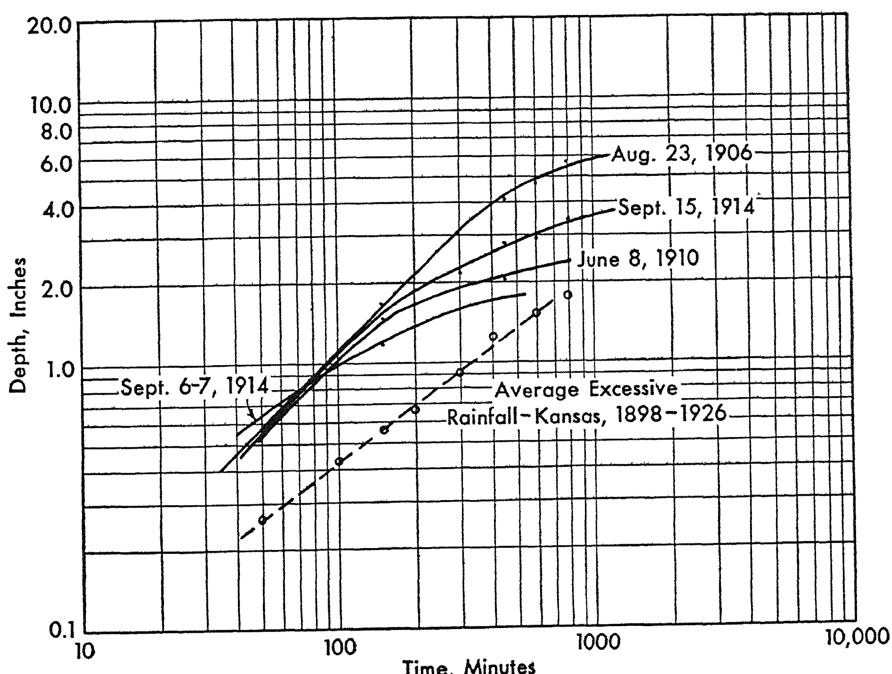



FIGURE 33. Depth versus Time for 3 Storms (Logarithmic Coordinates)




*Data from J. O. Jones*

FIGURE 34. Depth versus Time of Intense Rain, Kansas City, Mo. and Kan.

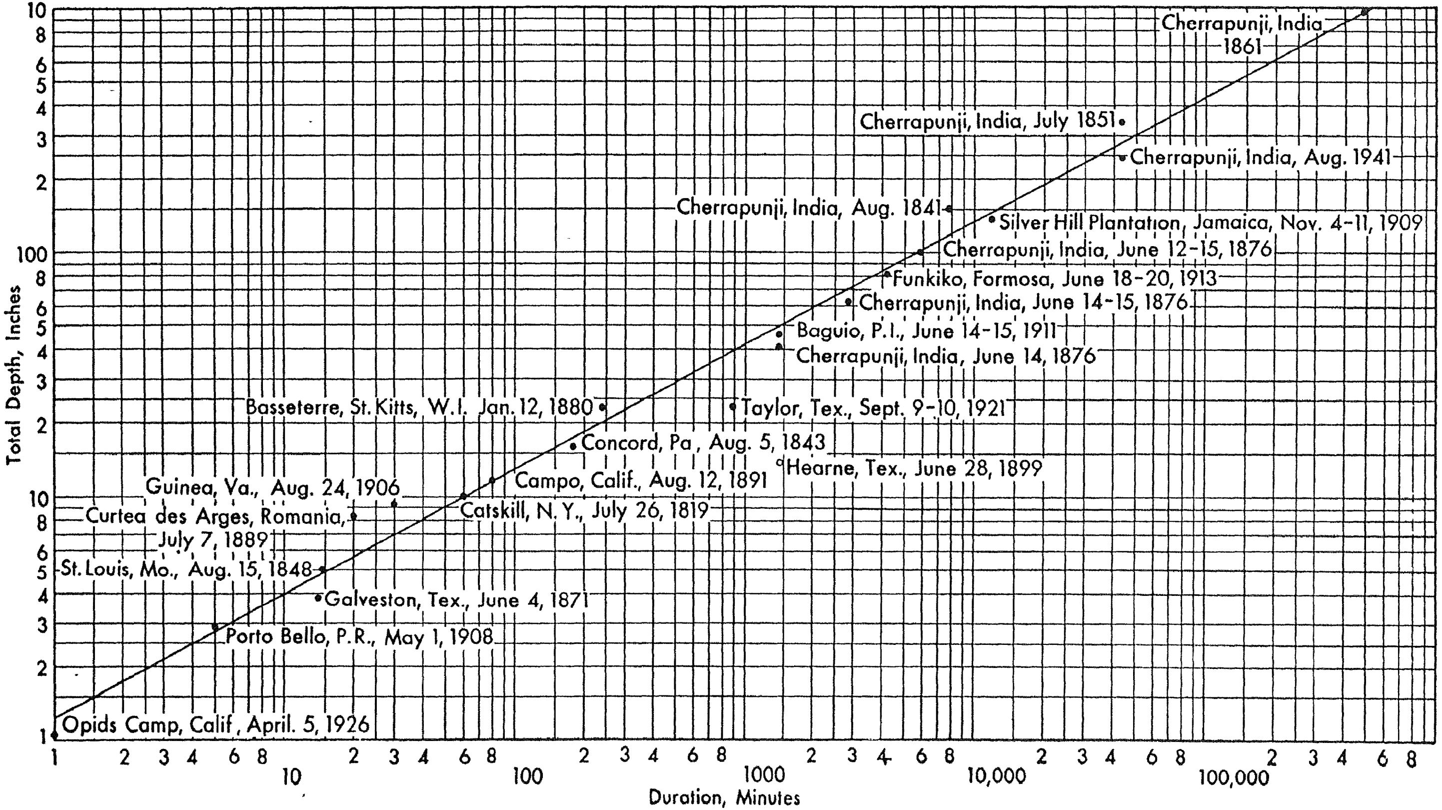
stantial answer in the data compiled and published by the Hydrometeorological Section of the Weather Bureau (97). Data of depth and time of the most intense rainfalls ever recorded were compiled from many parts of the world and were plotted on logarithmic coordinate paper. The resulting curve is shown in Figure 35. The curve is clearly a straight line as plotted, and although there is some scattering, the data follow the line closely for periods of time ranging from one minute to one year. Although the data for this figure came from many places, only those exposed to an ample supply of warm moist air plot near the curve. It can be noted that the places experiencing the longer intense rainfalls are located in the tropics subject to hurricanes or the monsoons of southern Asia.

The depth-area curves are used for determining the probable maximum precipitation to be expected for flood control, drainage, storm sewers, and similar purposes. They are therefore compiled from maximum observed depths regardless of whether or not the depths occurred in the same storm. These curves are, in fact, envelope curves intended to cover practically all observed rainfalls. They are frequently constructed to show the maximum for specified periods of time, so they are combined with frequency curves.

*Variations in Annual Precipitation.* Annual precipitation varies greatly from year to year from the many causes that affect the prevailing climate. This variation depends upon the type of precipitation of a given locality, being relatively greater in the more arid regions or on the interiors of continents. In regions of scanty rainfall this variation is in itself a matter of economic importance, since agriculture prospers or fails as rain is ample or scanty. Where the design of costly projects or the planning of operations depend for success on the evaluation of climatic conditions or hydrologic data it is of utmost importance that the soundness of the basic data be carefully examined and analyzed by the best available methods. Particularly, the variability of the means and possible range of variations should be known.

The data of annual precipitation can be treated as other data and may be examined or analyzed by methods of statistics. The average annual precipitation is a well-known and accepted quantity. The standard deviation $\sigma$ is an indication of the variability of data and is applicable to values of annual precipitation. It may be readily computed from the formula

$$\sigma = \sqrt{\frac{\sum x^2}{N}} \text{ or } \sigma = \sqrt{\frac{\sum (X)^2}{N} - M^2}$$




From the *Monthly Weather Review*

FIGURE 35. World Record Rainfalls

where the notation is the same as used heretofore. The standard deviation of the mean likewise can be computed by the formula

$$\sigma_m = \frac{\sigma_x}{\sqrt{N}};$$

from this the probable location of the mean of the universe can be found.

It should be noted that for comparative purposes the coefficient of variation is better than the standard deviation because in the former the effect of the difference in the magnitude of the mean is eliminated. This elimination is desirable for precipitation data in more arid regions where the annual precipitation tends to be more variable as well as smaller. Table 34 illustrates some data of annual precipitation in humid, subhumid, and semiarid climates with comparisons of standard deviations and coefficients of variation.

TABLE 34. CHARACTERISTIC STATISTICS OF ANNUAL PRECIPITATION AT SELECTED STATIONS

| STATION | YEARS OF RECORD* | MEAN ANNUAL PRECIPITATION | STANDARD DEVIATIONS | | COEFFICIENT OF VARIATION |
|---|---|---|---|---|---|
| | | *Inches* | $\sigma_x$ | $\sigma_m$ | *Per Cent* |
| Bismarck, N. Dak. | 66 | 16.26 | 4.06 | 0.50 | 24.9 |
| Cheyenne, Wyo. | 70 | 14.61 | 3.61 | .43 | 24.7 |
| Iowa City, Iowa | 70 | 35.26 | 5.04 | .60 | 14.3 |
| Kansas City, Mo. | 63 | 36.10 | 6.64 | .84 | 18.4 |
| Mexico, Mo. | 63 | 38.62 | 6.87 | .87 | 17.8 |
| Pierre, S. Dak. | 49 | 15.85 | 4.01 | .57 | 25.3 |
| Pueblo, Colo. | 52 | 11.51 | 5.29 | .73 | 46.0 |
| Omaha, Nebr. | 70 | 27.71 | 7.25 | .87 | 26.1 |
| Sheridan, Wyo. | 47 | 15.00 | 4.06 | .59 | 27.1 |
| Sioux City, Iowa | 59 | 26.41 | 6.77 | 0.88 | 25.6 |
| Wichita, Kans. | 52 | 29.22 | 7.68 | 1.07 | 26.3 |

* Full years of record, period ending 1940.

The first three columns are the statement of station name, years of record, and means of the samples. The standard deviation $\sigma_x$ of the sample consisting of the precipitation of the number of years of record, measures the dispersion or scattering of the observed data from the mean of the sample; the bigger the value of $\sigma_x$, the greater is the range of the observations above or below the mean.

The standard deviation of a distribution of data which approximates the normal symmetrical probability function also provides a means of testing the value of the individual years. A range of three times the standard deviation (see Figure 1) includes 99.7 per cent of

all observations. Then if the value of some observation (annual precipitation in this case) exceeds $3\sigma_x$, the chances are 1 to 333 or larger, that the observation is not due to chance variation, and it may logically be assumed to be caused by some blunder and should be discarded unless a thorough search establishes beyond any doubt the authenticity of the observation. For example, the annual precipitation at Omaha for the year 1869 is recorded as 64.52 inches. This gives a variation of 36.19 inches from the mean of 28.33 inches, which includes the year of 1869. This variation is approximately 4.3 times the standard deviation of 8.46 inches. The value for the year 1869 was therefore discarded from the computations used in this volume because it is *probably* erroneous, its chances of being correct being only approximately 9 in 20,000.

The standard deviation of the mean $\sigma_m$ of the universe can be used to indicate the probable range in value in which the mean may be expected to fall even though the value of the mean itself cannot be known. The reasoning followed to reach this conclusion is the same that was previously used to fix the probable location of the mean value for the precipitation of June at Omaha. The mean for Bismarck, for example, may be expected to lie between 14.76 and 17.76 inches.

The coefficient of variation, being a dimensionless value, provides a method of comparing on a common basis the variability of precipitation at various stations. The effect of the variation of the magnitudes of the means as well as the unit of measurement is eliminated. As indicated previously, it can be seen that stations with scanty precipitation tend to have greater values of these coefficients, but the tendency is not uniform.

*Fluctuations in Annual Precipitation.* Precipitation is subject to fluctuations of greater or shorter periods and to trends which may or may not be downward or upward portions of a very long fluctuation. These fluctuations are frequently referred to as cycles or cyclic changes; these terms, however, connote some regularity of change with time. This regular cyclic change cannot, however, be verified at present and for that reason the term "fluctuation" is used to denote a recurring increase or decrease with time without implying any regularity.

Trends are taken to be an average change in one direction. They are presumably an upward or downward change in precipitation, the extent and reversal of which are not known because of the shortness of our record.

Fluctuations and trends of precipitation may be the result of some unknown or undeterminable cause or may be due to accumulated

accidental or random variations. Long-time changes in precipitation may be superficially classified in three groups, namely,

1. Accidental or random fluctuations
2. Cyclic fluctuations
3. Trends

*Random Fluctuations*. Since there is much variation in the annual precipitation it may be expected that consecutive years will produce similar as well as dissimilar amounts. A chance event may happen singly or in random groups. That is, to use Fry's (68) expression, the number of units of precipitation may occur "collectively at random." This random occurrence must, however, be considered happening as variations about the mean which is controlled by the governing climatic factors of a given locality.

In order to compare a series of values obtained wholly by chance with a precipitation record, the data obtained from a succession of four-card "hands" are plotted in Figure 36. These data were obtained by dealing from a deck of playing cards from which the face cards and aces had been discarded, leaving all cards with 2 to 10 points; 4 cards therefore could have a value from 8 to 40 inclusive. The cards were returned and the deck reshuffled after each drawing. Seventy drawings were made. It was intended in the series to obtain data comparable in value to the precipitation at Omaha as expressed in inches, but the mean is somewhat less.

The data of the 70 drawings are not tabulated here but are shown graphically in Figure 36. The theoretical mean of their values is 24 while the computed mean is 24.73. The standard deviation of the sample of the universe is 0.60; the coefficient of variation is 20.2 per cent. The trend computed by the method of least squares is shown on the graph and there is practically no change from the mean. It will also be noted that the data show groups of highs and lows. This fluctuation can be ascribed only to chance since there is no other conceivable cause of such variation.

One example does not prove a case, but some comparison can be drawn between the random series and the statistical values of annual precipitation in Table 34. The standard deviations appear consistent with those in the table, being very close to that of Iowa City although the mean is less. The coefficient of variation is comparable to the group of Table 34. These comparisons indicate that chance probably is a large factor in the variations shown in Table 34.

*Cycles of Annual Precipitation*. By the term "cycles" the more or less regular recurrence of similar amounts of annual precipitation is understood. The variation from the mean should be similar in magni-

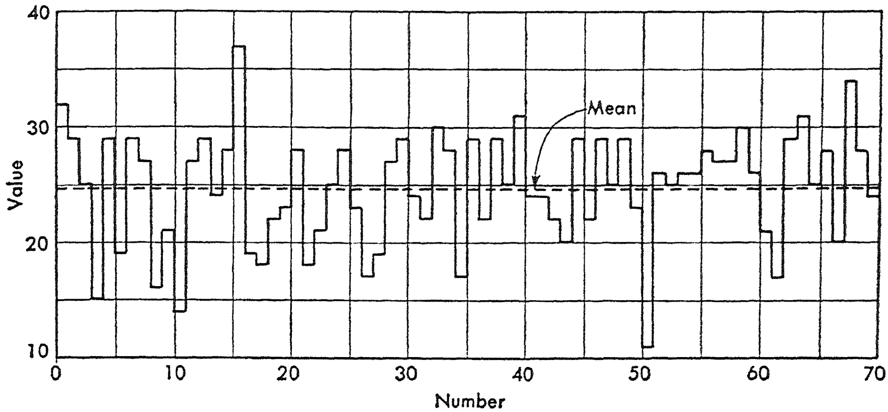



FIGURE 36. Fluctuations of Random Card Drawings

tude, and should be repeated at regular intervals of time. The subject of cyclic variation has been given considerable investigation and discussion and the opinions advanced have been far from unanimous. Agreement is lacking even on the subject as to whether or not identifiable cycles really exist in nonseasonal long-time precipitation. Various investigations have proposed a great number of cycles; according to Kincer (108), 138 had been proposed up to the year 1928, with lengths ranging from very short periods to 260 years. While not all adherents to the cyclic theory proffer a reason for such variation, the usual cause is assumed to be sunspots, although the course of action necessary to affect precipitation is vague. Humphrey (96) and Mills (137) state that the temperature of the earth tends to be lower at periods of maximum intensity of sunspots and higher when the intensity of sunspots is at a minimum.

Apparently the only clearly demonstrated effect of sunspots on meteorological phenomena is the effect on aurora borealis and similar magnetic or electric activity. That there is a close connection between sunspots and auroral displays has been amply demonstrated, but to date no recognizable causal relationship between the aurora borealis or similar phenomena and precipitation has been found. It must therefore be concluded that the effect of sunspots on precipitation must be sought through the variations in solar radiant energy.

The best results of investigations at present indicate that earth temperatures are lower during periods of high sunspot numbers when radiant energy is supposed to be high. The cause of this paradox is not fully explained and while further discussion is outside the scope of this book, the statement of the situation is sufficient to indicate that there can be no clearly demonstrable direct connection between sunspots and precipitation. Moreover, it has previously been shown that precipita-

tion is the result of a long chain of events, each subject to its own variations and each varying more or less independently. Therefore if there is any connection between sunspots and precipitation it is greatly obscured or possibly negated completely by the intervening atmospheric activity. Furthermore, if sunspots or even variations in solar energy cause variations in precipitation, those variations should be world-wide. The global extent of variations in precipitation has not been conclusively demonstrated and lack of suitably located rainfall stations render verification unlikely for some time. However, lack of demonstration does not mean that such world variations do not exist, for they are still possible.

Annual variation of course exists or is imposed on any possible cycles or fluctuations. The existence of cycles must be disclosed through averages of several successive years derived by methods discussed above. It can therefore be seen readily that so-called cycles are profoundly affected by the random magnitudes of annual precipitation.

There may be other causes which have not been recognized but which promote, affect, or modify the known fluctuations of annual precipitation. Mills (137) suggests that the positions of the planets may be a factor in causing fluctuation of earth temperatures. Humphrey (96) pointed out that volcanic eruptions sometimes eject vast quantities of fine material into the atmosphere, which would cut off and absorb solar radiation, thereby resulting in lower earth temperatures. Such phenomena would certainly inject great irregularity into any series of cycles. There may be other causes of variation not yet discovered, so that the net effect reduces us to dealing with an unknown number of independent variable factors, which is a situation common to other aspects of hydrology. The more irregularities that are introduced into the causes of precipitation, the closer precipitation data approach those of pure chance.

From a review of the literature on climatic cycles the following conclusions appear to be justified:

1. Fluctuations of various lengths exist in annual precipitation, the causes for which are not all known, except that they are built upon the random variations of annual precipitation plus whatever other causes may exist.

2. Small or short time fluctuations are more or less peculiar to individual stations or restricted localities and may not be felt at stations not far distant.

3. The longer and more pronounced fluctuations are felt synchronously over large areas.

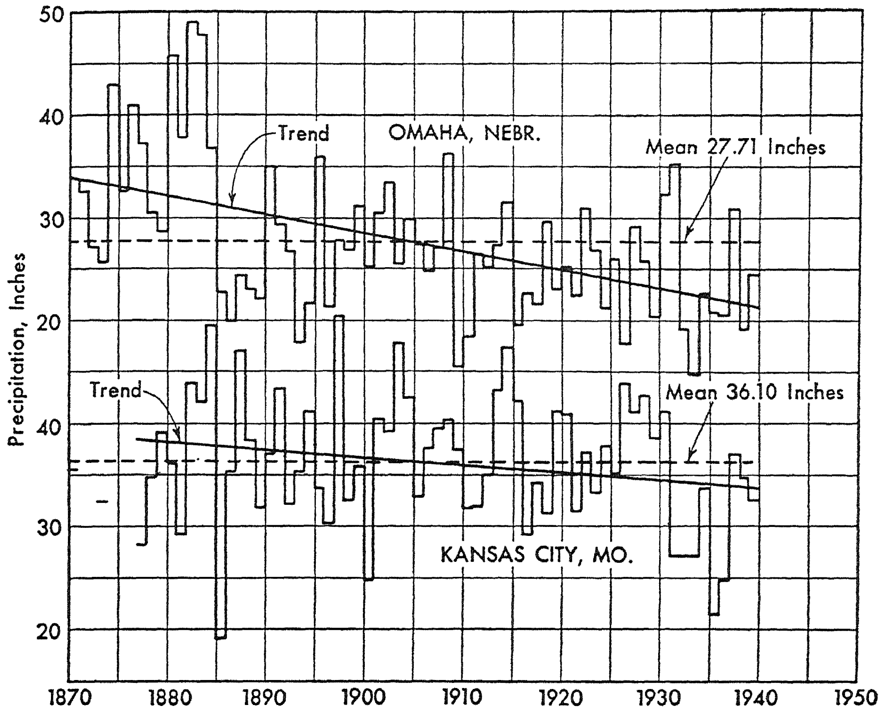



FIGURE 37. Trends of Annual Precipitation

*Trends of Precipitation.* Trends may be defined as diminishing or increasing average precipitation over a given period. The annual variations obscure the trend in many cases but in others it is plainly visible in a plotted graph. Figures 37 to 41 show examples of trends.

Trends of precipitation may be determined by a number of methods used in statistical work. Where there is a noticeable trend as in Omaha (Figure 37) a straight-line representation can be drawn by eye. This method, however, must be considered as approximate only. A somewhat definite trend can be depicted by taking the averages of the first and second halves of the record, plotting them at the quarter points of the record and drawing a straight line through the two points. This likewise is an approximation.

A method not uncommonly used is the development of the trend by means of moving averages. The annual precipitation of a number of years, which may be three to twenty, is averaged and plotted at the midyear of the group. Thus, assuming a series of $n$ years be taken from a record of $x$ years, the average

$$\frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

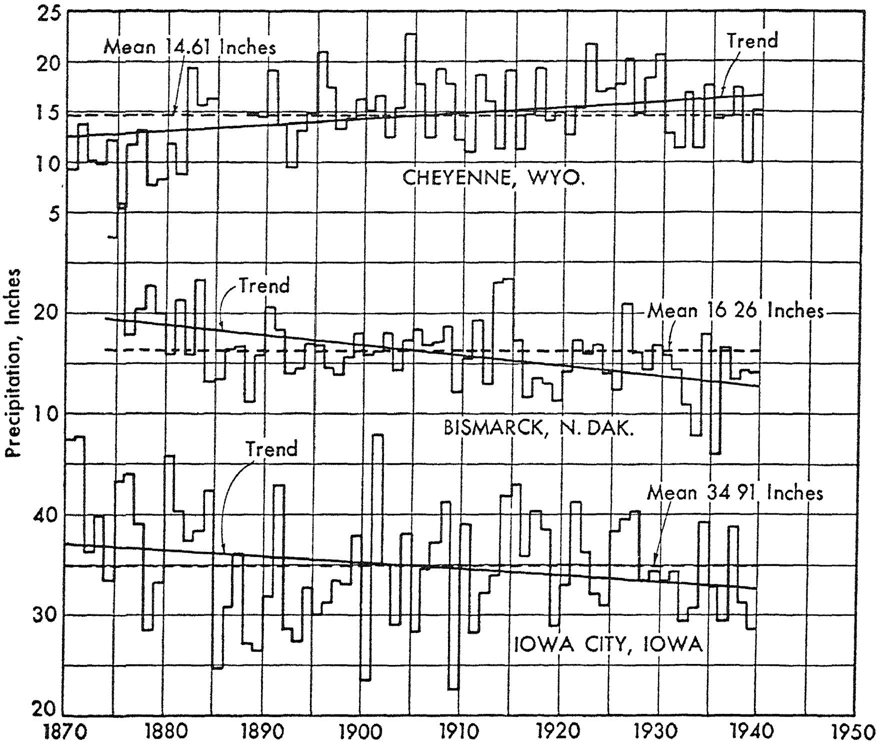



FIGURE 38. Trends of Annual Precipitation

is plotted at the middle year of the period. The next quantity taken is the average of the years $a_2$ to $a_{n+1}$ inclusive, thus,

$$a_2 + a_3 + a_4 + \cdots + a_n + a_{n+1}$$

which is plotted at the next successive year. This process is continued until the entire record $a_1$ to $a_x$ is covered.

The first effect of this process is the smoothing out of the annual variations, and not infrequently this result is all that is desired. In fact, using a small number of years, say three or five, will accomplish little more. However, the greater the number of years the more effectively will the short irregularities be smoothed out and the more prominently will the long trends be shown. Figure 39 shows the development of a trend in annual precipitation at Omaha, Nebr., by using an increasing number of years in the moving average.

The computation of the moving average may be systematized so that it can be readily accomplished either on an adding machine or without such an aid. For example, referring to Table 35 following

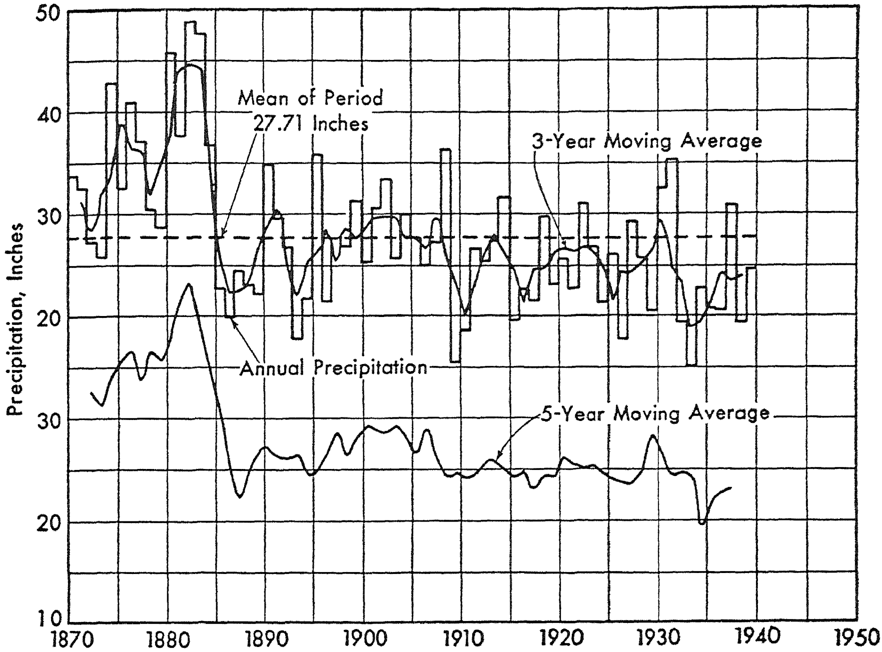



FIGURE 39. Moving Averages of Precipitation, Omaha, Nebr.

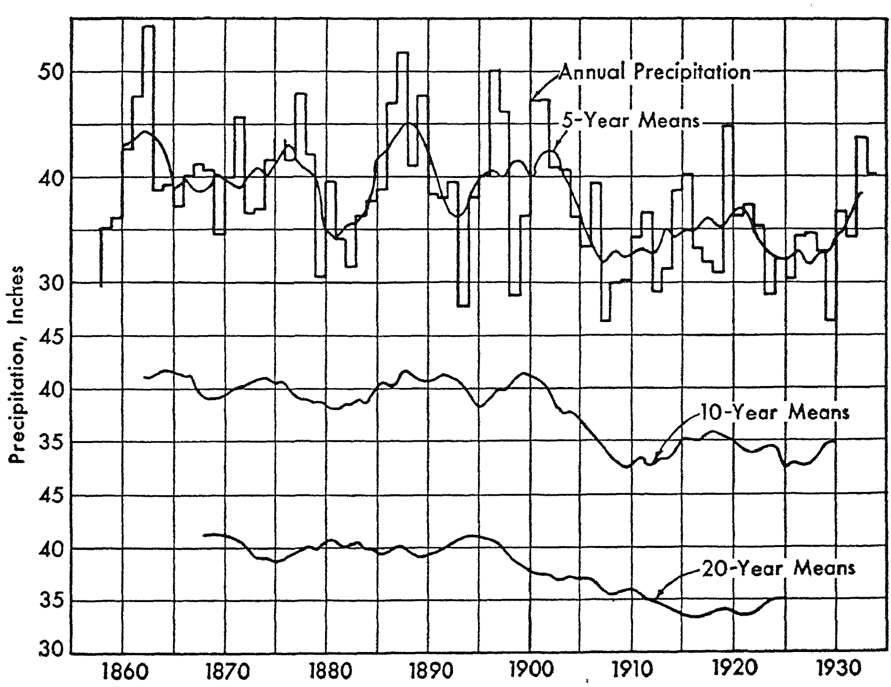



FIGURE 40. Trend by Moving Averages, Concord, N. H.

hereinafter, take a moving average of three years of the precipitation at Omaha, Nebr. The work would proceed thus:

TABLE 35. COMPUTATION OF A MOVING AVERAGE

| | YEAR | PRECIPITATION Inches | 3 YR. MEAN | ORDER |
|---|---|---|---|---|
| | 1871 | 33.81 | | |
| | 1872 | 32.48 | | |
| | 1873 | 27.04 | | |
| Total | | 93.33 | 31.11 | 1 |
| Deduct Year | 1871 | 33.81 | | |
| | | 59.52 | | |
| Add year | 1874 | 25.75 | | |
| Total | | 85.27 | 28.42 | 2 |
| and so forth. | | | | |

From Figure 40 it can be noted that a considerable number of years is needed in each average to develop the trend appreciably. This feature has the effect of reducing the length of the record which in most cases is not longer than necessary for studying trends. For making the most efficient use of a moderate record some other method would be desirable.

*Trends by Accumulative Deviations.* Another method of depicting trends of precipitation consists of plotting the accumulated sums of the deviations from the mean of the period. This method has the virtue of emphasizing the periods of excessive wetness or drouth. The procedure is simple; one simply finds the deviations from the mean by the formula

$$S - M = x,$$

where $x$ is the deviation and $M$ the mean of a series of observations $S$ of annual precipitation. The deviations are then added algebraically to each successive year of record and each total plotted against the year.

Two examples are shown in Figure 41. The abnormal wet period at the beginning of the record is shown conspicuously on the graph for Omaha. The drouth of the decade 1931–40 is shown prominently on both graphs.

*The Trend by Method of Least Squares.* For a given record the trend can be determined most accurately by the method of least squares. The trends of the records shown in Figures 37 and 38 have been computed by this method. The method is the same for all computations and hence only one computation is given.

*Areal Distribution.* Areal distribution of precipitation is commonly shown for engineering purposes by isohyetal maps on which lines depict equal depths of precipitation. This representation is similar to showing

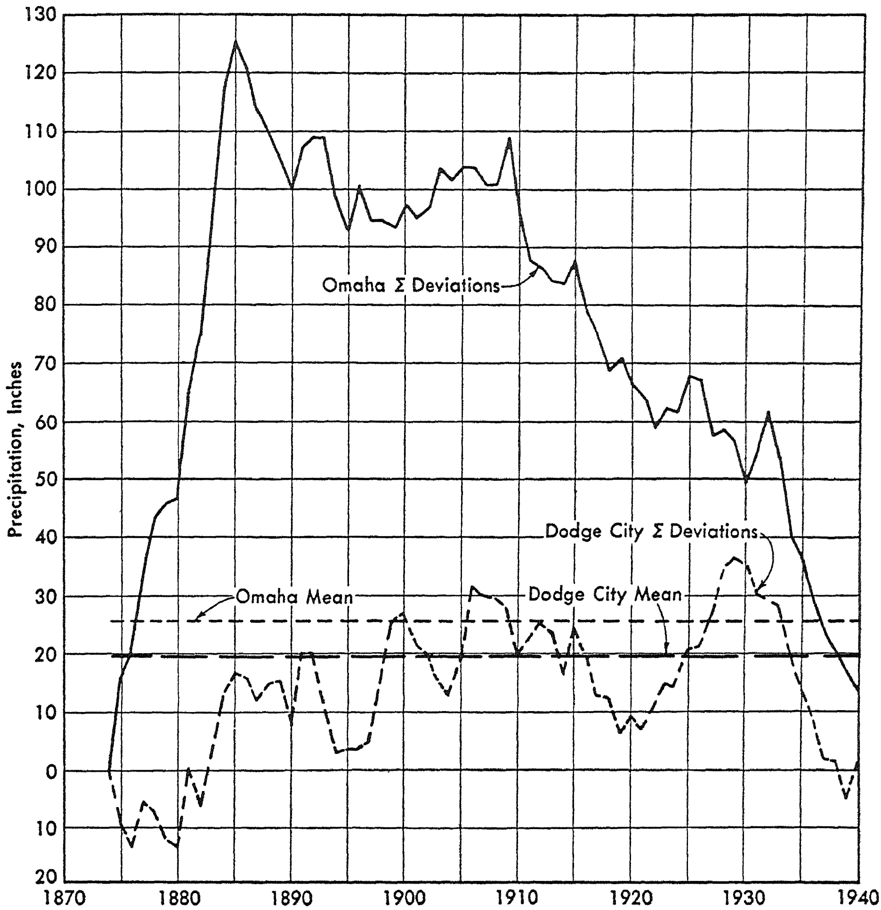



FIGURE 41. Trends by Accumulated Deviations, Omaha, Nebr., and Dodge City, Kan.

equal elevations of land surface by means of contours, and the method is peculiarly adaptable to computations of average depth or total volume of precipitation. On the other hand, it requires some knowledge of mathematics for ready interpretation and hence is less desirable for pictorial representation for nontechnical readers. For the latter purposes differential shading or crosshatching may be used. This method of representation is described in works on statistics such as Arkin and Colton (9) and needs no further discussion here.

Another method of determining average depths of precipitation is frequently used whenever it is not desired to show graphically the areal variations; this method is commonly called the Thiessen method.

*Isohyetal Maps.* Isohyetal maps depict precipitation by means of lines showing equal depths on the area involved. They are equally

TABLE 36. TRENDS OF ANNUAL PRECIPITATION, OMAHA, NEBR.

| YEAR | ORDER $T$ | PRECIPITATION Inches $P$ | $TP$ | $T^2$ |
|---|---|---|---|---|
| 1871 | 0 | 33.81 | 0 | 0 |
| 1872 | 1 | 32.48 | 32.48 | 1 |
| 1873 | 2 | 27.04 | 54.08 | 4 |
| 1874 | 3 | 25.75 | 77.25 | 9 |
| 1875 | 4 | 42.89 | 171.56 | 16 |
| 1876 | 5 | 32.51 | 162.55 | 25 |
| 1877 | 6 | 40.95 | 245.70 | 36 |
| 1878 | 7 | 37.05 | 259.35 | 49 |
| 1879 | 8 | 30.31 | 242.48 | 64 |
| 1880 | 9 | 28.52 | 256.68 | 81 |
| 1881 | 10 | 45.74 | 457.40 | 100 |
| 1882 | 11 | 37.68 | 414.48 | 121 |
| 1883 | 12 | 48.92 | 587.04 | 144 |
| 1884 | 13 | 47.68 | 619.84 | 169 |
| 1885 | 14 | 36.68 | 513.52 | 196 |
| 1886 | 15 | 22.67 | 340.05 | 225 |
| 1887 | 16 | 19.92 | 318.72 | 256 |
| 1888 | 17 | 24.22 | 411.74 | 289 |
| 1889 | 18 | 22.97 | 413.46 | 324 |
| 1890 | 19 | 22.08 | 419.52 | 361 |
| 1891 | 20 | 34.92 | 698.40 | 400 |
| 1892 | 21 | 29.44 | 618.24 | 441 |
| 1893 | 22 | 26.66 | 586.52 | 484 |
| 1894 | 23 | 17.82 | 409.86 | 529 |
| 1895 | 24 | 21.69 | 520.56 | 576 |
| 1896 | 25 | 35.90 | 897.50 | 625 |
| 1897 | 26 | 21.30 | 553.80 | 676 |
| 1898 | 27 | 27.84 | 751.68 | 729 |
| 1899 | 28 | 26.74 | 748.72 | 784 |
| 1900 | 29 | 31.20 | 904.80 | 841 |
| 1901 | 30 | 25.08 | 752.40 | 900 |
| 1902 | 31 | 30.48 | 944.88 | 961 |
| 1903 | 32 | 33.43 | 1,069.76 | 1024 |
| 1904 | 33 | 25.48 | 840.84 | 1089 |
| 1905 | 34 | 29.88 | 1,015.92 | 1156 |
| 1906 | 35 | 27.59 | 965.65 | 1225 |
| 1907 | 36 | 24.90 | 896.40 | 1296 |
| 1908 | 37 | 27.10 | 1,002.70 | 1369 |
| 1909 | 38 | 36.32 | 1,380.16 | 1444 |
| 1910 | 39 | 15.49 | 604.11 | 1521 |
| 1911 | 40 | 18.46 | 738.40 | 1600 |
| 1912 | 41 | 26.46 | 1,048.86 | 1681 |
| 1913 | 42 | 25.03 | 1,051.26 | 1764 |
| 1914 | 43 | 27.25 | 1,171.75 | 1849 |
| 1915 | 44 | 31.57 | 1,389.08 | 1936 |
| 1916 | 45 | 19.46 | 875.70 | 2025 |
| 1917 | 46 | 22.62 | 1,040.52 | 2116 |
| 1918 | 47 | 21.45 | 1,008.15 | 2209 |
| 1919 | 48 | 29.70 | 1,425.60 | 2304 |
| 1920 | 49 | 23.01 | 1,127.49 | 2401 |

TABLE 36. TRENDS OF ANNUAL PRECIPITATION, OMAHA, NEBR. *(Cont'd)*

| YEAR | ORDER $T$ | PRECIPITATION *Inches* $P$ | $TP$ | $T^2$ |
|---|---|---|---|---|
| 1921 | 50 | 25.29 | 1,264.50 | 2500 |
| 1922 | 51 | 22.46 | 1,145.46 | 2601 |
| 1923 | 52 | 30.95 | 1,609.40 | 2704 |
| 1924 | 53 | 26.83 | 1,421.99 | 2809 |
| 1925 | 54 | 21.12 | 1,140.48 | 2916 |
| 1926 | 55 | 25.96 | 1,427.80 | 3025 |
| 1927 | 56 | 17.66 | 988.96 | 3136 |
| 1928 | 57 | 29.09 | 1,658.13 | 3249 |
| 1929 | 58 | 25.69 | 1,490.02 | 3364 |
| 1930 | 59 | 20.35 | 1,200.65 | 3481 |
| 1931 | 60 | 32.28 | 1,936.80 | 3600 |
| 1932 | 61 | 35.20 | 2,147.20 | 3721 |
| 1933 | 62 | 19.20 | 1,190.40 | 3844 |
| 1934 | 63 | 14.90 | 938.70 | 3969 |
| 1935 | 64 | 22.57 | 1,444.48 | 4096 |
| 1936 | 65 | 20.73 | 1,347.45 | 4225 |
| 1937 | 66 | 20.51 | 1,353.66 | 4356 |
| 1938 | 67 | 30.87 | 2,068.29 | 4489 |
| 1939 | 68 | 19.20 | 1,305.60 | 4624 |
| 1940 | 69 | 24.50 | 1,690.50 | 4761 |
| Totals | 2415 | 1,939.40 | 61,808.08 | 111,895 |
| Mean | | 27.71 | | |

Total number of years equals 70 = $N$.

The formulas for the normal equations are as follows:

(1) $$\sum P = Na + b\sum T$$

(2) $$\sum TP = a\sum T + b\sum T^2$$

Substituting the values from the table:

(1) $$1{,}939.40 = 70a + 2{,}415b$$

(2) $$61{,}808.08 = 2{,}415a + 111{,}895b$$

Multiplying equation (1) by 34.5 to eliminate $a$,

$$\begin{array}{lrcl} (1) & 66{,}909.30 & = & 2{,}415a + \phantom{1}83{,}317.5b \\ (2) & 61{,}808.08 & = & 2{,}415a + 111{,}895.0b \\ \hline & 5{,}101.22 & = & 0 \quad - \ 28{,}577.5b \end{array}$$

$$b = -\frac{5{,}101.22}{28{,}577.5} = -\,0.1785$$

$$a = \frac{1939.40 - (2415)(-0.1785)}{70} = \frac{2370.48}{70}$$

$$= 33.86$$

$$P = 33.86 - 0.1785T.$$

For $T = 0$, $P = 33.86$, and $T = 70$, $P = 21.65$.

The above values for $T$ and $P$ are plotted on Figure 37, and a straight line is drawn to show the trend. Other examples of trends computed by the method of least squares are shown on Figures 37 and 38.

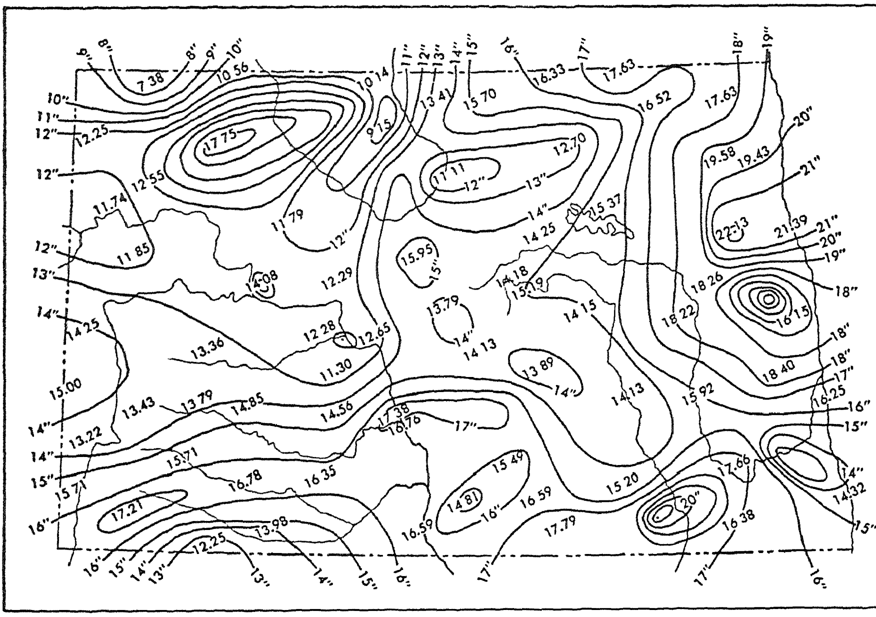



FIGURE 42. Isohyets of Annual Precipitation for North Dakota, 1930

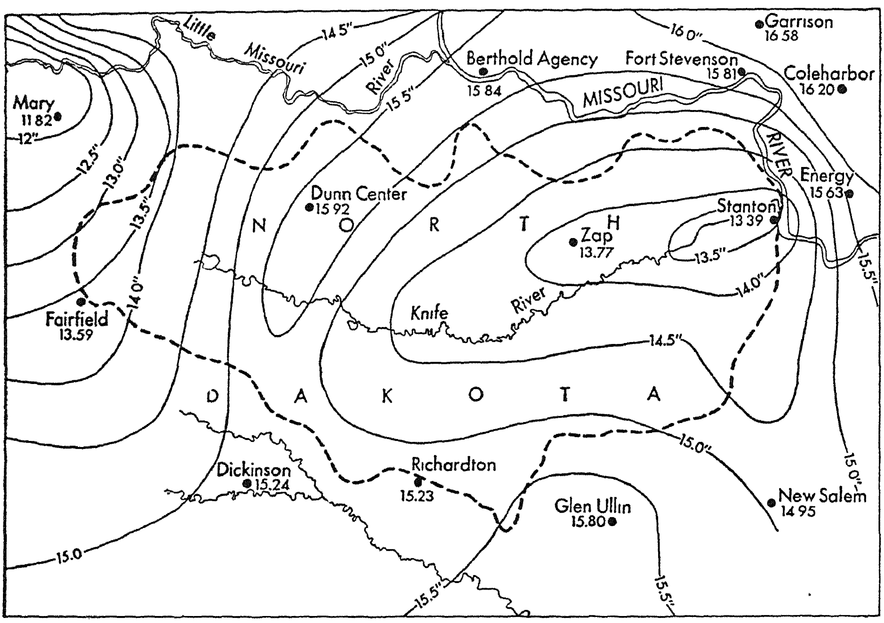



FIGURE 43. Isohyets of Mean Annual Precipitation, Knife River Basin, N. Dak.

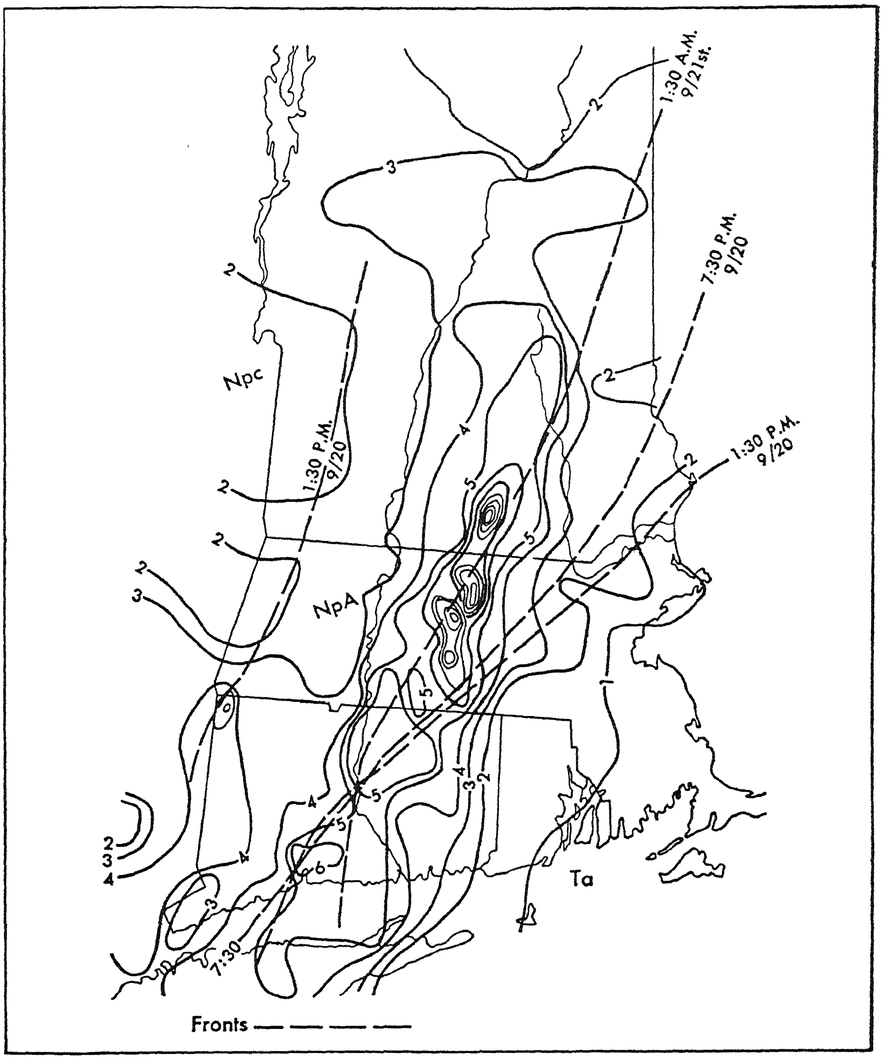



FIGURE 44. Maximum One-day Rainfall, 8 A.M. Sept. 20–8 A.M. Sept. 21, 1938

useful for depicting the means of any periods of time such as year, month, or day, and for showing the depths of precipitation for individual storms. The total volume of precipitation may be obtained from isohyetal maps by planimetering the areas and computing the quantities desired.

Figure 42 showing the precipitation for North Dakota for the year 1930 illustrates the isohyetal map for a given area. Figure 43 is an isohyetal map showing the mean annual precipitation over the Knife River watershed. The precipitation of a single storm is shown in Figures 44 and 45 which depict the intense rainfall on September 20,

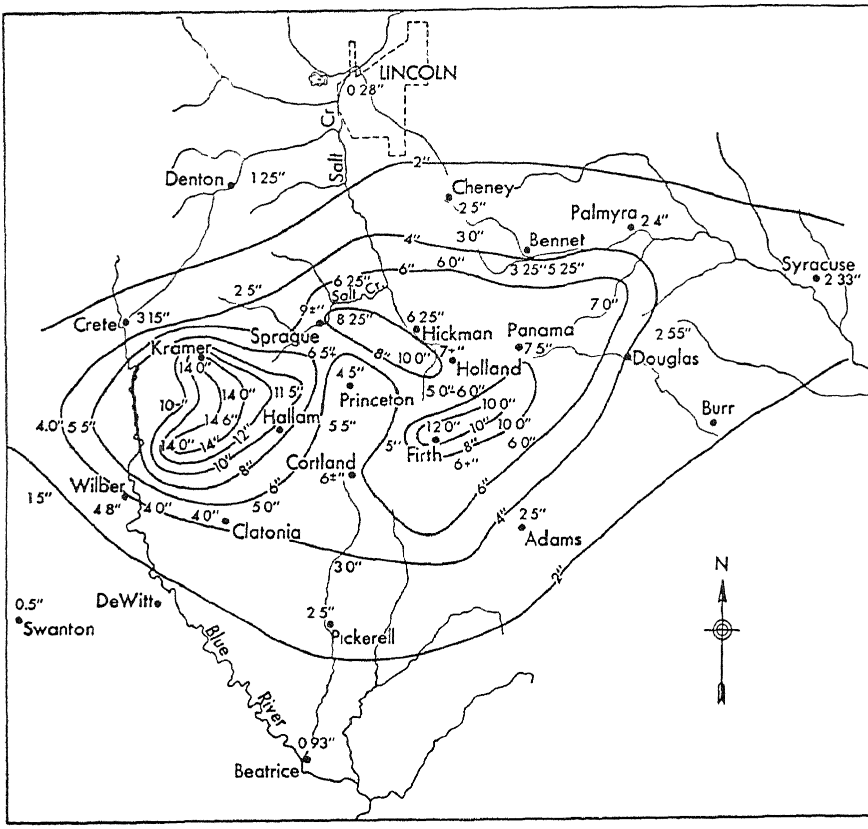



FIGURE 45. Isohyetal Map of Storm of May 10–11, 1942, near Lincoln, Nebr.

1938, in New England and on May 11, 1942, near Lincoln, Nebr. The distribution of the precipitation in the storm of March 1936 is shown in Figures 46 and 47.

As may be seen from the illustrations, an isohyetal map is constructed by plotting the data of precipitation at the geographical location of the precipitation stations; points representing the selected lines of equal depths are located proportionately between stations. The lines of equal precipitation, or isohyets, are drawn through them. Considerable judgment, experience, and knowledge of the terrain or areal peculiarities are required to construct representative isohyetal maps, since the pattern varies with different types of storms or with the nature of the data being used. Normals of long records will present a comparatively uniform or gradually varied pattern; a series of thunderstorms, on the other hand, will form a decidedly uneven and irregular pattern.

Measurement of the mean depth is obtained by measuring the areas within the various contours, summing the depth-area products, and

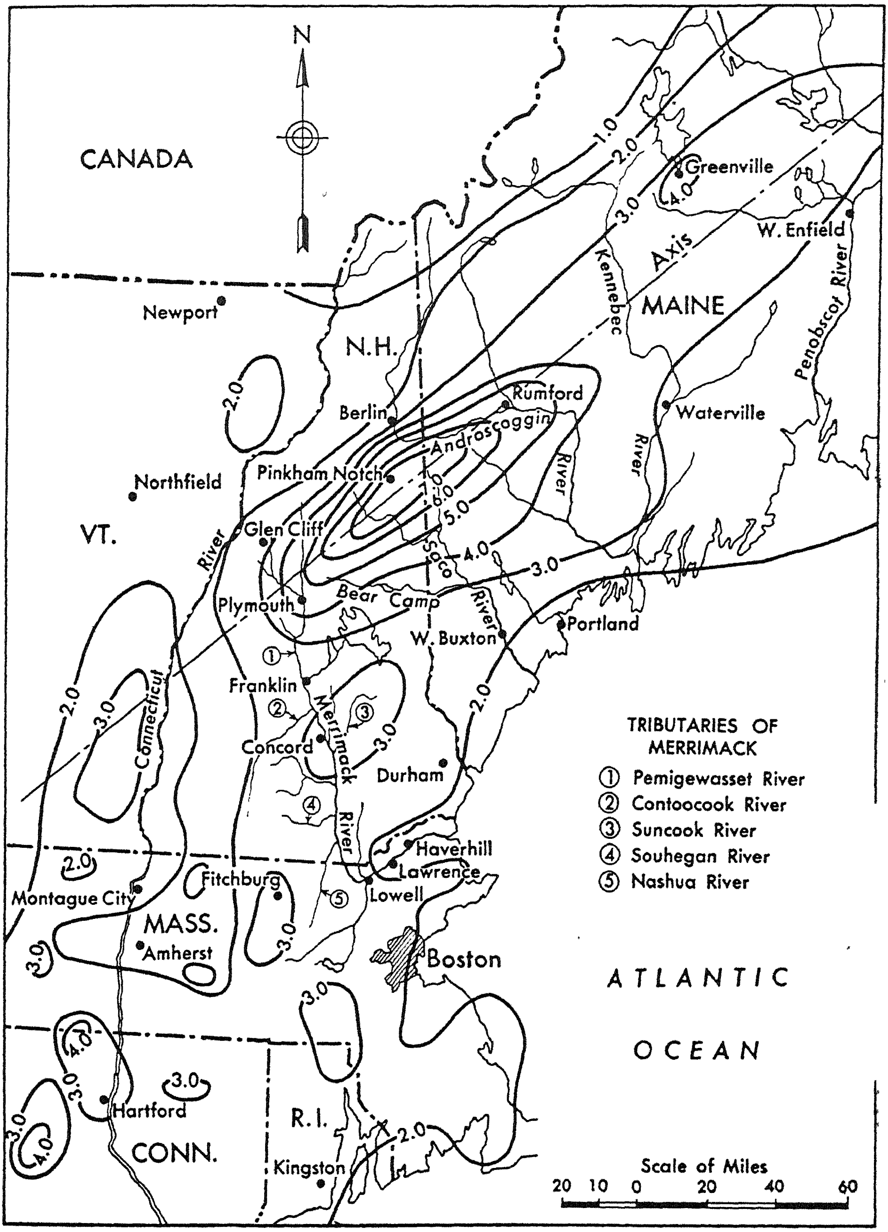



FIGURE 46. Isohyetal Map of New England, Storm of March 11–13, 1936

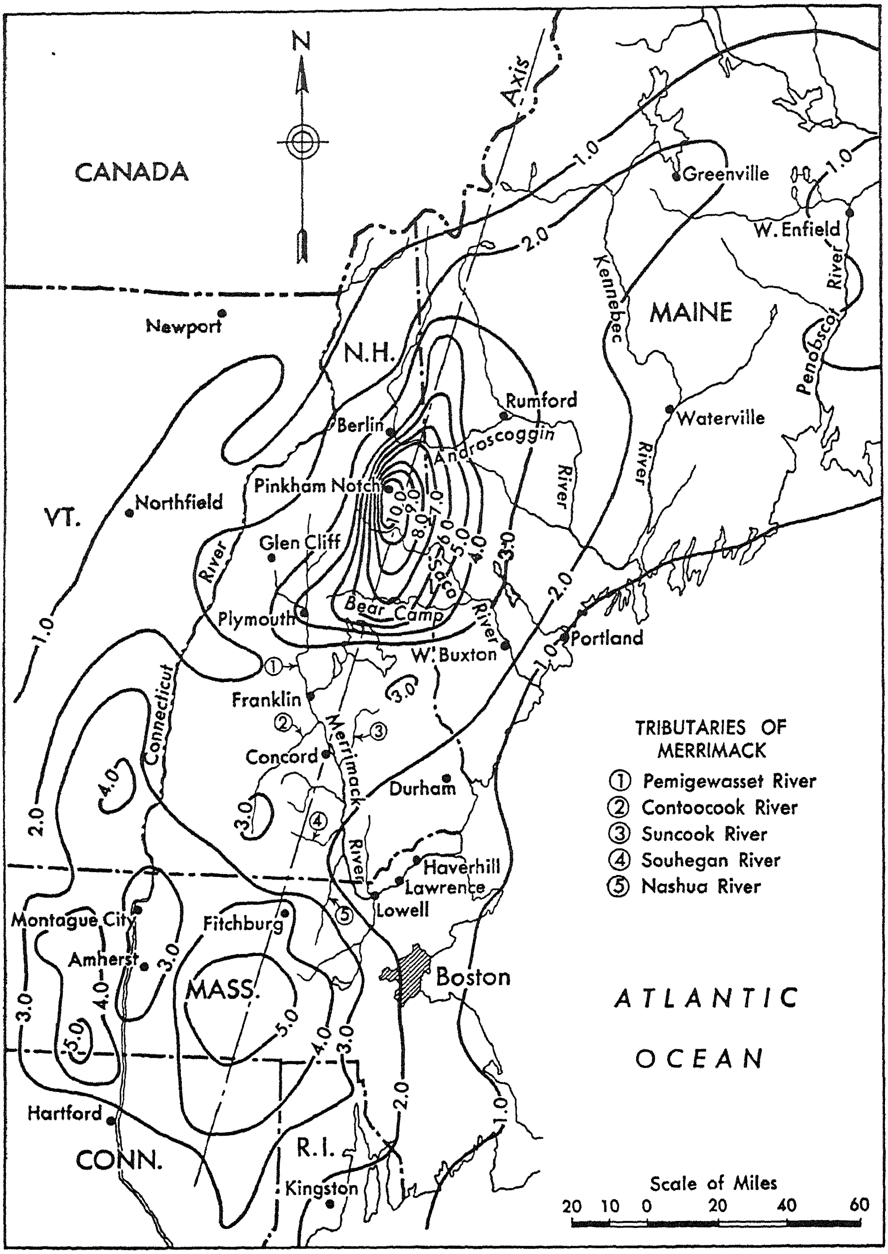



FIGURE 47. Isohyetal Map of New England, Storm of March 17–19, 1936

then dividing the total by the total area. The result is a weighted mean depth of precipitation; the accuracy of the result will, of course, depend upon the reliability of the data, number of precipitation stations, and the judgment of the worker in interpreting the pattern of the storm. The last factor is particularly important in deciding the areas of greatest intensity, for which usually all too few data are available; the contour of that area may be subject to rather wide errors even in the best of work. This situation is unfortunately aggravated by the sparsity of precipitation gaging stations.

*The Thiessen Method.* The term "Thiessen method" is used to denote a procedure for determining the mean depth of precipitation over a given area; it was first devised by Thiessen and later perfected by R. E. Horton. The fundamental principle followed to accomplish this purpose consists in "weighting" the value at each station by a suitable proportion of the area for which the mean is desired.

To initiate the procedure the data of precipitation, which may be depths of rainfall, snow, mean annuals, or any other values of similar type, are plotted on a map at the geographical location of the stations at which they were observed, as shown on the maps of the Knife River basin, N. Dak. (Figure 48.) Straight connecting lines are then drawn from each station to the adjacent ones, as from Mary to Fairfield, Fairfield to Dunn Center, Dunn Center to Zap, Dickinson, Richardton, etc. From the mid-points of these lines a second set of lines, the perpendicular bisectors of the lines connecting the stations, is drawn. These lines of the second set form the boundaries of the area allocated to each station as its weight, except where the area is limited by other lines such as the boundary of a drainage area.

In Figure 48 the object was to obtain the mean annual precipitation over the Knife River basin, hence all areas are limited by the watershed as well as mid-point boundary lines. The area apportioned to each station was measured and expressed as a percentage of the total area within the drainage basin; these percentages, being dimensionless values, are then used as weights for multiplying the data at the proper station. Thus the mean annual precipitation of the Knife River basin is found as shown in Table 37.

The principal advantage of the Thiessen method is ease of computation. This feature is particularly attractive where a large number of means are desired for the same area, as weights once found can be used for any number of computations. In such cases, too, extreme accuracy usually is not required and the method is to be recommended for the purpose.

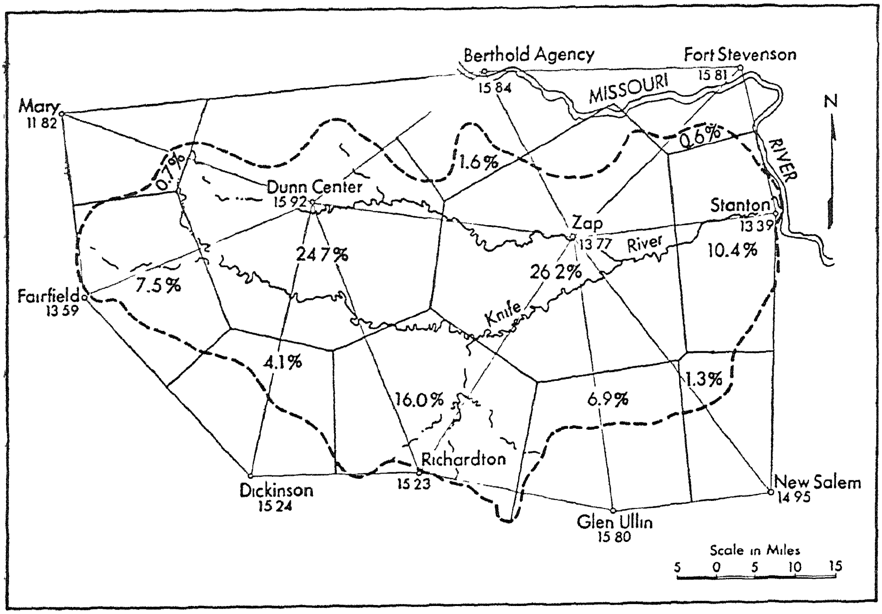



FIGURE 48. Mean Precipitation on Knife River Basin, N. Dak.

TABLE 37. MEAN ANNUAL PRECIPITATION, KNIFE RIVER BASIN, BY THIESSEN METHOD

| STATION | AREAL WEIGHT | PRECIPITATION, AT STATIONS *Inches* | WEIGHTED PROPORTION *Inches* |
|---|---|---|---|
| Fairfield | 0.075 | 13.59 | 1.02 |
| Mary | .007 | 13.82 | .10 |
| Dunn Center | .247 | 15.92 | 3.93 |
| Berthold Agency | .016 | 15.84 | .25 |
| Zap | .262 | 13.77 | 3.61 |
| Fort Stevenson | .006 | 15.81 | .09 |
| Stanton | .104 | 13.39 | 1.39 |
| New Salem | .013 | 14.95 | .19 |
| Glen Ullin | .069 | 15.80 | 1.09 |
| Richardton | .160 | 15.23 | 2.44 |
| Dickinson | 0.041 | 15.24 | 0.62 |
| Totals | 1.000 | | 14.73 |

*Study of Distribution of Storm Precipitation.* While isohyetal maps or weighted means suffice for many purposes, the areal distribution of precipitation requires further study. For this purpose depth-area curves are constructed, the location of storm axes are determined, and the distribution of storms is found. Of special importance are the depth-area curves by which the relationship of depth of precipitation and area covered are determined. This may be considered one of the fundamental relationships of storm study and one of greatest usefulness.

*Depth-Area Curves.* "Depth-area" curves are, as the name implies, curves that show the relationship between depth of precipitation and the area covered. They also have been called "time-area-depth" curves, but since time does not appear as a variable in any given curve, it appears desirable to designate them simply as "depth-area" curves. In them the average depth over a given area is commonly plotted against the area. Although not necessarily limited to depths and areas of storm precipitation, their utility is confined principally to storm precipitation and studies of storm runoff.

The first step in the formation of depth-area curves is the plotting of the storm precipitation on a suitable map having a conveniently large scale so that errors of measurement become inconsequential. Then the isohyetal lines are drawn. Next the area enclosed within the isohyetal lines is measured, preferably with a planimeter, and the areas and depths of precipitation computed; these values can then be plotted to form the desired curves.

Some preliminary thought should be given to the form of computations in order to reduce errors and facilitate checking. Table 38, taken from the report of the Miami Conservancy District (135), is an excellent and complete form.

TABLE 38. FORM OF COMPUTATION FOR DEPTH-AREA CURVE (STORM OF JULY 14–16, 1910, 3-DAY PERIOD)

| CENTER | ISO-HYETAL *Inch* | AREA ENCLOSED *Sq. Inch* | *Sq. M.* | BETWEEN ISOHYETALS *Inch* | AVERAGE DEPTH *Inch* | NET AREA *Sq. M.* | VOLUME *Inch-Miles* | TOTAL VOLUME *Inch-M.* | AVERAGE DEPTH *Inch* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | 22 | 0.004 | 5 | 23.7–22 | 22.8 | 5 | 110 | 110 | 22.8 |
| | 21 | 0.03 | 41 | 22–21 | 21.5 | 36 | 770 | 880 | 21.5 |
| | 20 | 0.06 | 82 | 21–20 | 20.5 | 41 | 840 | 1720 | 21.0 |

In most storms except the smallest, more than one peak, that is point of maximum precipitation, will very likely be found. These peaks will necessarily have to be measured separately until a common isohyetal is reached. In the above sample computation one peak has been designated "*A*" to distinguish it from the other peaks of the same storm.

Figure 49 shows the depth area curve of the storm of March 17–19, 1936, in New England, and Figure 50, a series of four storms that occurred in eastern Nebraska. Depth-area curves for series of storms in Texas is given in Figure 51.

*Ratio of Peak Precipitation to Storm Average.* As it is well known from common experience and as is shown by the isohyetal maps, the precipitation of a storm diminishes in all directions from one or more peaks, or points of maximum depths. The rate of decrease depends upon

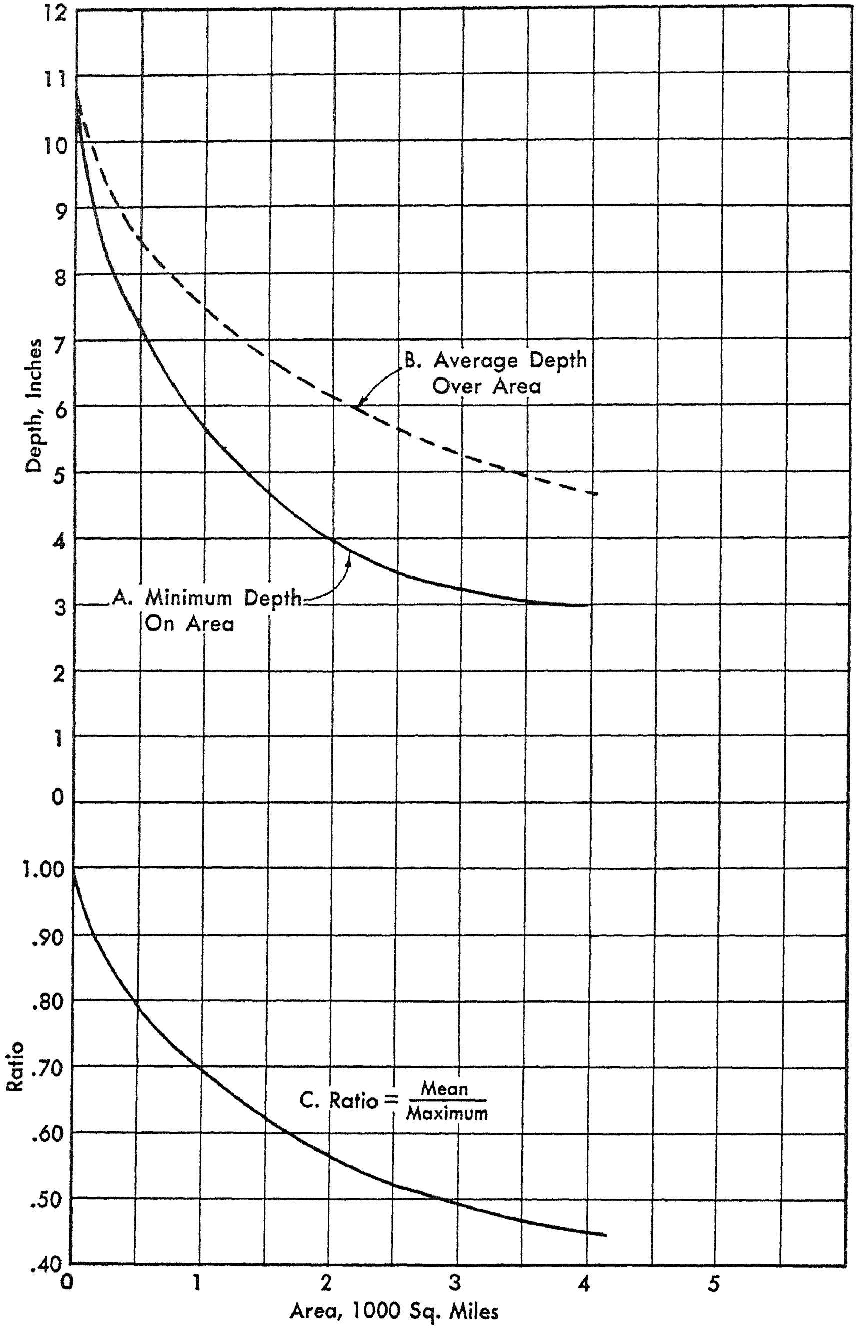



FIGURE 49. Depth-Area Curves, Storm of March 17–18, 1936 in New England

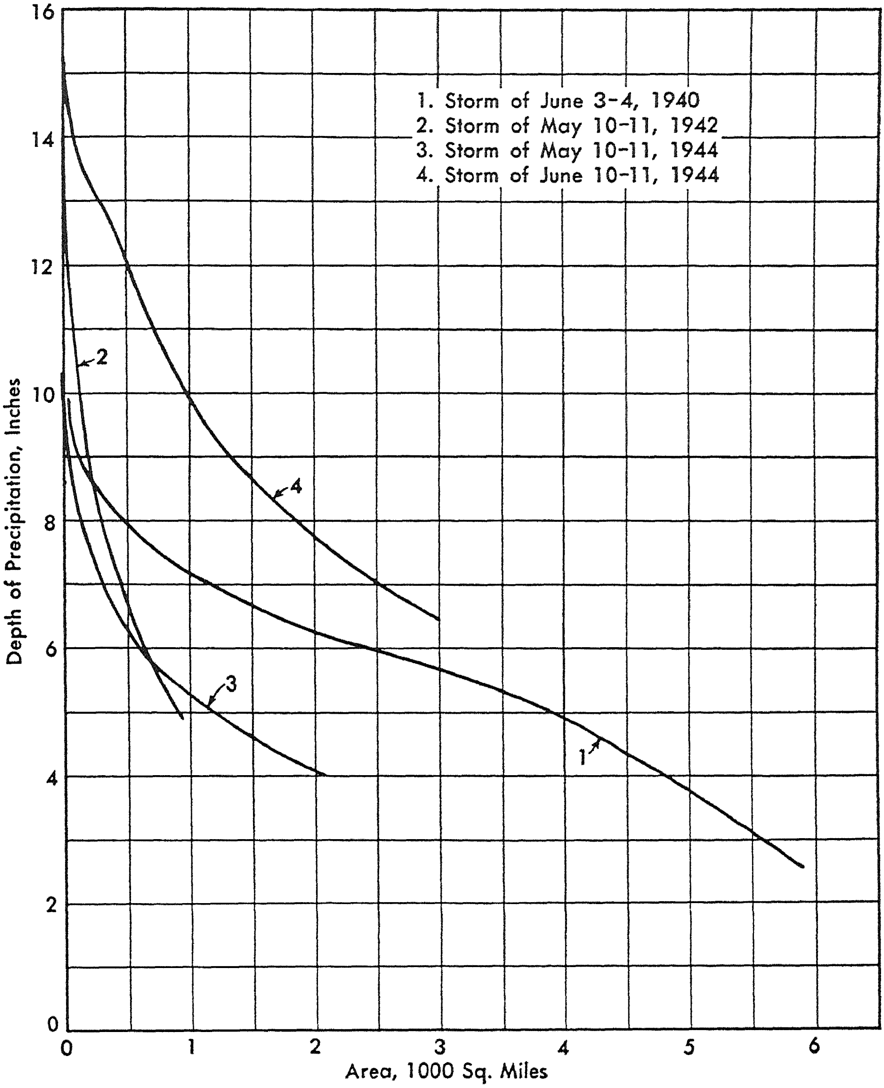



FIGURE 50. Depth-Area Curves, Nebraska Storms

many variable factors involving the type of storm, location and topography, distribution and movements of air masses, and practically every other causative agency operating in a storm. The isohyetal patterns of no two storms are closely alike although many may contain similar features.

The ratio of the peak distribution to the average indicates the rate at which the precipitation diminishes from the peak to the edges of the storm area. Curves similar to depth-area curves can be constructed by

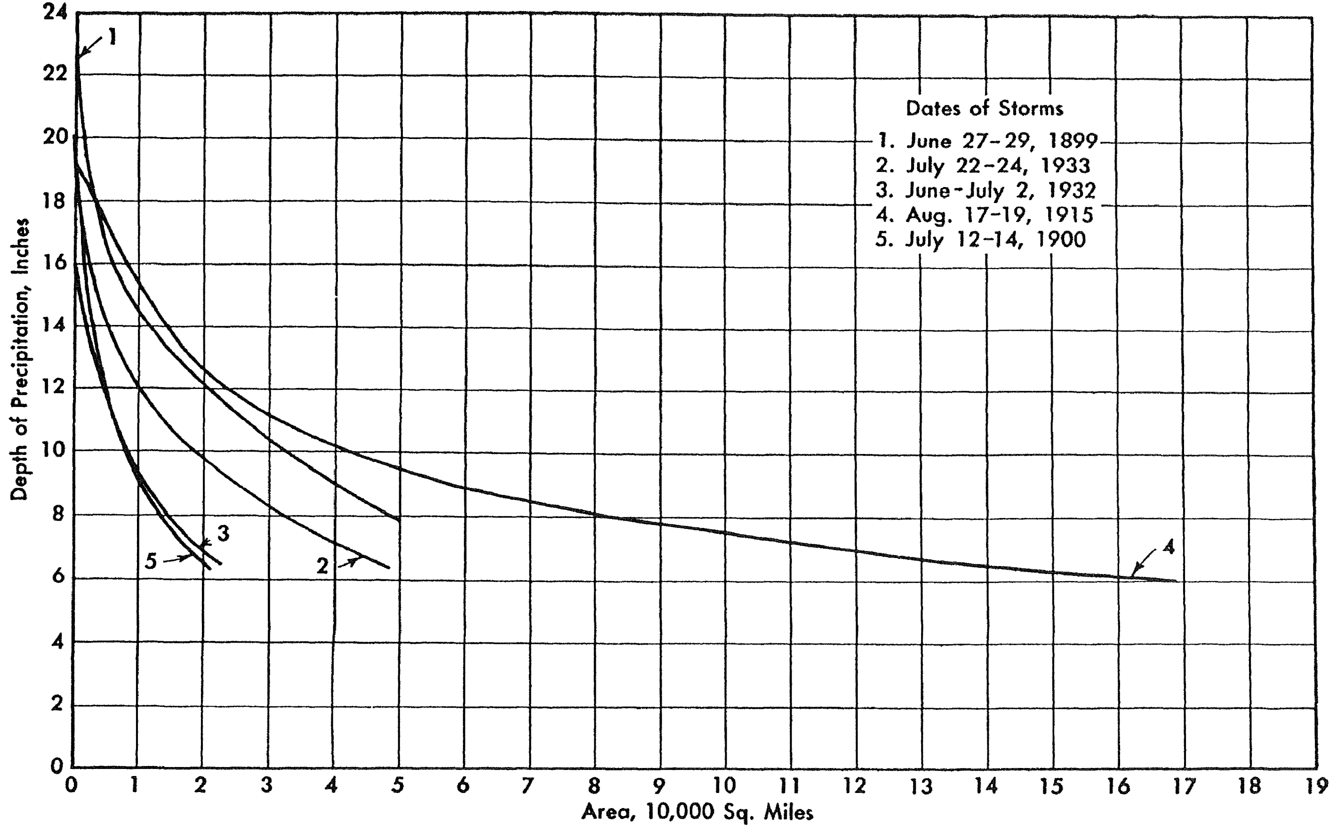



FIGURE 51. Depth-Area Curves, Five 3-Day Storms in Texas

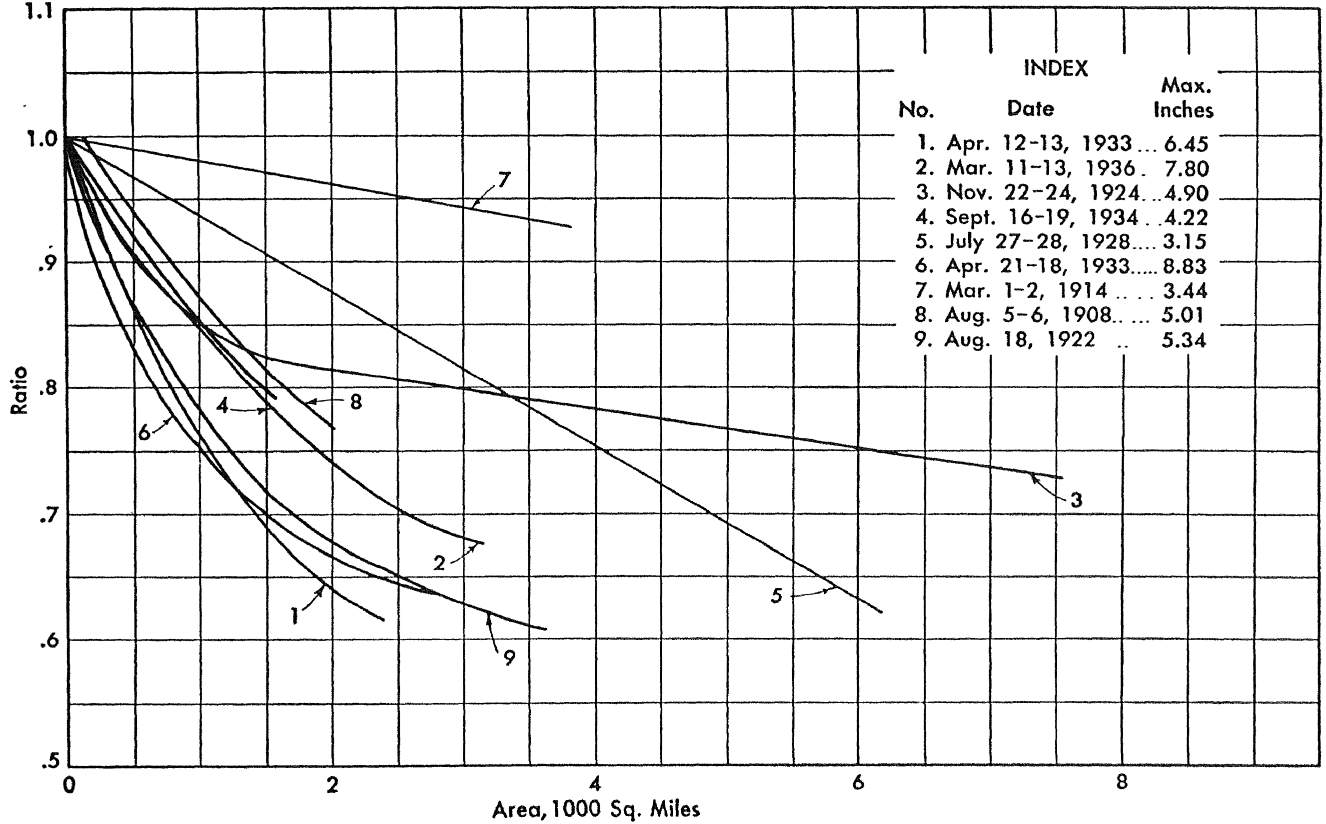



FIGURE 52. Ratio of Peak to Average Precipitation, New Hampshire Storms

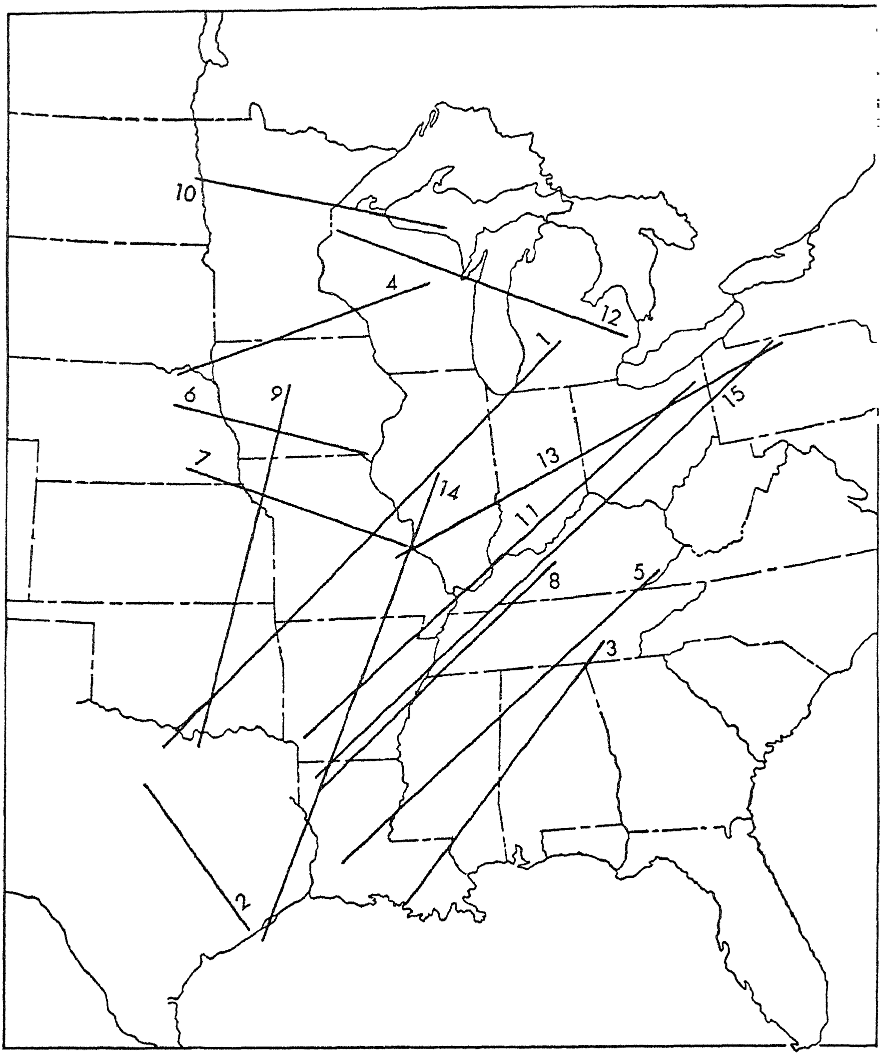



FIGURE 53. Direction of Axes of Large Storms in Central United States

dividing the peak depth by the average for a given area and plotting the ratios against the areas, as shown in Figure 49. A series of peak-average ratio curves for storms in New Hampshire is shown in Figure 52. These curves are useful for indicating possible reductions of average precipitation when the storm is transferred over other basins.

*Axes of Storm Rainfall.* It can be noted from the few samples of isohyetal maps that areas of heavy precipitation from great storms have elongated shapes. A line through the long dimension may be considered the axis of the storm rainfall. This feature is characteristic of the precipitation of all large storms involving extratropical or

frontal action, and is a feature to be considered in orienting a storm when it is desirable for study to transpose it over another basin. The axis is parallel to the general front.

In a given region the average direction of the isohyetal axes is parallel to the usual direction of the storm fronts. The direction varies somewhat in different regions. In New England the usual direction varies from north and south through southwest-northeast to east and west, indicating that the precipitation has come from warm air masses moving from east or south. Figure 53 shows a group of typical isohyetal axes in the central portion of the United States. Here the usual source of warm air is the Gulf of Mexico. The general direction of the axes is easterly to westerly and southwesterly. A few, however, lie approximately north of west to south of east; this tendency is more noticeable in the northern portion of the areas.

The storm axes in Figure 53 represent only a comparatively small number of the great storms in the central portion of the United States but they were selected as typical examples, except that close to the Gulf Coast there is a strong tendency for storm areas to be broad in comparison to length. The axes shown are for the storms listed below:

| | | | |
|---|---|---|---|
| 1. December 17–20, | 1895 | 9. October 20–24, | 1908 |
| 2. June 27–July 1, | 1899 | 10. July 20–22, | 1909 |
| 3. April 15–17, | 1900 | 11. October 4–6, | 1910 |
| 4. July 14–16, | 1900 | 12. July 20–24, | 1912 |
| 5. March 26–29, | 1902 | 13. March 23–27, | 1913 |
| 6. August 25–28, | 1903 | 14. August 17–20, | 1915 |
| 7. September 15–19, | 1905 | 15. January 20–25, | 1937 |
| 8. November 17–21, | 1906 | | |

*Shift of Precipitation.* The term "shift of precipitation" is used to designate a phenomenon sometimes referred to as "storm travel." This storm travel is apparent in the movement of a thunderstorm which progresses from one area to another. The importance of this progressive storm movement arises from the shift of precipitation, or more accurately, the shift of impact of precipitation.

Precipitation occurs as the storm is located over one area; when it moves, the area receiving the rainfall changes. If such a storm moves down a river valley, precipitation falls on the rising flood as it progresses downstream. To be noticeably effective in augmenting the flood the area covered by the storm must be relatively small, or the basin of the stream very large compared to the area covered by the storm. Otherwise precipitation is more or less uniform over the watershed. In relatively small storms such as thunderstorms, the shift of the impact of precipitation will probably follow the progressive movement of the storm.

But it does not follow that the precipitation accompanying the passage of an extratropical cyclone shifts with the movement of the area of low barometric pressure. These storms invariably arise from the convergence of two or more air masses of diverse origin. Many of these storms originate outside of the continental area; many others develop in, or on the edges of the mountainous areas. As they travel eastward they may be accompanied by precipitation or not, depending entirely upon the nature of the air masses involved.

Heavy precipitation can be caused only by the lifting of a warm moist tropical air mass over one of cold air. Such action normally occurs in an extratropical cyclone but is not confined to the center or low. The warm moist air is lifted along the entire front or fronts which extend through the low and is determined by the position, extent, and direction of travel of the moving front. The precipitation manifestly falls on the area over which the interaction of the air masses lifts the warm air, and the impact of rainfall shifts as the fronts move. This is illustrated by the fronts shown on the isohyetal map on Figure 44.

It is evident that the rate of travel of the storm as a whole is not necessarily the rate of shift of the impact of precipitation. The area of low barometric pressure marks the zone of convergence of the diverse air masses. It does not show the direction of the fronts leading away from the center nor their rate of travel; although the general direction can be inferred from the course of the isobars, the location of the front is shown more clearly by the trough of low pressure and by the drop in temperature. Because of these considerations, the shift of precipitation must be studied from some data other than those of the path of the storm.

The shift of impact of precipitation may be studied by means of the records of precipitation. Hourly records of automatic gages give the time of precipitation and a comparison of time for similar proportions of storm rainfall shows the shift of impact.

Hourly records of precipitation for as many recording precipitation gages as are available on or near the watershed in question must be obtained and used. These records are then computed and plotted in the form of cumulative rainfall curves. The data are summed up hour by hour; the total for each successive hour is divided by the total rainfall of the storm to give the percentage of total precipitation received to the end of the hour. These cumulative percentages are then plotted against the hour to which they were computed. Time is plotted as abscissas and percentages of rainfall as ordinates. All curves (to the

limit of convenient inspection) are plotted from the same origin, which for the time scale must be set prior to the beginning of the storm.

These curves have some properties that should be noted. For uniform meteorological conditions of a storm such as obtained through the passage of a single front they are similar in shape but differ, except in case of simultaneous precipitation, in time; that is, in a sense they are parallel. As in other summation curves a steep slope denotes a rapid rate of precipitation, and a flat slope a small rate. Discontinuities in the storms are shown by horizontal slopes. Differences in the abscissas of two curves for a given percentage represent differences in time of receipt of the percentage at the two stations. Curves lying one on another indicate simultaneous receipt of rainfall at the stations of the curves; this circumstance fixes the location of the frontal action which lies in the line of the two stations. The difference in time represented by the segment of abscissa is not the time of shift of precipitation between two stations, but from the difference in time of three stations and their known positions, the rate of shift of precipitation can be determined.

The method of computing the rates of travel is explained by referring to Figure 54. Points $P$, $H$, and $E$ represent three stations for which hourly records are obtainable. The distances are $P$ to $H = d_1$; $H$ to $E = d_2$; $E$ to $P = d_3$; the corresponding differences in time for the given percentage of the total rainfall are as follows:

$$H - P = t_1; E - H = t_2; E - P = t_3.$$

$M_1$ and $M_2$ mark parallel positions of the front as it passes through $E$ and $P$, respectively, but the direction of the front is still not known. The point $y$ marks the intersection of $PH$ extended to $M$; then the triangles $PHz$ and $yHE$ are similar. Then

$$yH : t_2 : : d_1 : t_1.$$

$$yH = \frac{d_1 t_2}{t_1}$$

The point $y$ may now be located and $xH$ can be drawn perpendicular to $M_2$. On the assumption that the front will move parallel to itself for the distance under consideration, it will traverse the distance $xH$ in the same time as $PH(t_1)$; then the rate of travel of the front is equal to $xH/t_1$.

The foregoing analysis is based on two assumptions; the first is that the front moves parallel to itself from position $M_1$ to $M_2$, and the second is that it moves at a uniform rate from position $M_1$ until it

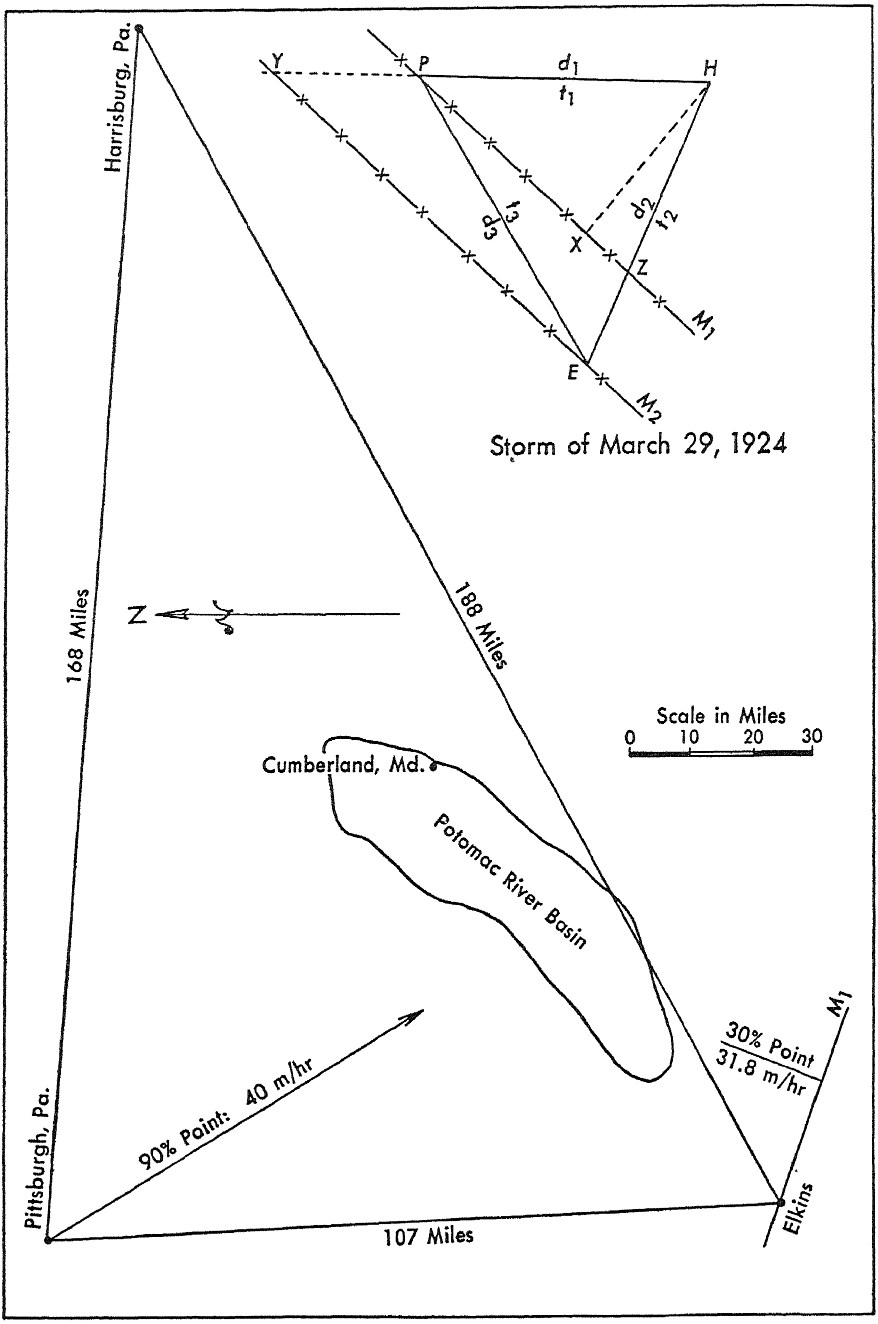



FIGURE 54. Shift of Precipitation, Storm of March 29, 1924

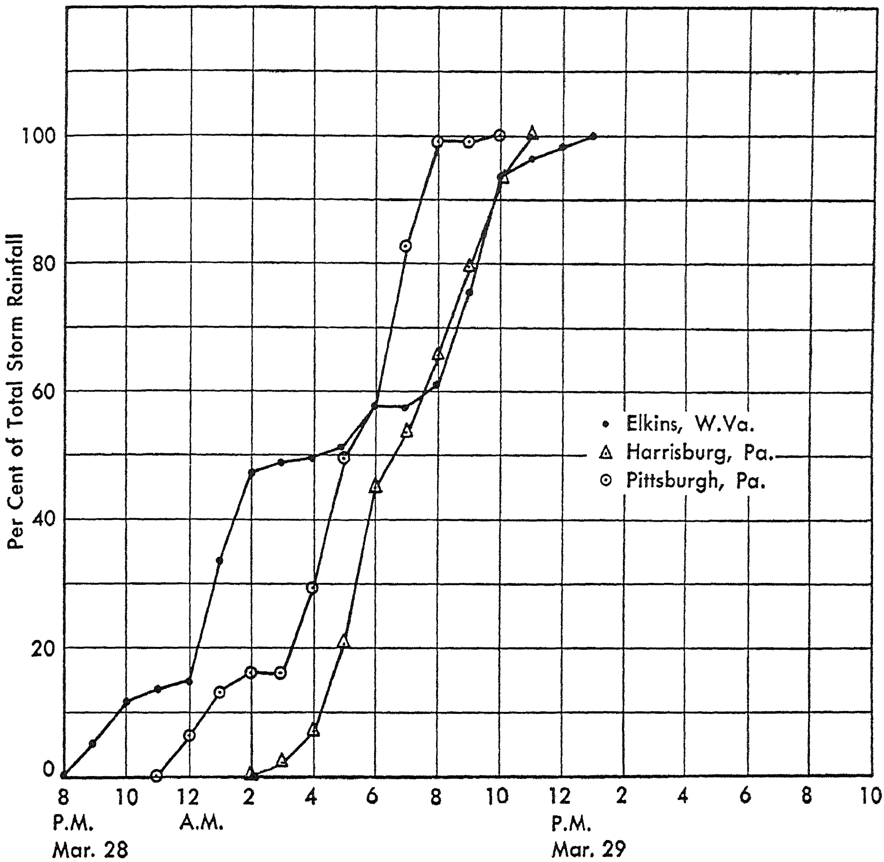



FIGURE 55. Accumulated Rainfall, Storm of March 29, 1924

passes point *P*. Neither of these assumptions is correct except approximately over small areas and through short periods of time, but making them provides a fair idea of the shift of the precipitation over a watershed. The method must be applied with a clear understanding that storms move, fronts change direction, and also that some fronts are made decidedly irregular by quasi-wave action. It is desirable therefore to use the above analysis on only small areas and continuous portions of a storm. The method will be applied to an example below in which the distances involved are rather large.

In order to determine the shifting of precipitation over the basin of the Potomac River above Cumberland, Md., the hourly records at Pittsburgh and Harrisburg, Pa. and Elkins, W. Va. are utilized for the storm of March 29, 1924. The summation curves of rainfall are shown in Figure 55. The computation for Harrisburg is given in Table 39 and is similar to those for the other stations.

TABLE 39. SUMMATION OF HOURLY PRECIPITATION, STORM OF MARCH 1924, HARRISBURG, PA.

| DATE AND HOUR | | RAINFALL *Inches* | SUMMATION *Inches* | PERCENTAGE OF TOTAL |
|---|---|---|---|---|
| 3/29/24 | 3 A.M. | 0.01 | 0.01 | 1.7 |
| | 4 | .03 | .04 | 6.9 |
| | 5 | .08 | .12 | 20.7 |
| | 6 | .14 | .26 | 44.8 |
| | 7 | .05 | .31 | 53.4 |
| | 8 | .07 | .38 | 65.5 |
| | 9 | .08 | .46 | 79.3 |
| | 10 | .08 | .54 | 93.2 |
| | 11 | 0.04 | 0.58 | 100.0 |

Figure 54 shows the stations in their positions relative to the basin. Applying the above formula at the time for 30 per cent of the rainfall, there are the following values:

$$d_1 = 168 \text{ miles}$$
$$t_1 = 1.40 \text{ hours (Pittsburgh to Harrisburg)}$$
$$t_2 = 4.55 \text{ hours (Elkins to Harrisburg)}$$
$$yH = \frac{(168)(4.55)}{1.40} = 546 \text{ miles.}$$

This result indicates that the rainfall is moving approximately northeasterly parallel to line $M_1$, practically down the basin, at about 31.8 miles per hour.

At the point of 90 per cent of rainfall Elkins and Harrisburg are both approximately 2.35 hours behind Pittsburgh, so that the rainfall is shifting parallel to a line through those points. The perpendicular line from Pittsburgh to that line divided by the elapsed time (2.35 hours) indicates a speed of approximately 40 miles per hour southeasterly.

Although this case is relatively simple, it illustrates a few of the changes during a storm that complicate this type of analysis. The cumulative rainfall curve for Elkins crosses both those for Pittsburgh and Harrisburg, while the latter remain substantially parallel to each other. The shift of rainfall veered from a northwesterly to a southeasterly direction. This is interpreted to mean that a warm front moved past Elkins while a cold front moved past Pittsburgh and somewhat later past Harrisburg and reached Elkins at approximately the 50 per cent point of rainfall. It may be safely inferred, however, that rain began falling on the upper (southwestern) end of the Potomac River basin and later fell substantially uniformly over the basin.

*Correlation of Precipitation Among Adjacent Stations.* Since precipitation varies from point to point during any storm and since even

annual averages of long records vary from place to place, the question of relationship between data of adjacent stations sometimes arises. This relationship is involved whenever a meteorologist attempts to extend or supplement a record at one station by data from nearby stations, or when an engineer wishes to utilize a given record in the study of precipitation for an area having no record. Frequently a general knowledge of the local climate, including the prevailing winds, usual air masses, and sources of moist air, will yield a satisfactory answer. But there is a limit to the distance over which the data of one station may be transposed. Moreover, the effect of relatively small topographical features frequently must be evaluated.

The question may be analyzed by methods of statistical correlation. These methods, which have been used for various purposes heretofore, are equally applicable to data of daily precipitation or annual totals. As may be properly inferred, however, the distance over which coefficients are comparable varies greatly among the types of data as well as the characteristics of the topography and climate between the selected stations.

To illustrate the correlation of annual precipitation between two not far-distant stations, the coefficient is computed below for the correlation of annual precipitation between Omaha and Lincoln, Nebr. The data for Omaha have been given previously; the data for

TABLE 40. ANNUAL PRECIPITATION FOR LINCOLN, NEBR.

| YEAR | PRECIPITATION *Inches* | YEAR | PRECIPITATION *Inches* | YEAR | PRECIPITATION *Inches* |
|---|---|---|---|---|---|
| 1878 | 34.55 | 1899 | 22.53 | 1920 | 26.19 |
| 1879 | 31.33 | 1900 | 33.72 | 1921 | 23.96 |
| 1880 | 22.70 | 1901 | 22.09 | 1922 | 24.77 |
| 1881 | 30.41 | 1902 | 41.22 | 1923 | 28.95 |
| 1882 | 29.52 | 1903 | 34.66 | 1924 | 21.91 |
| 1883 | 37.33 | 1904 | 27.72 | 1925 | 25.09 |
| 1884 | 26.89 | 1905 | 35.37 | 1926 | 26.24 |
| 1885 | 26.87 | 1906 | 34.08 | 1927 | 21.41 |
| 1886 | 28.50 | 1907 | 27.30 | 1928 | 27.88 |
| 1887 | 18.74 | 1908 | 35.69 | 1929 | 23.51 |
| 1888 | 25.70 | 1909 | 34.74 | 1930 | 20.74 |
| 1889 | 22.52 | 1910 | 31.33 | 1931 | 34.30 |
| 1890 | 15.12 | 1911 | 24.60 | 1932 | 26.19 |
| 1891 | 40.71 | 1912 | 22.39 | 1933 | 26.50 |
| 1892 | 29.62 | 1913 | 26.23 | 1934 | 17.23 |
| 1893 | 20.08 | 1914 | 40.02 | 1935 | 25.35 |
| 1894 | 19.15 | 1915 | 36.81 | 1936 | 14.09 |
| 1895 | 16.38 | 1916 | 23.05 | 1937 | 19.36 |
| 1896 | 38.07 | 1917 | 22.06 | 1938 | 28.35 |
| 1897 | 25.67 | 1918 | 22.26 | 1939 | 19.73 |
| 1898 | 28.10 | 1919 | 32.30 | 1940 | 23.17 |

Lincoln are listed in Table 40. Since the record of the latter city starts with 1878, the computation covers the period from 1878 to 1940.

Since there are considerable data to be used in this determination, it will save an appreciable amount of work to utilize a short method of computing the correlation coefficient. This method is based on the same formula but the elements are derived by statistical processes without finding each individual deviation.

Let $X$ = values of annual precipitation at Omaha, Nebr.
$Y$ = " " " " " Lincoln "
$p$ = the summation of the products of pairs of deviations from the mean

$$r = \frac{P}{(\sigma_x)(\sigma_y)}$$

$$P = \frac{\sum XY}{N} - (\bar{X})(\bar{Y})$$

$$\sigma_x = \sqrt{\frac{\sum X^2}{N} - (\bar{X})^2}$$

and
$$\sigma_y = \sqrt{\frac{\sum Y^2}{N} - (\bar{Y})^2}.$$

From the above mentioned data for Omaha, taken for the period 1878 to 1940, and the data for Lincoln, there are obtained the following values:

$$\sum X = 1703.57, \ \sum X^2 = 49{,}305.5443$$

$$\sum Y = 1703.30, \ \sum Y^2 = 48{,}625.3978, \text{ and } \sum XY = 47{,}796.5489.$$

From these values the following are derived:

$$\bar{X} = 27.04 \text{ in.} \qquad \sigma_x = 7.17$$

$$\bar{Y} = 27.04 \qquad \sigma_y = 6.38$$

$$p = 27.5138$$

and
$$r = \frac{27.5138}{(7.17)(6.38)} = 0.6015 \text{ (say 0.60)}.$$

Table 41 gives the correlation coefficient between Omaha, Nebr., and a number of precipitation stations on the surrounding central lowlands and adjacent Great Plains. Although there are no prominent topographic differences between the stations, it can be seen that there

is a noticeable variation in the value of the coefficient not accountable by distance alone. This variation is accounted for by differences in prevailing climate (65).

TABLE 41. CORRELATION COEFFICIENTS BETWEEN OMAHA AND SELECTED STATIONS

| STATION | PERIOD OF RECORD | MEAN *Inches* | COEFFICIENT | AIRLINE DISTANCE *Miles* |
|---|---|---|---|---|
| Algona, Iowa | 1871–1940 | 29.02 | +0.31 | 155 |
| Bismarck, N. Dak. | 1875–1940 | 16.26 | .45 | 450 |
| Blair, Nebr. | 1870–1940 | 28.48 | +.71 | 22 |
| Cheyenne, Wyo. | 1871–1940 | 14.61 | −.003 | 450 |
| Dodge City, Kans. | 1875–1940 | 19.87 | +.40 | 325 |
| Des Moines, Iowa | 1878–1940 | 31.87 | .72 | 125 |
| Fremont, Nebr. | 1878–1940 | 28.64 | .70 | 32 |
| Huron, S. Dak. | 1882–1940 | 19.26 | .37 | 245 |
| Kansas City, Kans. | 1878–1940 | 36.10 | .28 | 170 |
| Pueblo, Colo. | 1889–1940 | 11.51 | .26 | 455 |
| Rapid City, S. Dak. | 1888–1940 | 17.21 | .25 | 415 |
| Sheridan, Wyo. | 1894–1940 | 15.00 | .12 | 660 |
| Wichita, Kans. | 1889–1940 | 29.22 | +.41 | 265 |

Correlation of daily precipitation can be obtained by the same methods except that the use of the correlation table as illustrated in Table 6 will prove advantageous because of its labor-saving capability when used with large numbers of data.

Since daily precipitation depends entirely upon storm characteristics high correlation can be expected only between stations spaced at relatively close distances. In widespread storms of general orographic rains or perhaps some frontal action, correlation can be expected to be good. On the other hand, light showers or convective thunderstorms cover relatively little area and may miss even the nearest adjacent station; where such precipitation is common, correlation may be low.

# 6 FREQUENCY OF PRECIPITATION

*Definition of Frequency of Precipitation.* Fundamentally, "frequency of precipitation" denotes its occurrence in relation to time, but the term is commonly used with some limitation as to magnitude and period. The term can properly be applied to the number of days in a year on which rain occurs, or stated inversely, the number of days between recurring rains. More commonly, the term "frequency" is used with a stated or implied amount of precipitation in such a way that a given depth may be expected to be equaled or exceeded in a designated number of years or other unit of time.

*Need for Rainfall Frequency.* The greatest need for data of rainfall frequencies is felt in various economic or related studies. The capacities of sewerage and drainage works are usually designed on the basis of a certain depth of rainfall to be expected during a selected period of time. Flood control works are frequently designed according to a similar criterion, particularly if the loss of human life is not to be guarded against and the protective works are proposed primarily as insurance against property loss. Farm terraces and highway and railway culverts are other structures that are designed in consideration of the frequency of certain rainfall intensities.

Rain interferes with many outdoor events such as baseball and football games, circuses, fairs, and similar exhibits. In this case the magnitude above a given minimum is important, and the frequency of the rainfall damaging the venture is a measure of the risk to be taken. Loss due to rain may or may not be covered by insurance, but in any case there is a definite risk which must be taken by someone.

*Data for Frequency.* Data for computing the frequency of precipitation consist in depths or amounts of rain (or snow in its water equivalent) for various periods of time. The length of the unit of time selected varies with the purpose for which the frequency studies are to be used. For small areas as used in sewer design, frequencies are needed for

short periods of time varying from five minutes to two hours. For such work as drainage and flood control the unit may be a day or multiples of a day. Frequency of precipitation in annual amounts is needed for long-range planning for water supply, irrigation, and power development. These types of frequencies will be considered in the foregoing order, after first discussing some empirical methods of determining frequencies of short periods of time, such as an hour or fraction thereof.

*Some Empirical Methods.* The importance of knowing what rates of rainfall may be expected and how often they may occur is great enough to induce many investigators to devise means of analyzing available data to determine the frequency of such rates. Each investigator usually has produced his own formula which was derived from the data of a specific locality, and the constants at least would be expected to vary from place to place. However, the usual formula is of the form

$$i = \frac{K}{t + a} \qquad \text{or} \qquad i = \frac{K}{t^a}$$

where $i$ is the amount of precipitation to be expected in the specified interval of time, $t$ the time in minutes, and $K$ and $a$ are constants. Meyer (133) lists 22 investigators who have proposed rainfall formulas of the above types for various localities in the middle and eastern portions of the United States. Each formula was devised for a given locality and for the solution of specified problems, such as for "maximum rainfall," "basis of design flood," or "once in a given number of years."

Mr. Meyer himself prepared a series of 35 formulas to determine the amounts of intense rainfall in time intervals of less than two hours, and a similar series of 35 formulas to determine the hourly rates to be expected in specified intervals of years. These formulas are applicable to the portion of the United States east of the Rocky Mountains, which he divided into five areas corresponding roughly to the exposure to tropical maritime air masses. For each area a formula, differing in the value of its constants only, is devised for frequency intervals of 1, 2, 5, 10, 25, 50, and 100 years. The formula to obtain the amounts of precipitation is of the form

$$i = \frac{Kt}{t + a}$$

the notation being as given above.

The formula designed to give the hourly rates in inches of precipita-

tion to be expected has a constant $B$ for each situation, as follows:

$$R = \frac{B}{t + a}$$

in which $R$ is the rate in inches per hour. The coefficients for the formula for the amount of precipitation are given in Table 42. Note that there are five areas designated I, II, III, IV, and V, which are shown by Meyer (133) on a map not reproduced here.

TABLE 42. COEFFICIENTS IN MEYER'S FORMULAS FOR INTENSE RAINS

| FREQUENCY | $K$ | | | | | $a$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | AREA | | | | | AREA | | | | |
| *Once In Years* | *I* | *II* | *III* | *IV* | *V* | *I* | *II* | *III* | *IV* | *V* |
| 1 | 2.42 | 1.67 | 1.20 | 1.00 | 1.00 | 23.0 | 18 | 13 | 15 | 13 |
| 2 | 3.00 | 2.18 | 1.60 | 1.40 | 1.25 | 24.5 | 21 | 16 | 16 | 13 |
| 5 | 3.67 | 2.85 | 2.03 | 1.80 | 1.50 | 27 | 23.5 | 18 | 17.5 | 13 |
| 10 | 4.60 | 3.57 | 2.50 | 2.20 | 1.75 | 32 | 26 | 19.5 | 19 | 13 |
| 25 | 5.91 | 4.20 | 3.02 | 2.67 | 2.10 | 40 | 28 | 21 | 20 | 14 |
| 50 | 7.50 | 4.81 | 3.60 | 3.10 | 2.53 | 50 | 30 | 23 | 21 | 16 |
| 100 | 10.00 | 5.41 | 4.27 | 3.50 | 3.00 | 65 | 32 | 25 | 22 | 18 |

These coefficients are shown graphically in Figure 56.

*Miami Conservancy Investigations.* A particularly notable study of the frequency of intense precipitation occurring in periods of one to six days was made in the investigations for the flood control projects of the Miami Conservancy District. This study covered the eastern half of the United States which was divided into quadrangles two degrees of latitude wide by two degrees of longitude in length. By combining the records of all precipitation stations of a quadrangle into one record, the frequency of the greatest precipitation and the depth of the second, third, and lower orders of magnitude were found. Isopluvial lines were drawn on maps to show depths of rainfall for given frequencies. The results were published (135) as charts showing the isopluvial lines for the given frequencies.

*Frequency of Intense Rainfall at Boston, Massachusetts.* In a study of the frequency of intense precipitation at Boston, Mass., Sherman (164) introduced a slightly different formula in the form

$$i = \frac{K}{(t + b)^d}$$

in which $K$, $b$, and $d$ are constants, $t$, the time in minutes, and $i$ the rate of precipitation in inches per hour. He found that for the data he used, $b$ and $d$ equaled 7.0 and 0.7, respectively, and that $K = 16F^{0.27}$, where $F$ was the frequency in years. On this basis he found that the frequency of rainfall rates in inches per hour covering various periods of time from

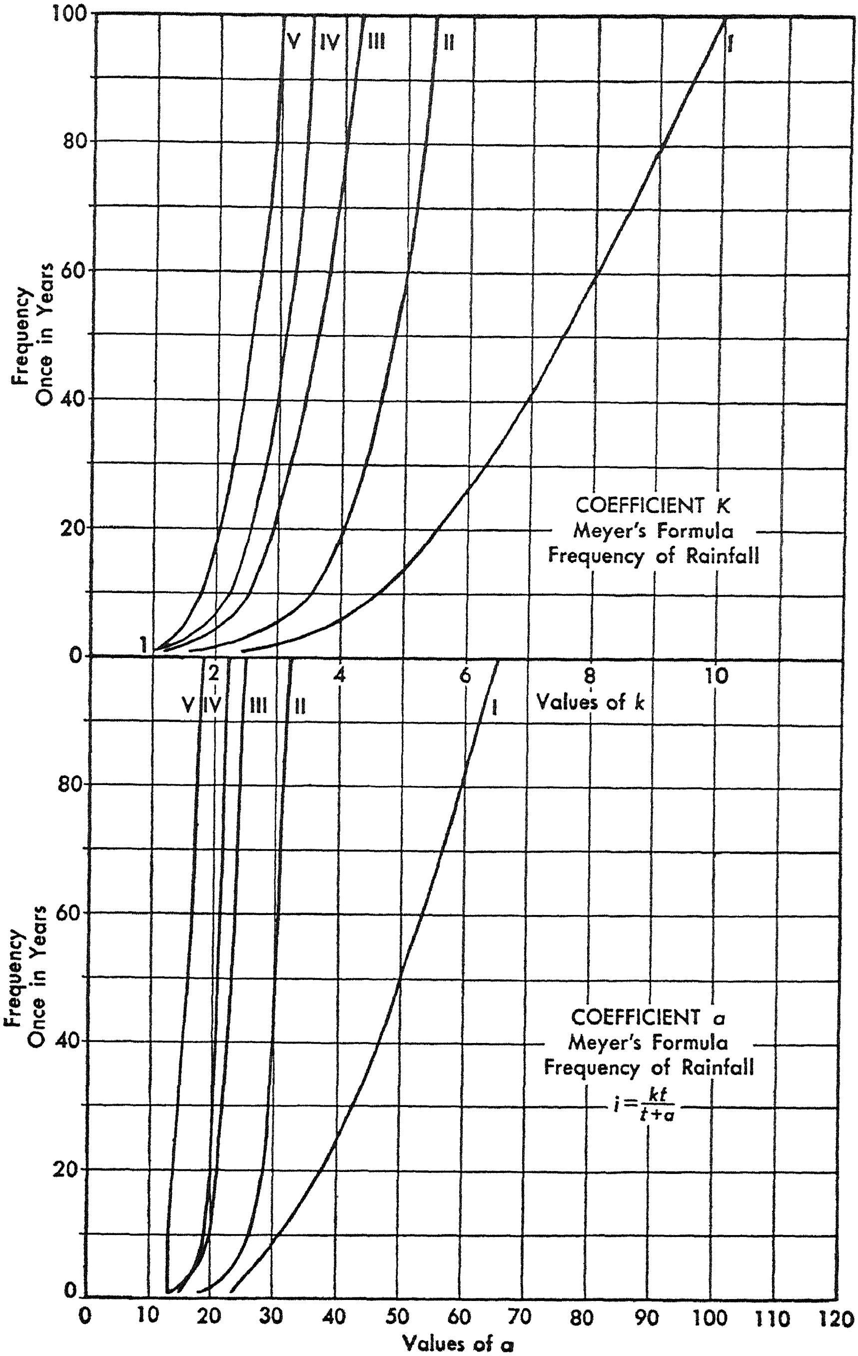



FIGURE 56. Coefficients for Meyer's Formula for Frequency of Rainfall

0.1 to 70 hours could be represented by a series of parallel curves. By the use of the above formula Sherman found that frequencies could be determined for storms of longer duration than the one or two hours to which previous investigators had limited their study.

*Rainfall Studies in New York.* Studies of intense rainfall were made by Bleich (21) who used and compared all three of the foregoing types of formulas and in addition determined expected frequencies by means of the type III frequency curve of Karl Pearson. His data and studies were limited to periods of rainfall of 120 minutes. He concluded that the form of

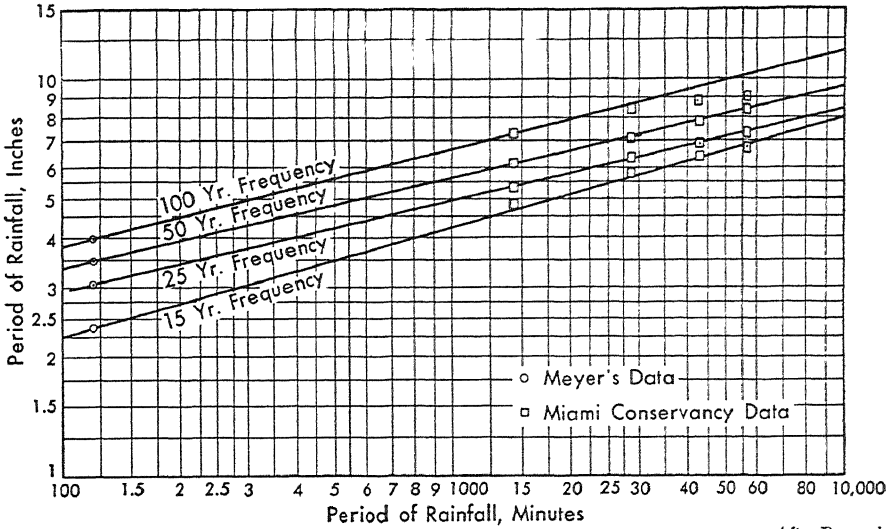
$$i = \frac{K}{(t + b)^d}$$

was to be recommended for use in New York. However, it is to be noted that his numerical coefficients were derived by the method of least squares and for that reason should fit best the particular set of data from which they were derived.

*Frequencies of Intermediate Periods.* With the exception of the Boston studies by Sherman and the studies by the Miami Conservancy District, the investigations described in the foregoing paragraphs were limited to duration of rainfall of not over two hours. Since the shortest period considered in the Miami investigations was one day, there is a gap between 120 and 1440 minutes not covered by a general formula, Sherman's investigations being limited to Boston. Nevertheless, data of rainfall and consequent runoff for intermediate periods in the gap are needed in design, for example, of storm sewers, flood control, and drainage.

Bernard (18) undertook to fill in the gap from the previously published data of the Miami Conservancy District reports and Meyer's work, discussed above. The total expected precipitation for periods of 120 minutes from Meyer's data and 1, 2, 3, and 4 days (1440, 2880, 4320, and 5760 minutes) from the Miami data for various frequencies was plotted on logarithmic paper; it was found that all data plotted substantially on a straight line. An example is shown in Figure 57. From this step there was finally obtained a series of constants to fit in formulas of the exponential type, $i = C/t^n$. This process was repeated for each of the quadrangles covering the eastern portion of the United States in the Miami investigation. The two coefficients $C$ and $n$ were then plotted on maps covering the same region. These maps were published in the paper referred to above (18).

*Yarnell's Investigations.* An extensive investigation of frequency of intense rainfall was made under the direction of David L. Yarnell of the




*After Bernard*

FIGURE 57. Relationship of Meyer's and Miami Conservancy Studies, Quadrangle 15E

United States Department of Agriculture. The results were issued as a publication (200) of that Department.

In this investigation 28,077 rainstorms covering all the United States were analyzed. Intensity-frequency diagrams were prepared for each station by plotting rainfall in inches against frequency in years. Data were taken from these diagrams to plot on maps the isohyetal lines representing rainfall to be expected for a given duration of rain and a given frequency in years. These representations of frequency and intensity, which were the final results of the investigation, were published in the form of the isohyetal maps. The expected rainfalls for given frequencies were derived by purely empirical operations, but since the maps take into consideration an appreciable area, the depths should in general be more reliable than figures derived by empirical methods from data at one station. The maps could not, however, take into account all variations of local topography which, as was shown in the previous chapter, can have important effects.

*Utility of Empirical Rainfall Formulas.* That there is an insistent demand for means of estimating expected rainfall is shown by the many studies that have been made for that purpose. The demand has come from the needs of sewerage design, storm sewerage, farm drainage, and more lately, interior drainage behind flood control works and flood protection from the smaller drainage areas. The results of the studies have provided the best available answer for specific local design purposes, in spite of the variation in formulas and coefficients.

Nevertheless, in spite of their great utility the above methods are subject to the defects inherent in all empirical methods. They are limited in their application to the data from which they have been derived, since there is no unifying theory upon which they are based and by which they can be compared and evaluated as to suitability for estimating the occurrence of future rainfall. It should be noted that all these methods are for the purpose of estimating rainfall to be expected in the future; there is no need for determining past expectancies which are a matter of record.

*Defects of Observations.* There are some aspects of the above methods common to all frequency studies that should be kept in mind when using the predicted rainfalls. The data of depths of rainfall are observed for fixed and arbitrary consecutive periods of time, such as an hour or a day. The amount falling within that period is treated as if uniformly distributed over the period. Thus 1.0 inch may be the rainfall recorded for one hour, whereas it actually may have fallen within half that time with no rain falling for the remainder of the time. Similarly a rain may be ascribed to one day while falling within 10 or 12 hours. This discrepancy is inherent in the data because of the methods of observation and cannot readily be eliminated by methods of analyses; it can be offset by considering for design purposes the higher rates falling in shorter periods of time.

The arbitrary periods of time of observation fixed by the clock or calendar also frequently split a period of heavy rainfall. For example, a period of intense rainfall may last from 1:35 to 2:45 P.M. It would be recorded in hourly periods as falling from 1:00 to 2:00 P.M. and from 2:00 to 3:00 P.M. The two separate hours would necessarily each be credited with lower rainfall than occurred, say, from 1:35 to 2:35. This factor tends to reduce the *recorded* intensities of precipitation; the defect would be inherent in any data taken for consecutive uniform periods of time.

Another consideration that should always be kept in mind in using any method of analysis is the sparsity of rain-gaging stations which permits many of the peaks of rainstorms to occur amidst them so that the maximum precipitation is not recorded. For example, note the 14-inch storm near Lincoln, Nebr., May 11, 1942, described in the preceding chapter. A "bucket survey" of intense storms and subsequent adjustment of expected rainfall is the only practicable answer to this situation at present. In an attempt to produce a method of analysis that will place the estimating of precipitation on a sound theoretical basis, consideration will now be given methods of statistical analysis as applied to precipitation.

*Basis of Theoretical Probability.* The applicability of theoretical formulas to the determination of frequency of precipitation is a matter that must be settled finally by judgment and verified whenever possible by mathematical processes applied to all known pertinent facts after these are assembled and analyzed. The basis for using such methods lies in the causes of precipitation itself, and a review of those factors is not out of place at this time.

The source of precipitation is the moisture in the air. It has previously been shown that the amount of atmospheric moisture is variable and that data of humidity may be analyzed by statistical methods including the computation of frequencies. The daily values of moisture content have likewise been shown to be substantially independent of those for the preceding and succeeding days. Furthermore, it is seen that it is possible to experience high humidity without precipitation, since precipitation requires in addition other factors for its occurrence.

Some type of storm or atmospheric disturbance is necessary to produce precipitation. At a given point or locality there is generally a variety of storms which can cause precipitation, each of which can result in varying depths of rain or snow. Not all types of storms are equally effective in precipitating moisture because each has different possibilities of collecting, converging, and lifting moist air masses. Variations of wind velocities and directions, slopes of fronts, convection, topography, and thermal conditions of land surface all enter into the production of the storm precipitation. All these factors will vary in general independently of each other and when operating, will have variations in their effectiveness.

Chance variation is seen to be the outstanding characteristic in the deposition of atmospheric moisture. A given quantity of precipitation is, for all intents and purposes, the result of chance variations of the independent causative factors. This conclusion does not abrogate the law of casuality, since that cannot conceivably be abrogated, but it means that the various causes of precipitation operate independently, that they have a wide range of variation, and that a proper combination of the various necessary factors, each having its effective magnitude, is necessary to produce a given amount of precipitation. It is also possible that a like amount may be produced by a somewhat different combination of factors varied in the proper degree of magnitude to be equally effective. However, it seems probable that the number of combinations that would produce exactly the same amount of precipitation, no more and no less, must be small compared to the number that would produce different amounts of precipitation.

The foregoing description points out that the fortuity involved in

causing precipitation approximates the conditions of substantial independence postulated for the fundamental theorems of probability. The final effect, precipitation, is plainly a compound event dependent upon the happening in sequence or simultaneously of a number of causative events more or less independent. Furthermore, the total precipitation of any given storm happens only once so that the set of causative events that actually occurs can happen only once and other similar events that would cause different precipitation must fail. Hence the probability of the final event is the product of the probabilities of the favorable and unfavorable causative events. It is beyond the scope of this book to enter into an adequate discussion of the development of the theory of probability, for which the reader is referred to some treatise such as Fry's (68), but the theory will be utilized in the following paragraphs for developing the basis of frequency of precipitation.

The application of statistical methods may be approached by the following lines of reasoning. Let $p_1, p_2, p_3, \cdots p_n$ be the probabilities of occurrence of the various factors contributing to any given amount of precipitation, and $q_1, q_2, q_3, \cdots q_n$ be the probabilities of occurrence of the factors failing to contribute. Out of a total of $m$ meteorological factors operating a sufficient number must occur to produce the given precipitation; that is, there must be $n$ factors with favorable probabilities $p_1, p_2, p_3, \cdots p_n$. There may also be a number of causative factors with probabilities $q_1, q_2, q_3 \cdots q_{(m-n)}$, not operating or not operating in a sufficient degree to produce the precipitation. The events with favorable and unfavorable probabilities then constitute a chance combination, the compound probability of which may be expressed thus,

$$P(C_n^m) = (p_1 \times p_2 \times p_3 \times \cdots \times p_n)(q_1 \times q_2 \times q_3 \times \cdots \times q_{(m-n)}).$$

$C_n^m$ is the combination of $m$ things taken $n$ at a time and $P(C_n^m)$ is the probability of the combination.

We may assume an average value $p$ for the various probabilities of factors contributing to the observed rain or other precipitation, and an average value $q$ for the probabilities of the factors failing to contribute. The individual values of all the various probabilities may not approximate the average since there will very likely be a considerable variation among them. An arithmetical average may not be the proper one since it may be unduly influenced by one or more extreme values, but we may suppose that such cases can be cared for by a geometric mean. As we shall not perform any arithmetical work of this sort, difficulties of computation will be side-stepped. Theoretically we need only mean values of $p$ and $q$ such that the product of the means each raised to a

power equal to its number of factors will equal the product of the individual $p$'s and $q$'s themselves. Assuming that this is obtainable, we can then express the compound probability as

$$P = C_n^m p^n q^{(m-n)}.$$

This equation is the well-known binomial law and is the starting point for a large part of the theory of probability.

Fry (68) has shown that from the above equation there can be derived the so-called "normal probability function," when $m$ substantially equals $n$. Likewise the Gram-Charlier series can be obtained from the above binomial law. Starting with the normal probability function, Fisher obtains one logarithmic transformation and Slade obtains another. These functions are all listed in Chapter 1, together with references to publications in which the derivation may be found.

Before proceeding to apply the foregoing formulas it will be well to examine a number of distribution histograms of precipitation data. The data of rainfall are divided into two types, one consisting of the number of days of rain per annum and the other showing the variation of magnitude. These distributions are very different and should be separated in any theoretical treatment of frequency of precipitation.

*Typical Distribution of Number of Rainy Days.* Typical distributions of the number of rainy days occurring in a series of years are shown in Figures 58 and 59. Figure 58 shows two stations in North Dakota and the histograms are of the number of rainy days observed for six months only, April to September inclusive. The other figure is constructed from data of two New England stations; both histograms include data of the full year and thus include data for precipitation in the form of snow. The mean number of days is seen to be close to the axis of each histogram. In all cases the number of years is relatively small so that close fits to any given theoretical norm could not be expected, although the shape in histograms approximates the bell-shaped curve of the normal probability function. This conclusion is supported in Figure 60 by the plotting on probability coordinates on which the points fall approximately on a straight line.

For the figures shown, the statistical parameters are as given in Table 43.

TABLE 43. PARAMETERS OF DISTRIBUTION OF RAINY DAYS

| | BISMARCK | DICKINSON | DURHAM | GREENVILLE |
|---|---|---|---|---|
| Mean | 51.0 | 51.3 | 103.9 | 145.2 |
| Standard Deviation | 8.7 | 8.5 | 15.9 | 18.5 |
| Coefficient of variation | 0.17 | 0.17 | 0.14 | 0.13 |

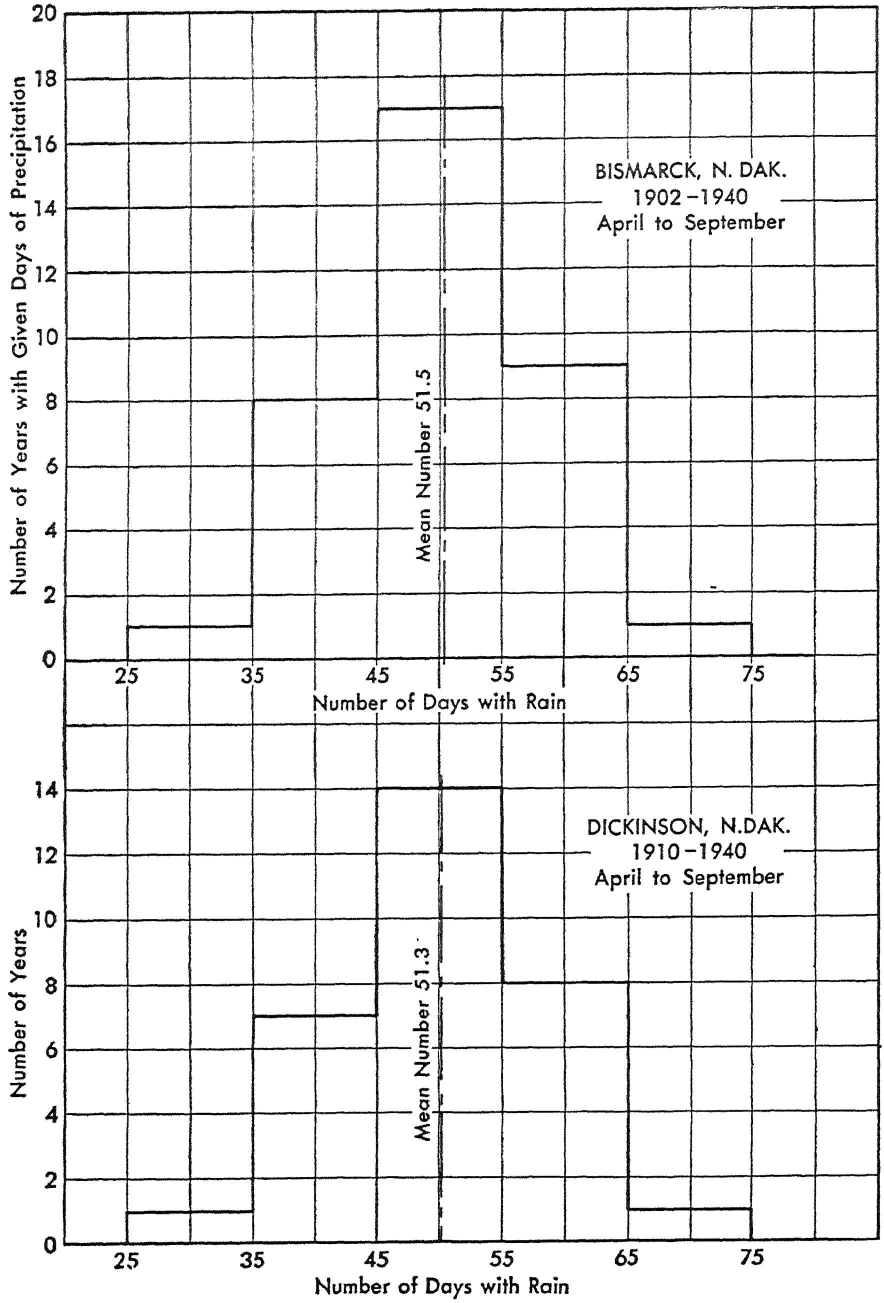



FIGURE 58. Histograms of Number of Rainy Days

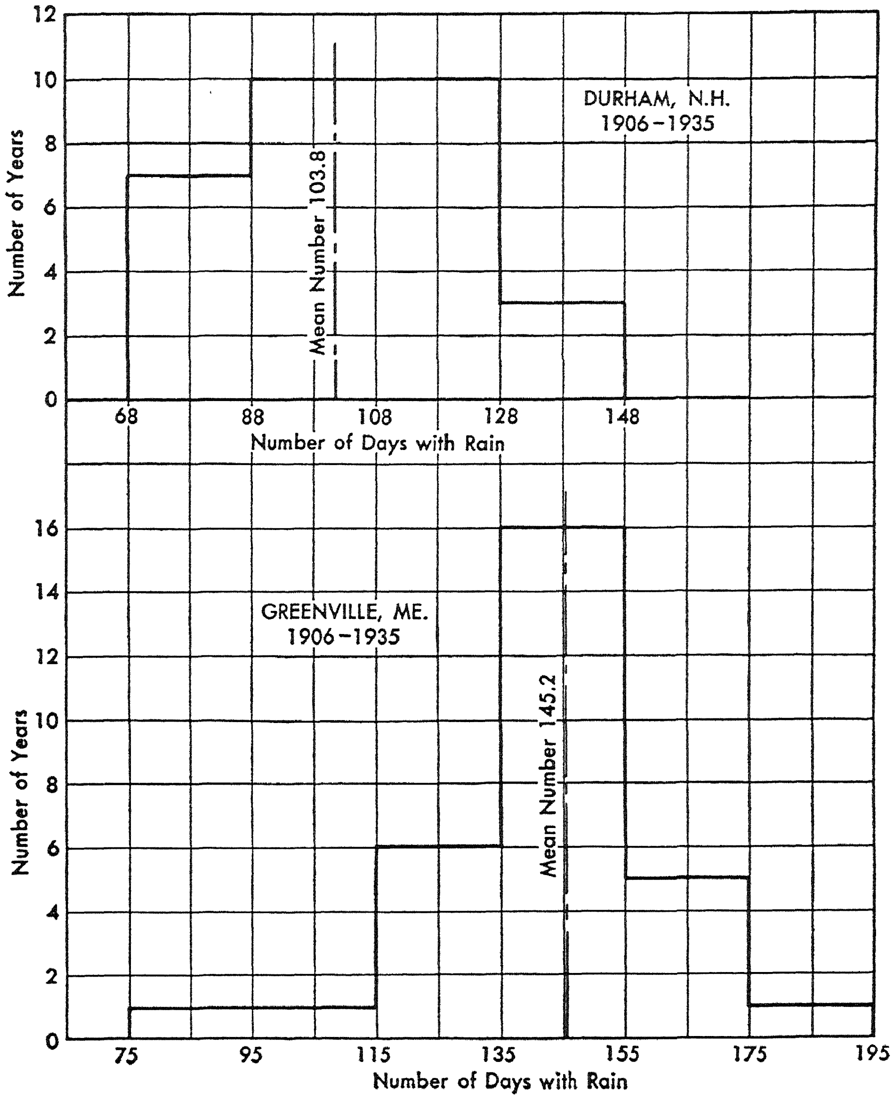



FIGURE 59. Histograms of Number of Rainy Days

The paucity of data resulting from the short record does not justify further tests to determine goodness-of-fit, but for the present it can be assumed that the above distributions approximate the normal law. While four examples do not prove a case, it may be reasonably inferred from the known variations in precipitation that distributions of the annual number of days of rain follow approximately the normal law. No practical purpose is served, however, by computing the frequency of a given number of days of rain in one year and therefore no such determinations are made.

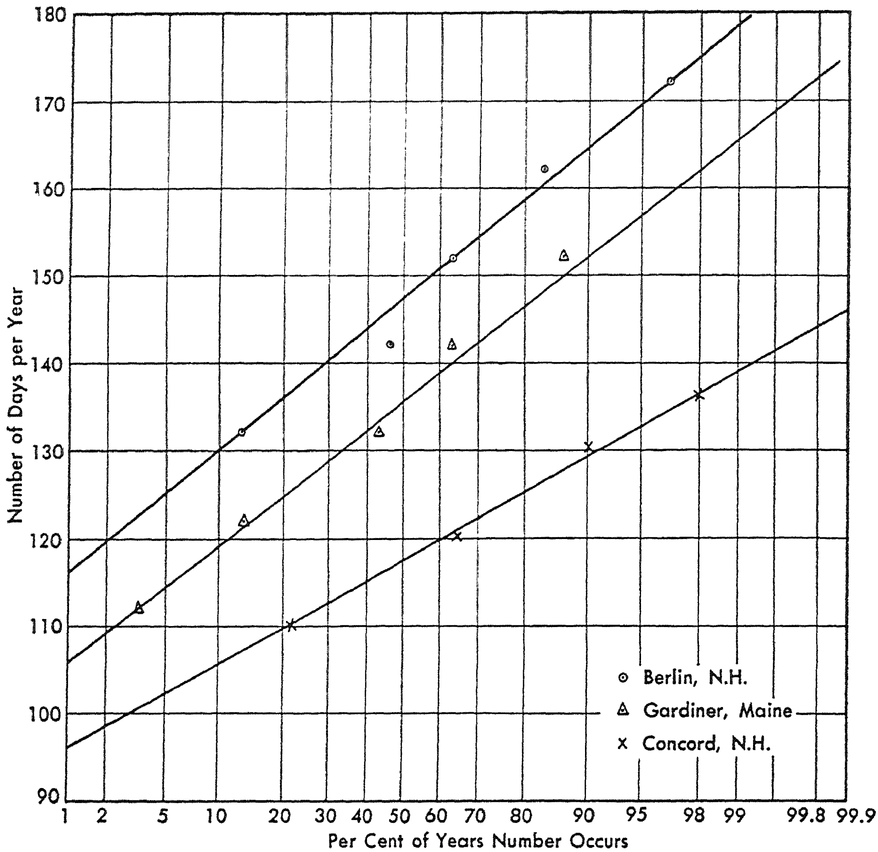



FIGURE 60. Frequency of Number of Rainy Days per Year

*Typical Distributions of Depths of Daily Precipitation.* The second type of distribution histograms are those of the magnitudes of daily precipitation. The data for these histograms, shown in Figures 61–67, were obtained from precipitation stations in various parts of the United States between the Atlantic seaboard and the Rocky Mountains, and as noted on the figures, a part of them include data for the entire year and the remainder cover only the six warm months from April to September.

The histograms of the magnitude of daily precipitation are conspicuously different from those of the number of rainy days; the largest group, the class with the largest number of days, contains those with the least precipitation. The resulting distributions are therefore of extreme skew; in fact, these distributions are the most skewed type to be found in hydrology. In the examples given above in those from the eastern region the mean daily precipitation is somewhat less than the

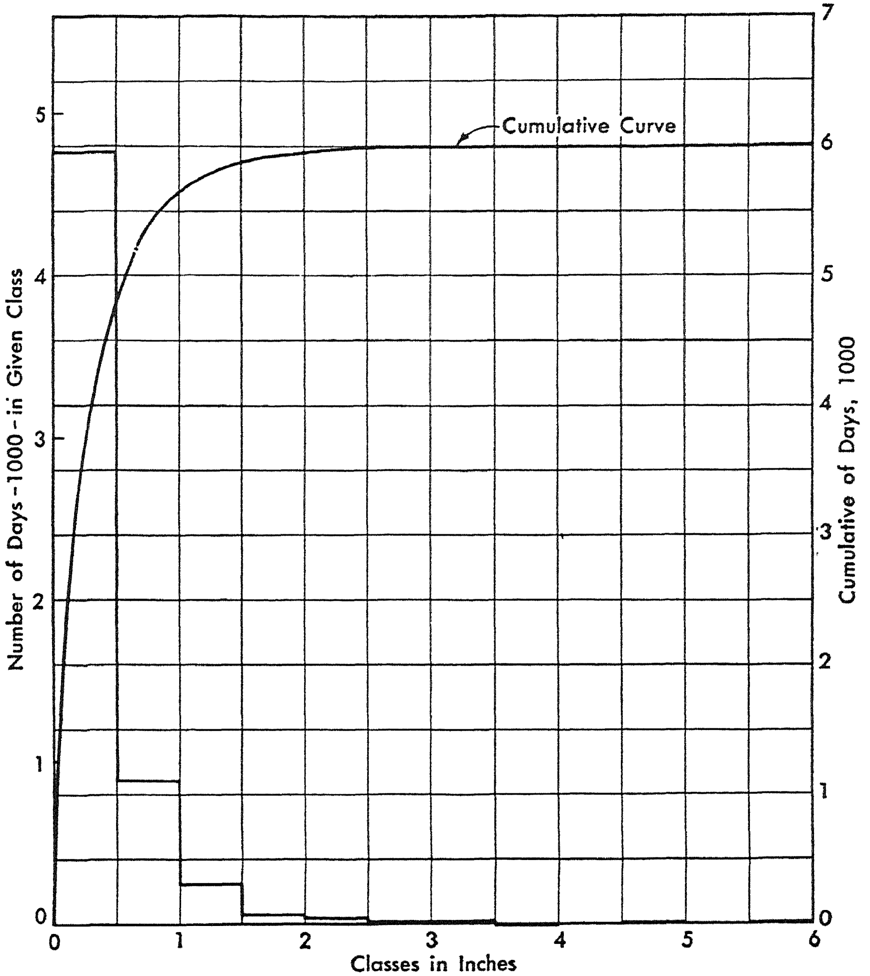



FIGURE 61. Histogram of Days of Precipitation, 1885–1935, Concord, N. H.

upper limit of the first class, and from the western areas the mean daily rainfall approximates the first upper class limit.

For comparative study a number of the above distributions have been plotted on logarithmic probability paper. These plottings are shown in Figures 68–72.

*Theoretical Functions for Distributions of Daily Precipitation.* The extreme skew of the foregoing distributions limits the number of theoretical functions by which frequency can be computed; the normal probability function cannot be used. The characteristic of the data used to compute frequency is a continuous variable; in other words, each magnitude is not a discrete unit but is continuous with the next. For this reason a function containing a continuous variate should be

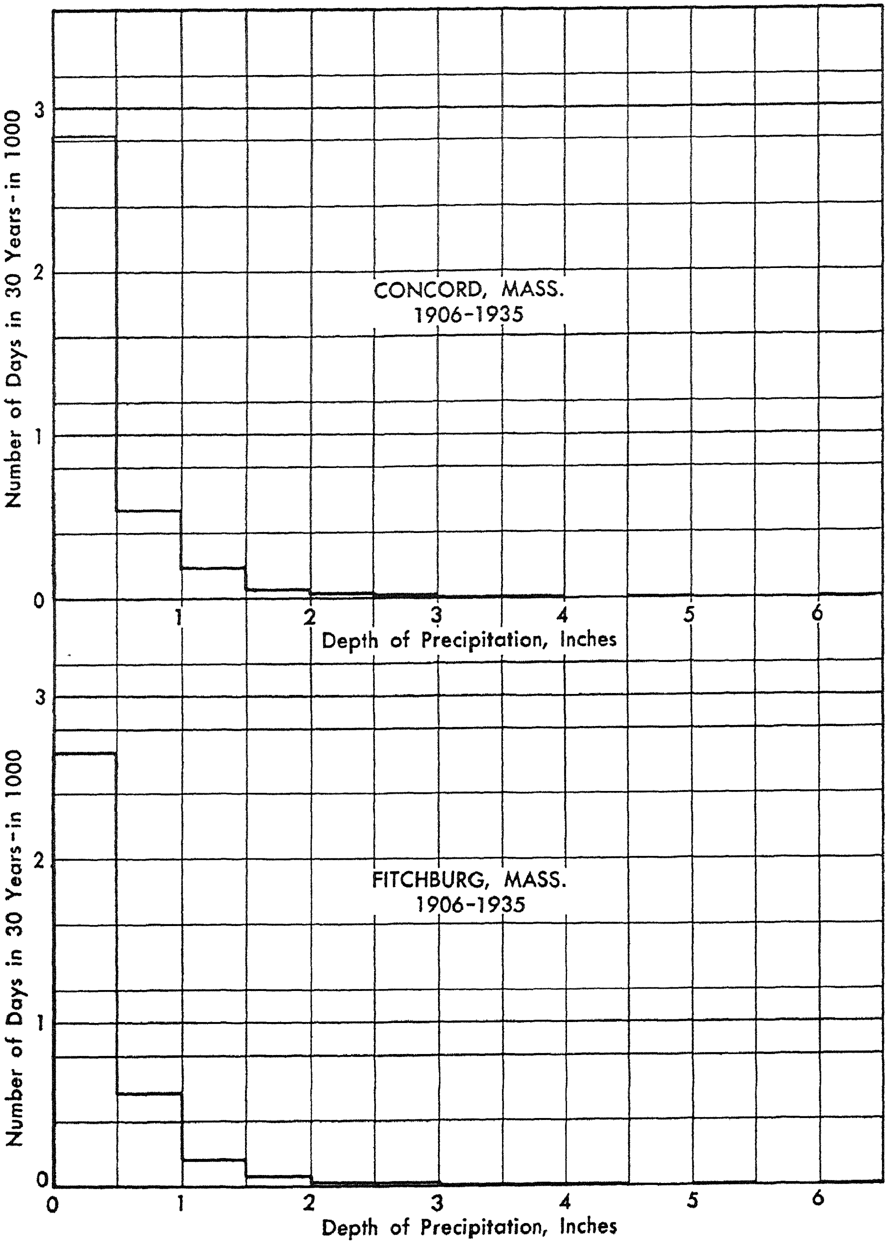



FIGURE 62. Histograms of Days of Precipitation

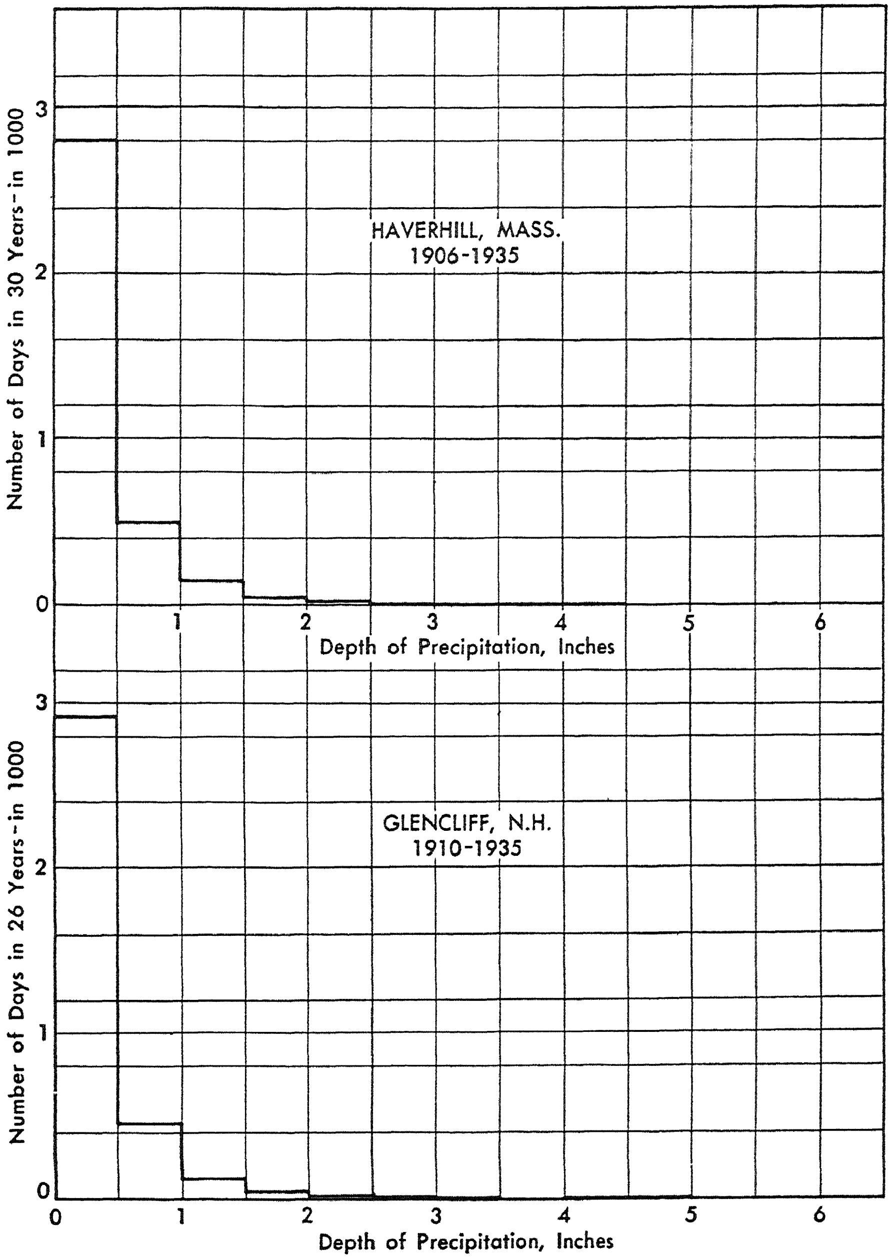



FIGURE 63. Histograms of Days of Precipitation

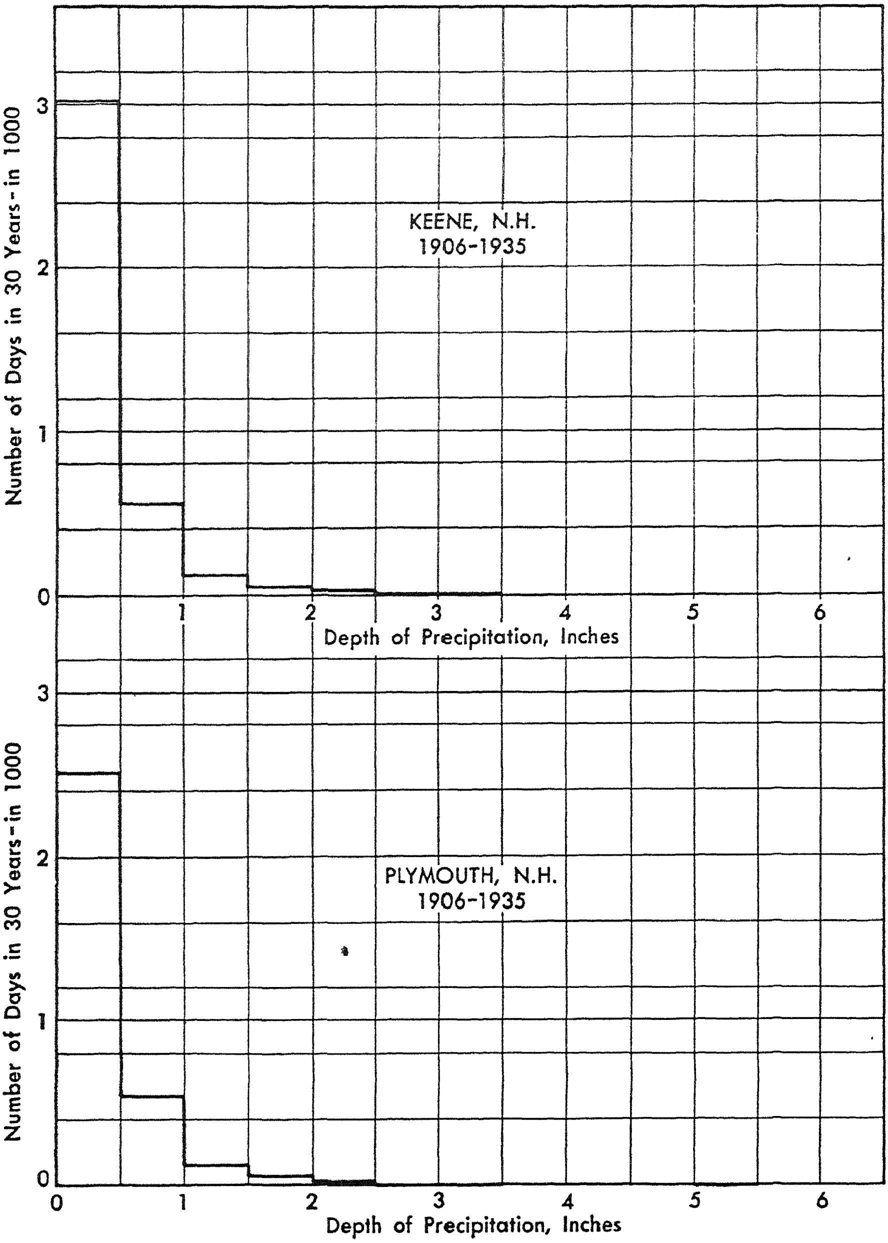



FIGURE 64. Histograms of Days of Precipitation

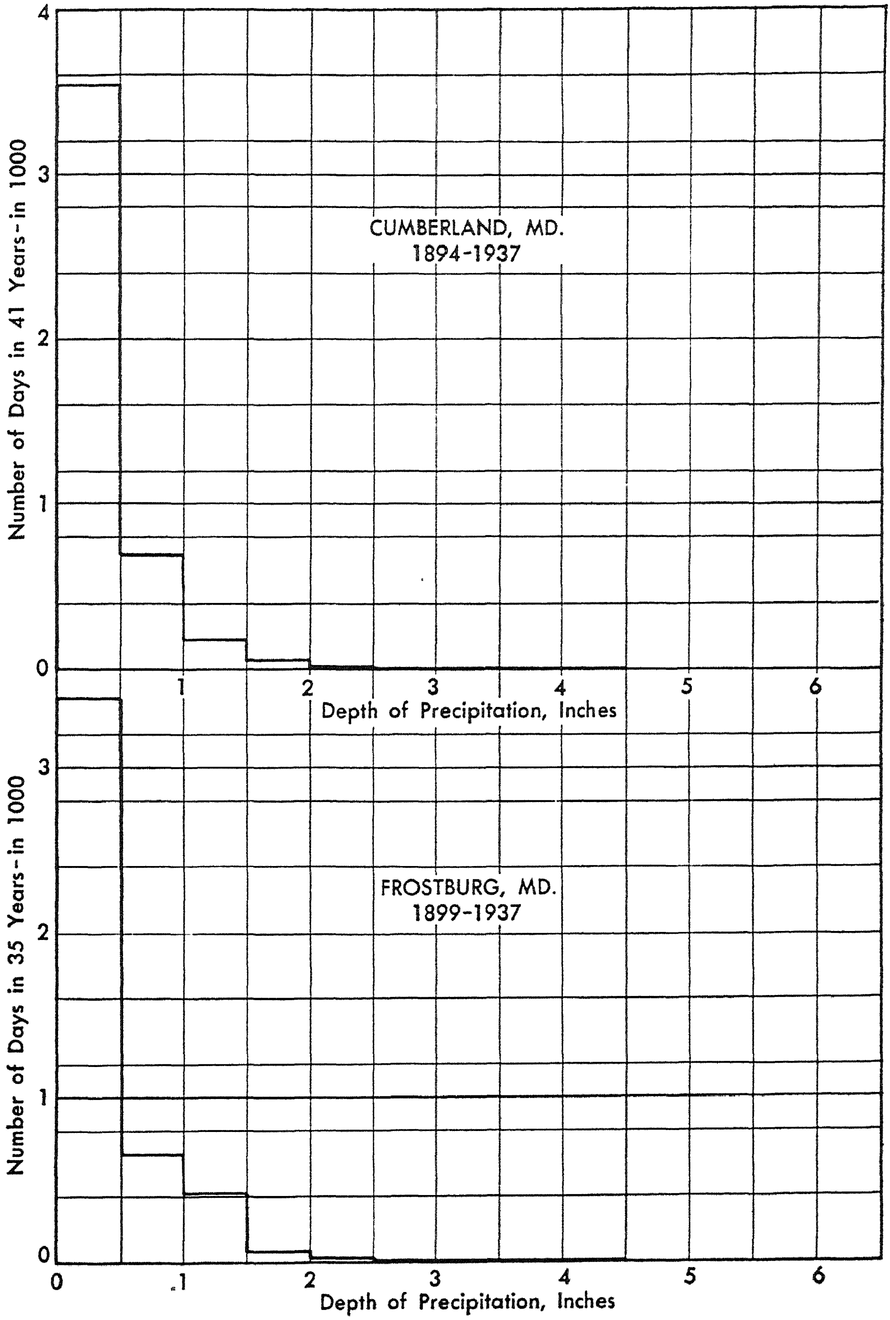



FIGURE 65. Histograms of Days of Precipitation

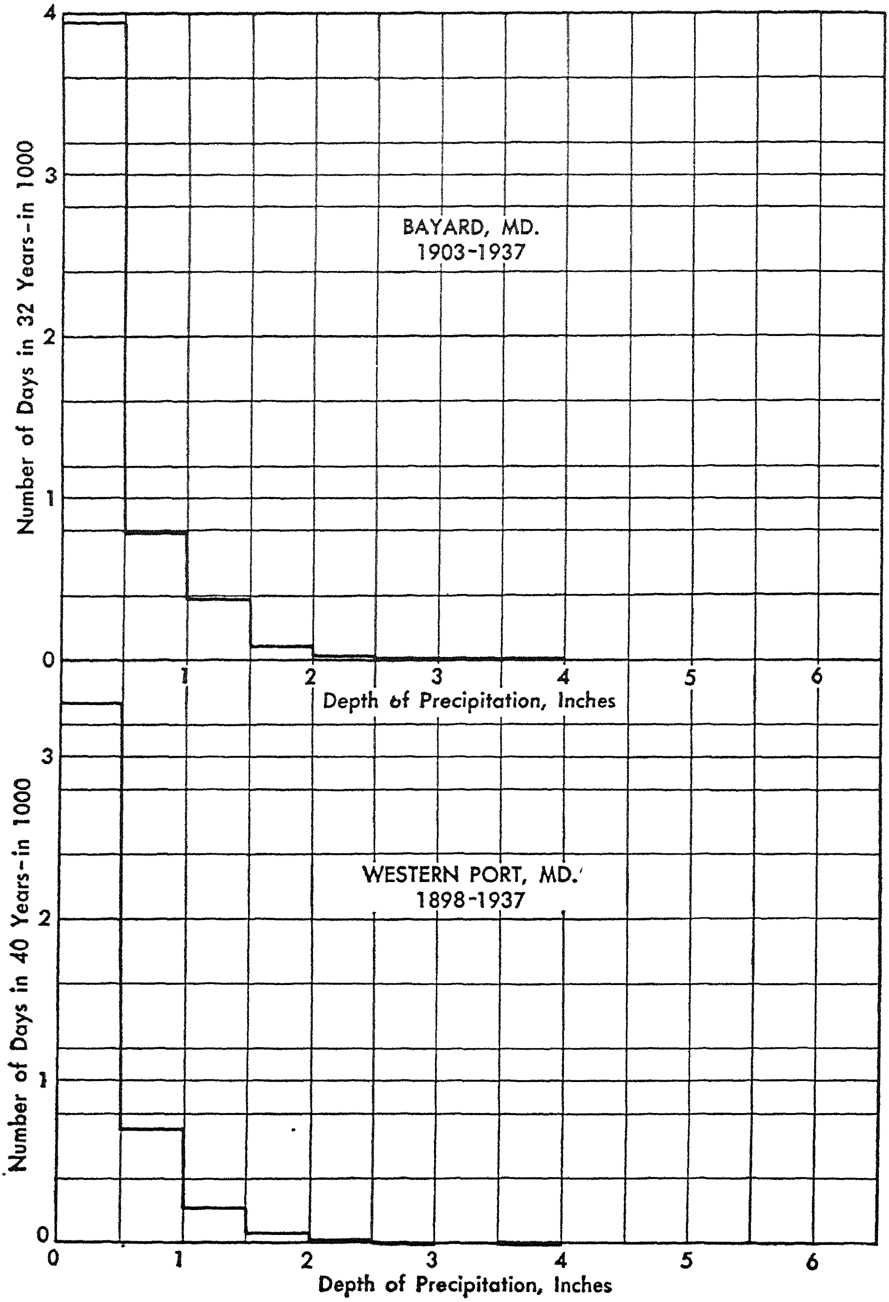



FIGURE 66. Histograms of Days of Precipitation

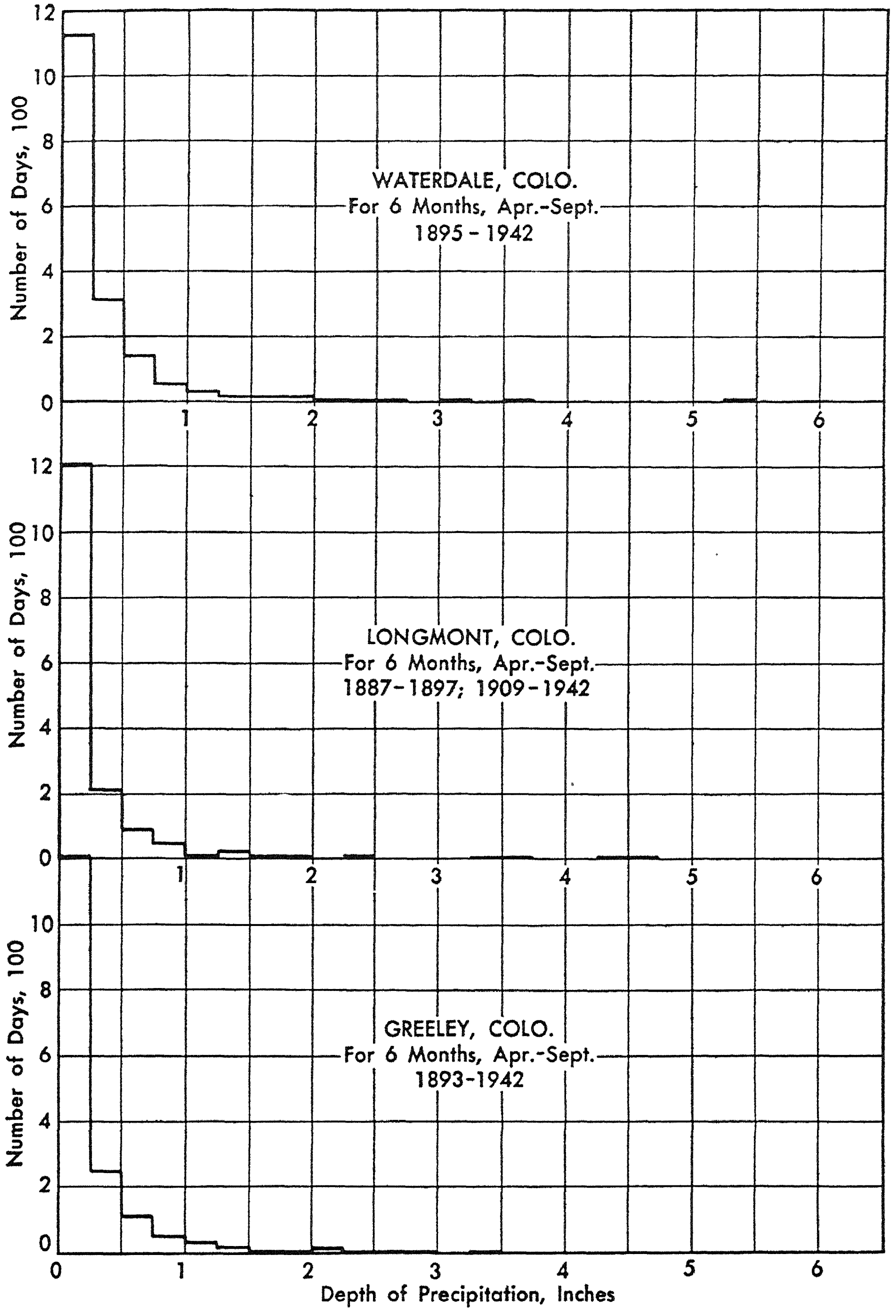



FIGURE 67. Histograms of Days of Precipitation

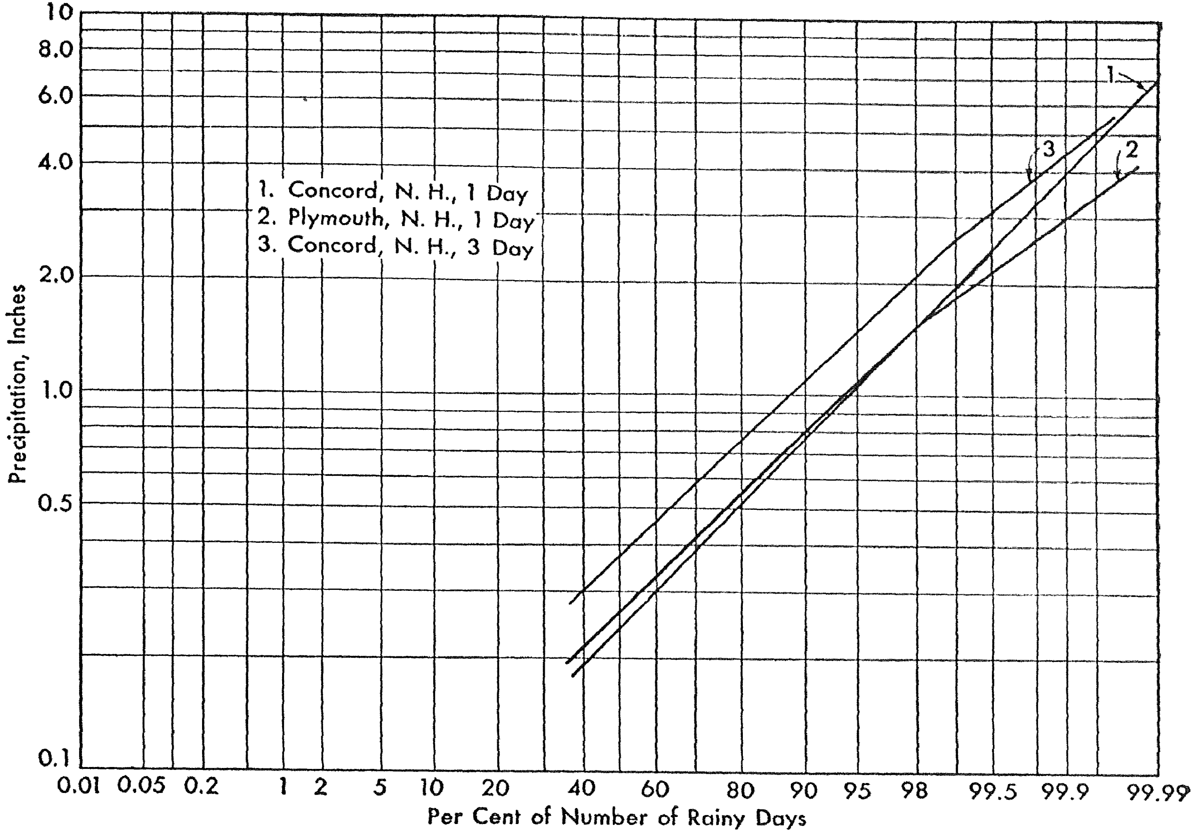



FIGURE 68. Frequency Distribution of Precipitation

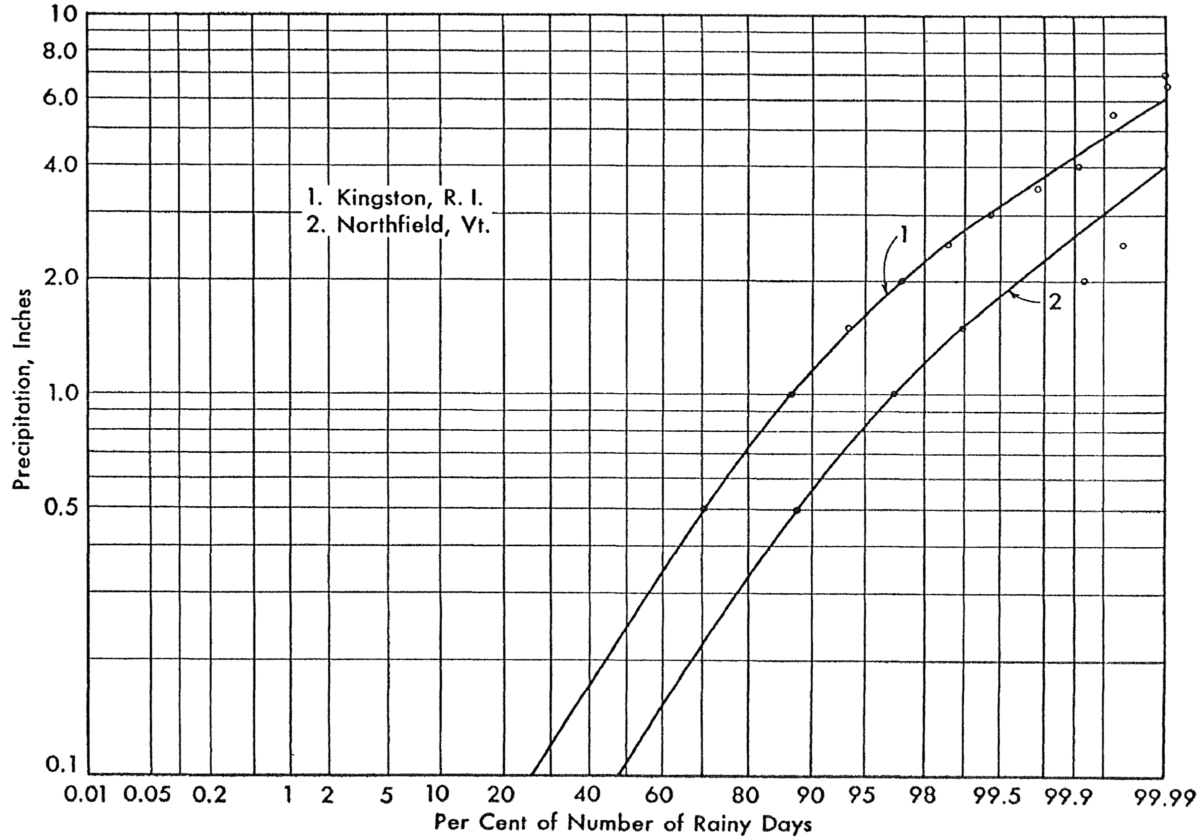



FIGURE 69. Frequency Distribution of Precipitation

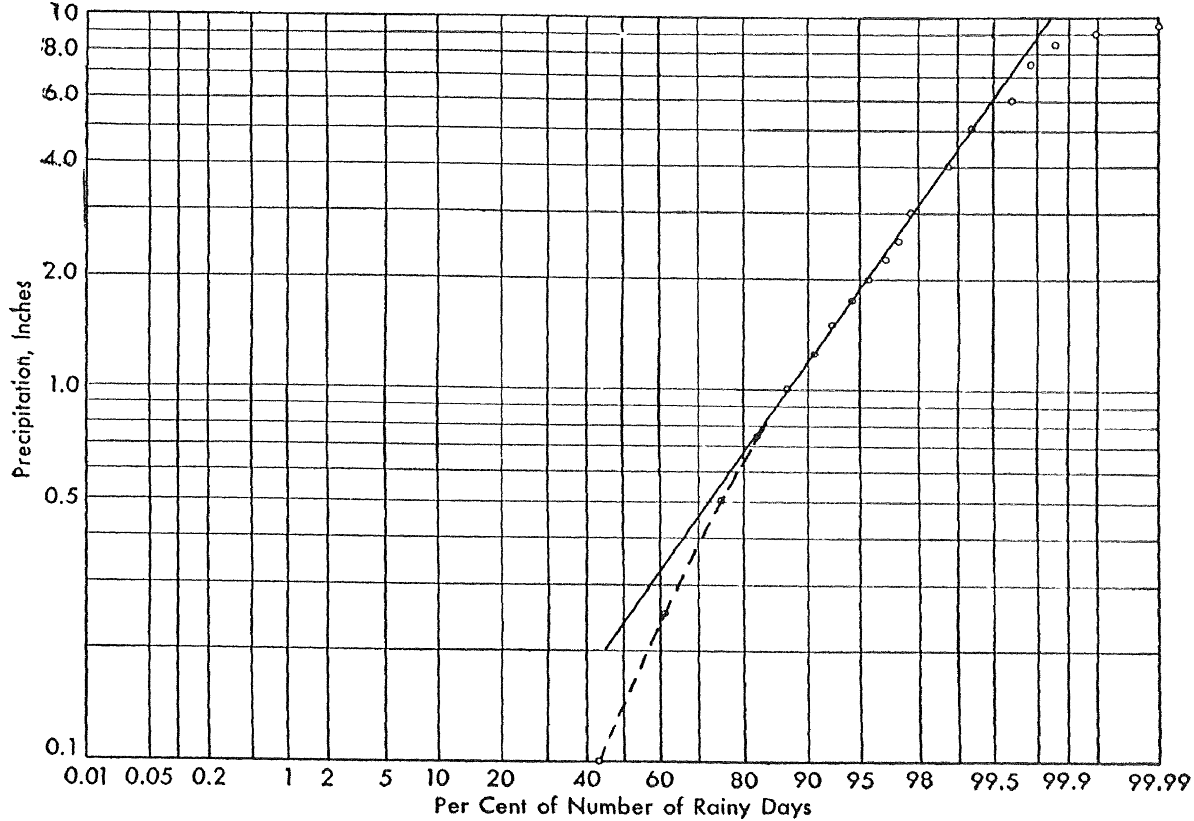



FIGURE 70. Frequency Distribution of Precipitation, Fort Lauderdale, Fla.

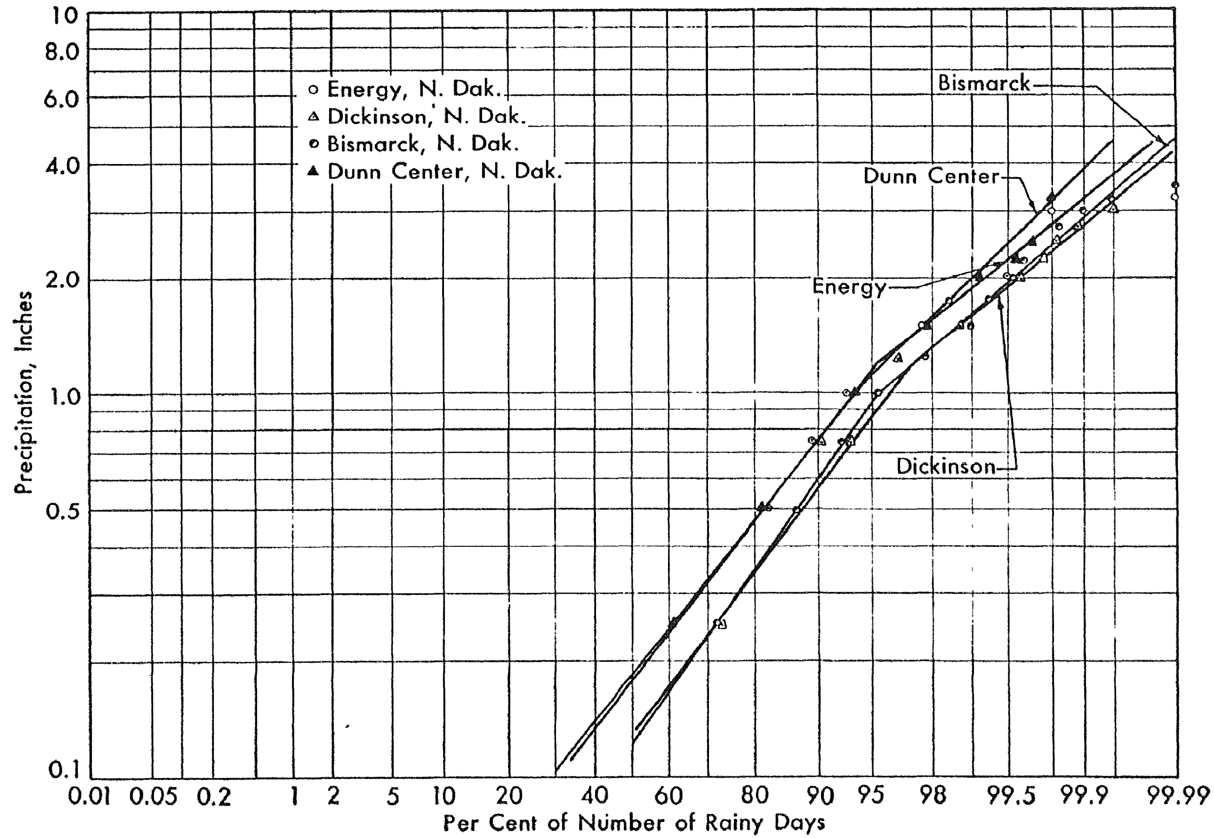



FIGURE 71. Frequency Distribution of Precipitation

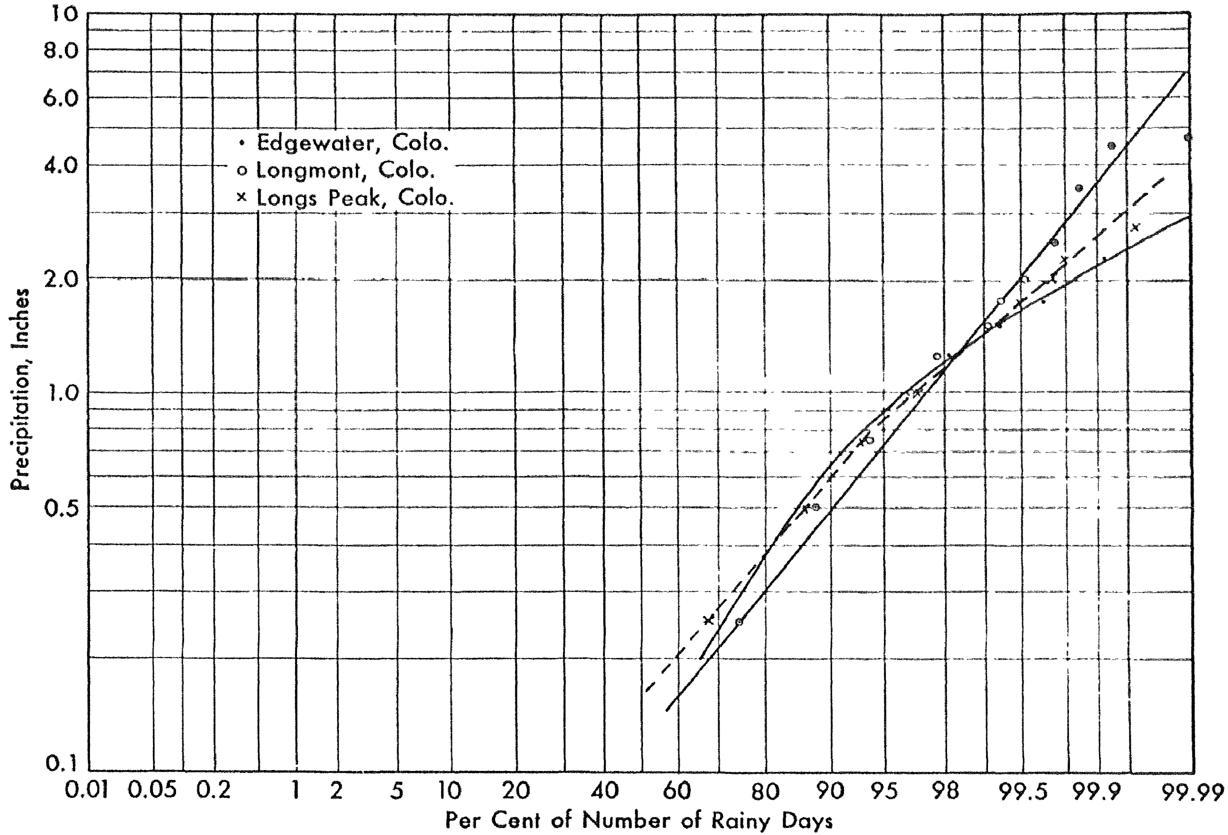



FIGURE 72. Frequency Distribution of Precipitation

used: this eliminates Poisson's function. Although a Gram-Charlier series provides for considerable skew, it is not enough for the distributions under discussion. This leaves only those functions with a logarithmic transformation of the variate or their equivalent. The two functions with logarithmic transformations are those presented by Fisher (59) and Slade (169). The methods developed by H. A. Foster (67) which are based on Karl Pearson's probability functions will cover abbreviated portions of the distributions, but the published tables will not cover the entire range of data. All the frequencies shown hereinafter will be based on Slade's partly bounded function.

These functions have been described in Chapter 1 and the theory underlying them need not be discussed here. However, one complete computation will be given to illustrate the application of the theory.

*Computation of Frequency of One-Day Precipitation, Concord, New Hampshire.* This computation of one-day precipitation at Concord, N. H. will be made by Slade's partly bounded asymmetrical function, which in the integral form is

$$P = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{c \log \frac{d(x+b)}{t^2+1}} e^{-u^2}\, du.$$

The notation has been given in Chapter 1, except $P$ which equals the probability, $t$ a parameter derived from the moments, and $u$ which represents the function. The problem now is to determine the values of the term $c \log d(x + b)/(t^2 + 1)$, which forms the argument $z$ for entering the tables of the probability integral. Because of space limitations no attempt will be made to explain or derive the equations and relations used in computing the argument, but the various steps will be illustrated by giving the details of the frequency computation for the station at Concord, N. H.

*Data of Precipitation at Concord, New Hampshire.* The amounts of precipitation reported each day with 0.01 inch of rain or more of the 51-year record at Concord, N. H. were obtained from the *Weather Bureau Bulletins, Climatological Data,* and were classified into groups of one-half inch increments. The classes and number of days in each group form the basic data from which the statistical moments were computed. The computation of the moments is given in Table 44.

In order to simplify computation, the power sums in columns 4, 5 and 6 were computed about an axis through the origin. Thus $\sum x^p f(x)/N$ is the $p$'th moment about the origin. The transformations following Table 44 are employed to compute the moments about the axis of the arithmetic mean.

TABLE 44. COMPUTATION OF STATISTICAL MOMENTS

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| CLASSES OF PRECIPITATION IN INCHES | AVERAGE OF CLASS $= X$ | NUMBER OF OBSERVATIONS IN EACH CLASS $= f(x)$ | $xf(x)$ | $x^2f(x)$ | $x^3f(x)$ |
| 0 to 0.5 | 0.25 | 4,760 | 1,190.00 | 297.5000 | 74.3750 |
| 0.5 to 1.0 | 0.75 | 881 | 660.75 | 495.5625 | 371.6718 |
| 1.0 to 1.5 | 1.25 | 245 | 306.25 | 382.8125 | 478.5156 |
| 1.5 to 2.0 | 1.75 | 55 | 96.25 | 168.4375 | 294.7656 |
| 2.0 to 2.5 | 2.25 | 38 | 85.50 | 192.3750 | 432.8438 |
| 2.5 to 3.0 | 2.75 | 13 | 35.75 | 98.3125 | 270.3594 |
| 3.0 to 3.5 | 3.25 | 8 | 26.00 | 84.5000 | 274.6250 |
| 3.5 to 4.0 | 3.75 | 1 | 3.75 | 14.0625 | 52.7344 |
| 4.0 to 4.5 | 4.25 | 0 | 0 | 0 | 0 |
| 4.5 to 5.0 | 4.75 | 1 | 4.75 | 22.5625 | 107.1719 |
| 5.0 to 5.5 | 5.25 | 0 | 0 | 0 | 0 |
| 5.5 to 6.0 | 5.75 | 1 | 5.75 | 33.0625 | 190.1094 |
| Totals | | 6,003 | 2,414.75 | 1,789.1875 | 2,547.1919 |
| Symbol | | $N$ | $\sum xf(x)$ | $\sum x^2f(x)$ | $\sum x^3f(x)$ |

$$M = \sum \frac{xf(x)}{N} = \frac{2,414.75}{6,003} = 0.40226$$

$$U_2 = \sum \frac{x^2f(x)}{N} - (M)^2 = \frac{1,789.1875}{6,003} - (.40226)^2 = 0.13624$$

$$U_3 = \sum \frac{x^3f(x)}{N} - 3\,\frac{x^2f(x)}{N}\,M + 2(M)^3$$

$$= \frac{2547.1919}{6,003} - 3\,\frac{(1,789.1875)}{6,003}(.40226) + 2(.40226)^3 = 0.19483.$$

Referring to the expression given above by which the values of $z$ are to be found, it will be seen that the values of the parameters $c$, $d$, $b$, and $t$ must be determined. For convenience the quantity $d/(t^2 + 1)$ may be set equal to $K$, which then eliminates $d$ as a separate value and simplifies the computations.

After computing the moments, the following quantities which are used to compute the parameters are found in the following order:

$$(U_2)^{1/2} = (0.13624)^{1/2} = 0.36911;\ (U_2)^{1.5} = 0.05029$$

$$D = \frac{U_3}{(U_2)^{1.5}} = \frac{0.19483}{(0.13624)^{1.5}} = 3.87413;\ \frac{D}{2} = 1.93706.$$

The parameter $t$ is found from the cubic equation $t^3 + 3t - D = 0$, in which $t = A^{1/3} + B^{1/3}$.

$$A = \frac{D}{2} + \sqrt{\left(\frac{D}{2}\right)^2 + 1} = 4.11701$$

$$B = \frac{D}{2} - \sqrt{\left(\frac{D}{2}\right)^2 + 1} = -0.24289$$

$$t = (4.11701)^{1/3} + (-0.24289)^{1/3} = 0.97880$$

$$(t^2 + 1) = 1.95805 \text{ and } \log_e (t^2 + 1) = 0.67195.$$

Then $$b = \frac{(U_2)^{1/2}}{t} = \frac{0.36911}{0.97880} = 0.37710.$$

The value of the parameter $c$ is combined for convenience with the values of two constants, one of which, $\sqrt{2}$, is contained in the standard probability tables of the basic function and whose value is not included in the integration of the above function. The logarithms used in the probability function are those to the base $e$, but since the usually obtainable tables are those of the common logarithms to the base 10, it is necessary to multiply all values by the modulus of the two bases, 2.30258; this can most readily be accomplished by including it in the value with the parameter $c$.

This combined value is therefore

$$2.30258\sqrt{2}c = \frac{2.30258}{(\log_e(t^2 + 1))^{1/2}}.$$

Substituting,

$$2.30258\sqrt{2}c = \frac{2.30258}{(0.67195)^{1/2}} = 2.80895$$

$$d = \frac{t(t^2 + 1)^{1.5}}{(U_2)^{1/2}}.$$

Then, by substitution,

$$K = \frac{(t^2 + 1)^{1/2}}{b}.$$

Then

$$K = \frac{(1.95805)^{1/2}}{0.37778} = 3.71069.$$

The computation of the argument, $c \log d(x + b)/(t^2 + 1)$ of the variate is shown in Table 45. Starting with the values of $X$, con-

veniently selected to coincide with upper class limits of the data, the argument in column 5 is developed in the successive steps indicated in columns 2, 3, and 4. The values of $f(z)$ are obtained from standard probability tables (preferably large tables such as Pearson's) with argument in column 5. It will be noted that the values of $X$ are absolute, that is, they are total depths of precipitation; a double step is made in column 2 to find the value of $X$ from the mean and to change the origin of the calculations from zero to the mean.

TABLE 45. COMPUTATION OF PRECIPITATION PROBABILITY

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $X$ | $x = X - M + b$ | $Kx$ | $\log_{10}(Kx)$ | $z = 2.80895$ (*Col. 4*) | $f(z)$ |
| 0 | | . | | . | . . . |
| 0.5 | 0.47484 | 1.76198 | 0.24690 | 0.6935 | 0.75600 |
| 1.0 | .97484 | 3.61733 | .55840 | 1.5685 | .94162 |
| 1.5 | 1.47484 | 5.47267 | .73820 | 2.0736 | .98094 |
| 2.0 | 1.97484 | 7.32802 | .86499 | 2.4297 | .99243 |
| 2.5 | 2.47484 | 9.18336 | .96300 | 2.7050 | .99658 |
| 3.0 | 2.97484 | 11.03871 | 1.04292 | 2.930 | .99831 |
| 3.5 | 3.47484 | 12.89405 | 1.11039 | 3.119 | .99909 |
| 4.0 | 3.97484 | 14.74940 | 1.16877 | 3.284 | .99944 |
| 4.5 | 4.47484 | 16.60474 | 1.22021 | 3.428 | .99969 |
| 5.0 | 4.97484 | 18.46009 | 1.26623 | 3.557 | .99981 |
| 5.5 | 5.47484 | 20.31543 | 1.30783 | 3.674 | .99988 |
| 6.0 | 5.97484 | 22.17078 | 1.34578 | 3.780 | .9999216 |
| 6.5 | 6.47484 | 24.02612 | 1.38068 | 3.878 | .9999476 |
| 7.0 | 6.97484 | 25.88147 | 1.41299 | 3.969 | .9999640 |
| 7.5 | 7.47484 | 27.73681 | 1.44306 | 4.053 | .9999747 |
| 8.0 | 7.97484 | 29.59216 | 1.47178 | 4.136 | 0.9999822 |

The values in column 6, Table 45, give the probability that a given depth of precipitation will not be exceeded: the element of time does not yet enter the picture. However, frequency curves are commonly required to show the magnitude to be expected to exceed a selected value in a given time, and it is now necessary to make one more step to meet this requirement. The data for the frequency curve may be obtained from column 6 in Table 45, as shown in Table 46. To find the probability that a given depth of precipitation exceeds the selected values of $X$, the values of $f(z)$, column 6, Table 45, are subtracted from 1.0; these results are shown in column 2, Table 46. Then the values of column 2, being the quantities $(1 - f(z))$, are multiplied successively by the average number of days per year, which was found to be 117.7; the results are given in column 3, Table 46, and they may be plotted against the respective values of $X$ to construct the frequency curve as shown in Figure 73.

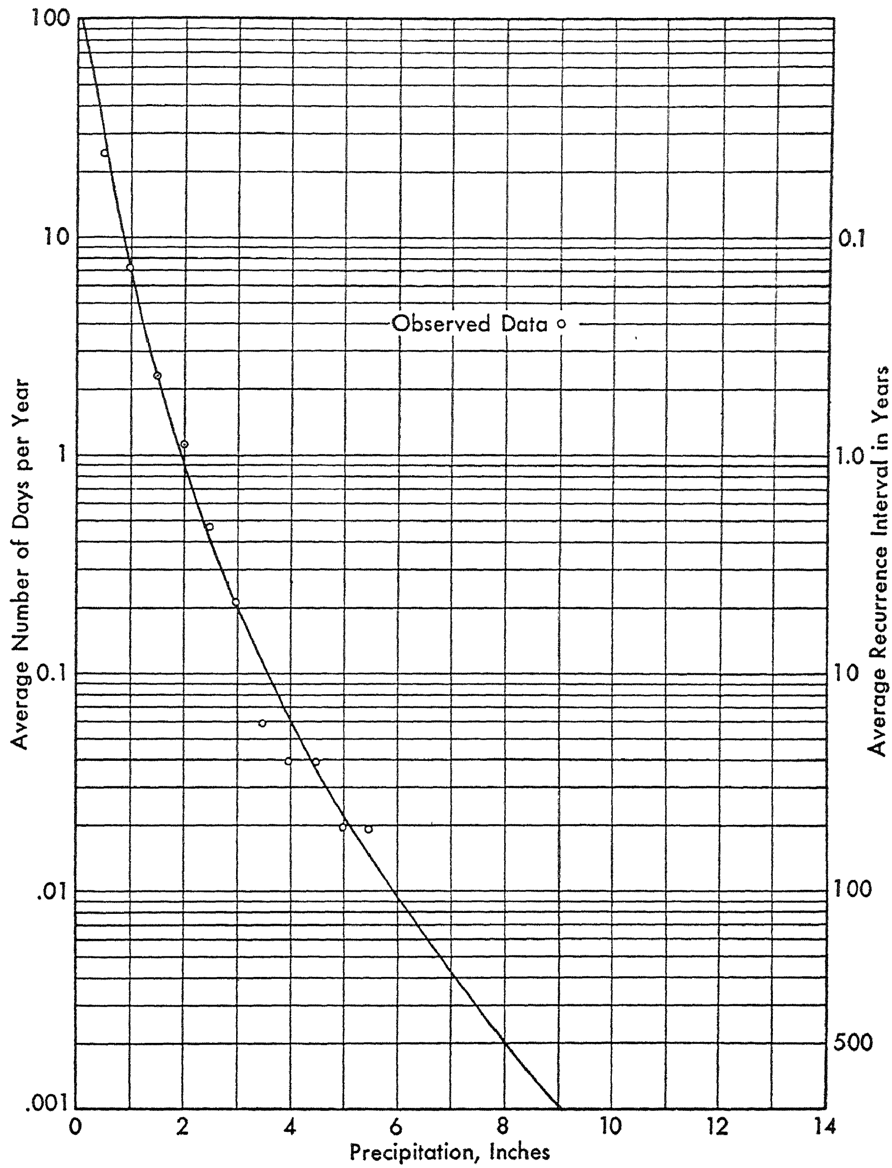



FIGURE 73. Frequency of One-Day Rainfall, Concord, N. H., 1885–1935

TABLE 46. FREQUENCY OF PRECIPITATION, CONCORD, N. H.

| 1 | 2 | 3 |
|---|---|---|
| $X$ | $1 - f(z)$ | NUMBER OF DAYS PER YEAR EQUAL TO OR GREATER THAN $X$ |
| 0 | 1.00000 | 117.7 |
| 0.5 | 0.24400 | 28.55 |
| 1.0 | .05838 | 6.83 |
| 1.5 | .01906 | 2.230 |
| 2.0 | .00757 | 0.886 |
| 2.5 | .00342 | .401 |
| 3.0 | .00169 | .198 |
| 3.5 | .00091 | .105 |
| 4.0 | .00056 | .0655 |
| 4.5 | .00031 | .0353 |
| 5.0 | .00019 | .0222 |
| 5.5 | .00012 | .01404 |
| 6.0 | .0000784 | .00917 |
| 6.5 | .0000524 | .00613 |
| 7.0 | .0000360 | .00421 |
| 7.5 | .0000253 | .00296 |
| 8.0 | 0.0000178 | 0.00206 |

*Accuracy of the Frequency Curve.* The computations of the frequency curve should, of course, be checked as they are made. There still remains the question of how accurately the computed frequency checks with the observed. The first check, and one that can be readily made, is to compute from $f(z)$, column 6, Table 45, the number of days not exceeding the various values of $X$, and compare them with

TABLE 47. COMPARISON OF COMPUTED AND OBSERVED DAYS OF RAIN

| 1 | 2 | 3 |
|---|---|---|
| $X$ | NUMBER OF DAYS EQUAL TO OR LESS THAN $X$ | OBSERVED FREQUENCY *for comparison* |
| 0 | . . . | . . . |
| 0.5 | 4538 | 4760 |
| 1.0 | 5653 | 5641 |
| 1.5 | 5889 | 5886 |
| 2.0 | 5958 | 5941 |
| 2.5 | 5983 | 5979 |
| 3.0 | 5993 | 5992 |
| 3.5 | 5997.5 | 6000 |
| 4.0 | 5999.6 | 6001 |
| 4.5 | 6001.1 | . . . |
| 5.0 | 6001.86 | 6002 |
| 5.5 | 6002.28 | . . . |
| 6.0 | 6002.53 | 6003 |
| 6.5 | . . . | . . . |
| 7.0 | . . . | . . . |

the observed number under the same classes. The total number of rainy days observed at Concord, N. H. in the period 1885–1935 is 6003; this number multiplied by the successive values of $f(z)$ gives the theoretical number of days to be compared with the observed number as given in columns 2 and 3, Table 47. This comparison gives a quick rough check which can readily be combined with the computation of the precipitation probability.

*Test for Goodness-of-fit.* Statistical methods furnish their own test of accuracy of a distribution such as that obtained by the method followed in the foregoing paragraphs; this is the goodness-of-fit test. To utilize this test it is necessary to compute the number of days in each class to form the theoretical distribution for comparison with the observed one. The theoretical distribution is computed by subtracting each number in column 2, Table 47, from the bigger number on the line below. The computation for the goodness-of-fit is given in Table 48 below.

TABLE 48. GOODNESS-OF-FIT OF FREQUENCY CURVE, CONCORD, N. H.

| $X$ | NUMBER OF DAYS FROM $f(z)$ | OBSERVED NUMBER | DEVIATION | DIVERGENCE |
|---|---|---|---|---|
| 0 | ... | ... | ... | ... |
| 0.5 | 4532 | 4760 | −228.0 | 11.47 |
| 1.0 | 1121 | 881 | +240 | 51.43 |
| 1.5 | 236 | 245 | − 9 | .34 |
| 2.0 | 69 | 55 | + 14 | 2.84 |
| 2.5 | 25 | 38 | − 13 | 6.76 |
| 3.0 | 10.0 | 13 | − 3 | .90 |
| 3.5 | 4.5 | 8 | − 3.6 | 2.72 |
| 4.0 | 2.1 | 1 | + 1.1 | .58 |
| 4.5 | 1.5 | 0 | + 1.5 | 1.50 |
| 5.0 | 0.66 | 1 | | |
| 5.5 | 0.42 | 0 | − 0.67 | 0.34 |
| 6.0 | 0.25 | 1 | | |

Total $\chi^2 = 78.84$

With 11 classes for $n'$ and $\chi^2 = 78.84$, a very poor fit is indicated. However, the first two classes equaling 58.85 make up the most of the value of chi-square. By regrouping the days into classes of one-inch increments and recomputing, a much better fit is found; on this basis $\chi^2 = 9.07$ and $n' = 6$, which gives $P' = 0.11$. Other aspects also must be considered. Since the signs of the deviations are mixed it may be reasonably concluded that there is no constant error throughout the range of magnitude.

It is likely that the large divergences of the first two classes given in Table 48 are due, in part at least, to the large class increment, 0.5

inch, which is greater than the average daily precipitation. Two increments to a daily rainfall of 1.0 inch would include some very marked changes in the curve, since the histogram of variations must rise from zero to the peak and fall considerably within that range. These changes could not be followed closely by a computation from 1/2-inch increments. Referring now to Figure 68 it will be seen that there is a break and change of direction of the plotted points between 2.0 and 3.0 inches, indicating the existence of a compound frequency curve which would produce relatively large deviations at that point as the theoretical function attempted to fit itself to the observed data. In view of these aspects it may be concluded that the fit of the function used above is satisfactory for general purposes. This conclusion is supported by the improved fit obtained by using one-inch classes and by the plotted points of observed data shown on Figure 73, although these plotted points are not a sensitive test from a mathematical viewpoint.

*Frequencies of Precipitation in New England.* A number of frequency curves were computed from stations scattered over New England but centered around New Hampshire. Except for Concord, N. H., for which a frequency curve covering the period 1885–1935 was computed above, the data used were obtained during the 30-year period, 1906–1935. The results of these computations provide an opportunity

TABLE 49. CHARACTERISTIC VALUES OF PRECIPITATION FREQUENCY CURVES FOR NEW ENGLAND

| STATION | MEANS, $M$ *Inches* | MOMENTS $U_2$* | $U_3$* | SKEW $D$ | NUMBER OF DAYS PER YEAR |
|---|---|---|---|---|---|
| Amherst, Mass. | 0.42251 | 0.159 | 0.240 | 3.82465 | 122.6 |
| Concord, Mass. | 0.41814 | 0.154 | 0.239 | 3.97087 | 121.0 |
| Fitchburg, Mass. | 0.42478 | 0.166 | 0.298 | 4.37878 | 115.1 |
| Haverhill, Mass. | 0.40248 | 0.136 | 0.183 | 3.62828 | 117.1 |
| Hartford, Conn. | 0.41583 | 0.158 | 0.224 | 3.58178 | 125.9 |
| Kingston, R. I. | 0.51557 | 0.289 | 0.496 | 3.18003 | 107.6 |
| Concord, N. H.** | 0.38113 | 0.117 | 0.175 | 4.36925 | 118.6 |
| Durham, N. H. | 0.43131 | 0.179 | 0.345 | 4.66042 | 104.3 |
| Glencliff, N. H. | 0.38110 | 0.117 | 0.157 | 3.97113 | 137.2 |
| Keene, N. H. | 0.38082 | 0.098 | 0.102 | 3.28398 | 125.2 |
| Nashua, N. H. | 0.42118 | 0.158 | 0.249 | 3.96671 | 114.9 |
| Plymouth, N. H. | 0.41006 | 0.128 | 0.154 | 3.36762 | 108.4 |
| Cavendish, N. H. | 0.40406 | 0.138 | 0.197 | 3.82870 | 121.6 |
| Northfield, N. H. | 0.33147 | 0.075 | 0.137 | 7.22656 | 123.8 |
| Gardiner, Maine | 0.39672 | 0.124 | 0.160 | 3.65671 | 136.8 |
| Greenville, Maine | 0.37928 | 0.118 | 0.183 | 4.56042 | 145.2 |
| Rumford, Maine | 0.39686 | 0.142 | 0.221 | 4.10051 | 125.4 |

* Values approximate only, computed by slide rule and not used for the construction of the frequency curves.

** For 30-year record.

to compare characteristic values of frequency curves, since the climate of the area is fairly homogeneous except for the effect of topography and distance from the coast, and the length of record is generally identical. These characteristic values are given in Table 49.

The climate of New England is typically marine, particularly along the coast, having moderately warm summers and cold winters so that precipitation in winter is usually snow. The region is traversed by polar continental, tropical Atlantic, and modified continental and maritime air masses throughout the year, and in winter by polar Atlantic masses also. It is subject to frequent extratropical cyclones, relatively infrequent thunderstorms, and rarely is visited by a tropical hurricane. Precipitation is unusually uniform throughout the year.

Frequency curves for the stations listed above are shown on Figures 74–78.

*Frequencies of Precipitation in the Middle Atlantic States.* The characteristics of the frequencies of daily precipitation for five stations in the Middle Atlantic states are shown below in Table 50. Precipitation in this region is plentiful throughout the year although the stations shown are located some distance from the coast. The winters are cold enough so that precipitation in that season is frequently snow. The region is subject to invasion by polar continental and tropical maritime air masses, both direct and modified. It is frequently traversed by extratropical cyclones, thunderstorms are frequent in summer, and it is occasionally affected by tropical hurricanes although it does not receive the full violence or precipitation of these storms.

TABLE 50. CHARACTERISTIC VALUES OF PRECIPITATION FREQUENCY CURVES FOR MIDDLE ATLANTIC STATES

| STATION | MEANS, $M$ *Inches* | MOMENTS $U_2$ | $U_3$ | SKEW $D$ | NUMBER OF DAYS PER YEAR |
|---|---|---|---|---|---|
| Bayard, Md. | 0.43066 | 0.14157 | 0.14369 | 2.69739 | 148.3 |
| Cumberland, Md. | 0.40278 | 0.12950 | 0.16551 | 3.55156 | 110.4 |
| Frostburg, Md. | 0.42007 | 0.14848 | 0.18666 | 3.26249 | 125.9 |
| Piedmont, W. Va. | 0.42211 | 0.13515 | 0.13705 | 2.7566 | 110.0 |
| Westernport, Md. | 0.41142 | 0.11857 | 0.11161 | 2.73353 | 109.5 |

The frequency curves for the above stations are shown on Figures 79 and 80.

*Frequencies of Precipitation in the Southern Midwest.* The characteristics of frequencies of daily precipitation in the southern midwest area of the United States are listed in Table 51. The region covered extends over northern Texas, southern Oklahoma, and southwestern Arkansas. Precipitation is abundant in this region but varies considerably with the seasons; in winter it occasionally may be snow but is usually rain.

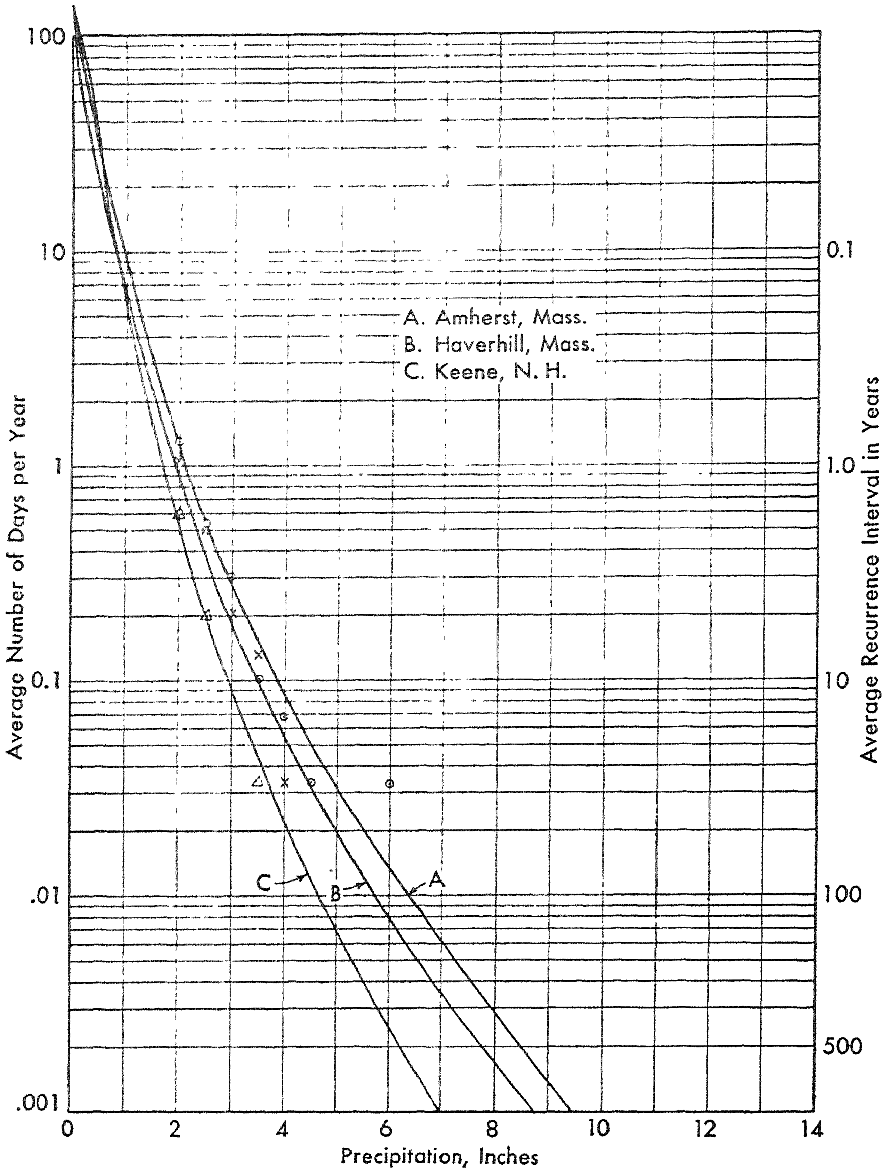



FIGURE 74. Frequency of One-Day Rainfall

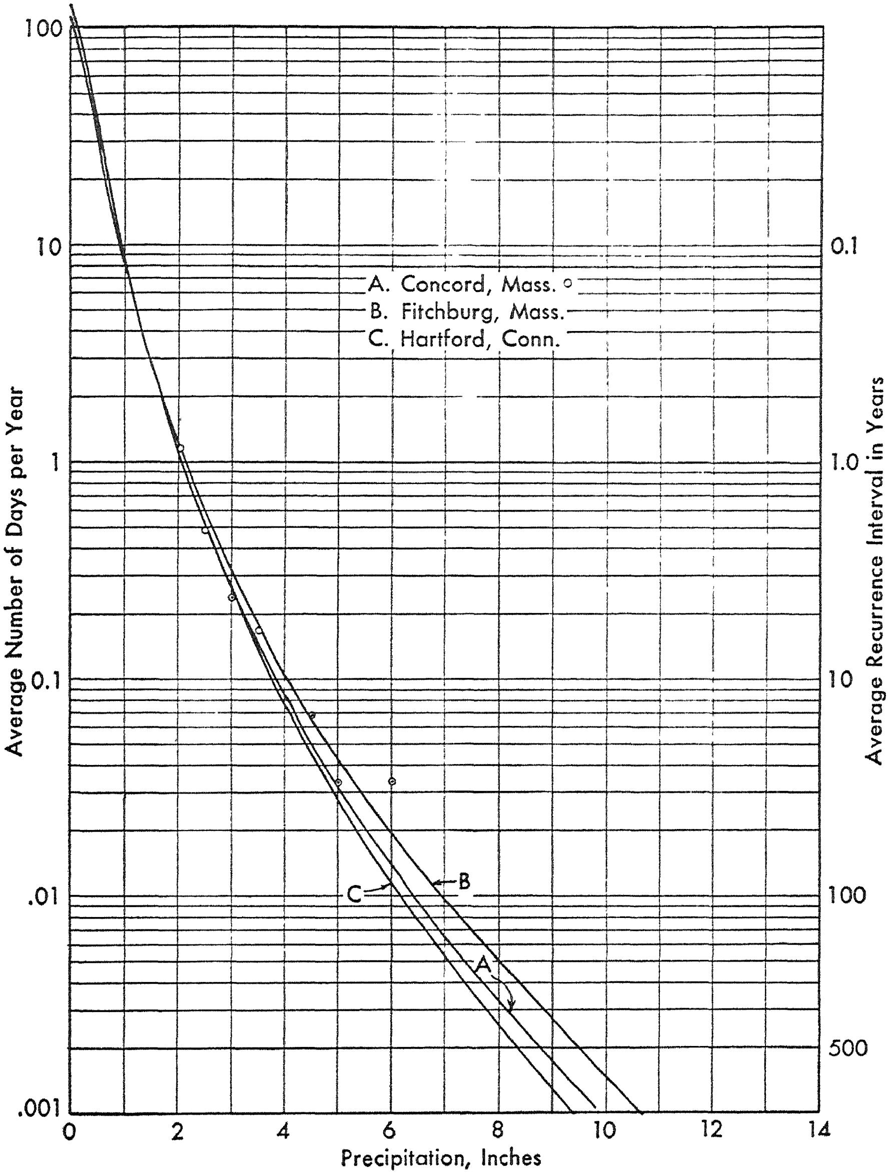



FIGURE 75. Frequency of One-Day Rainfall

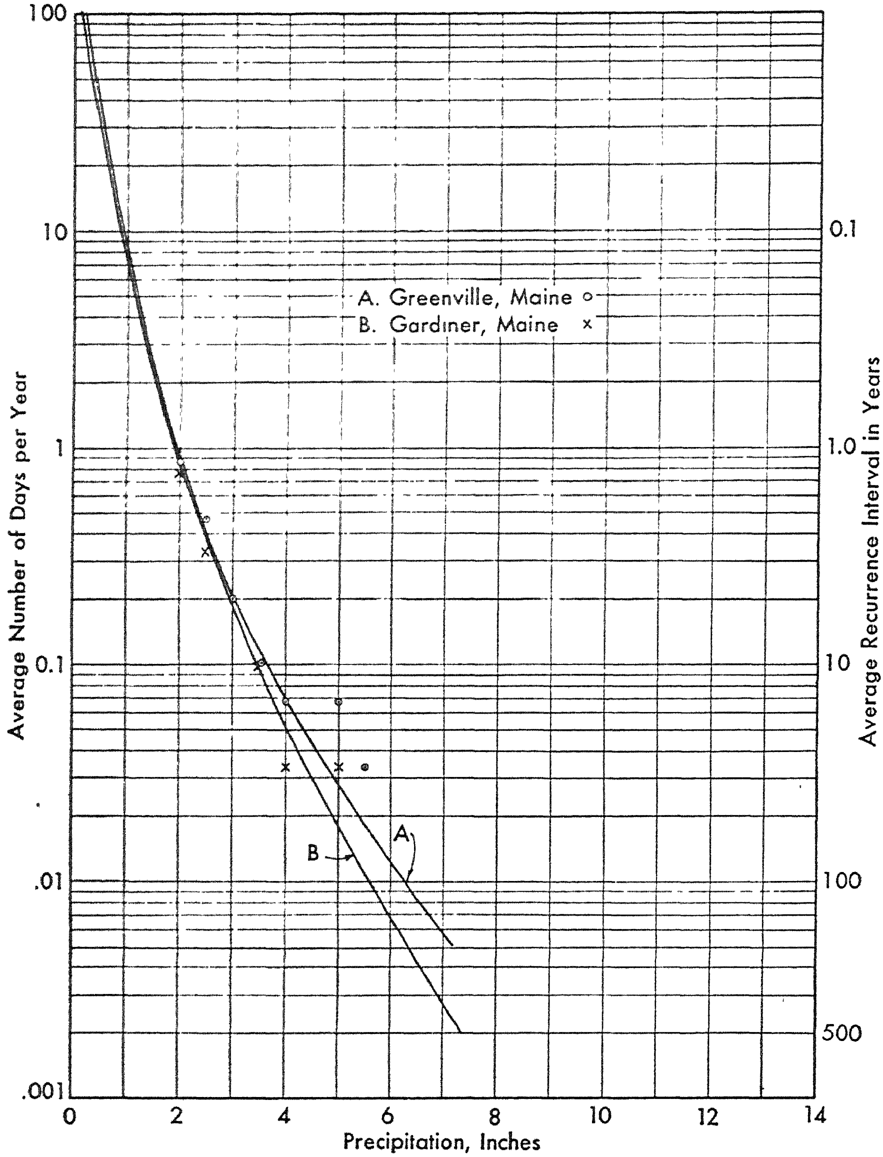



FIGURE 76. Frequency of One-Day Rainfall

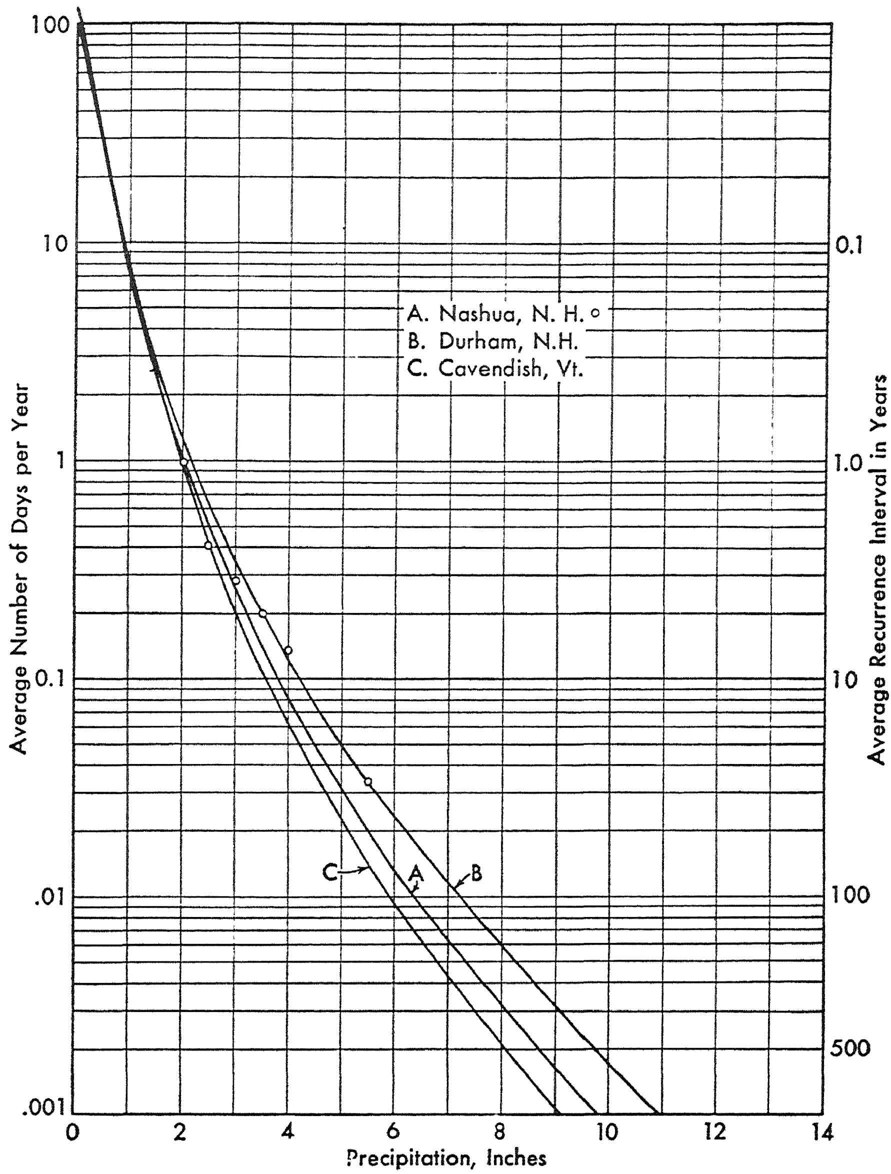



FIGURE 77. Frequency of One-Day Rainfall

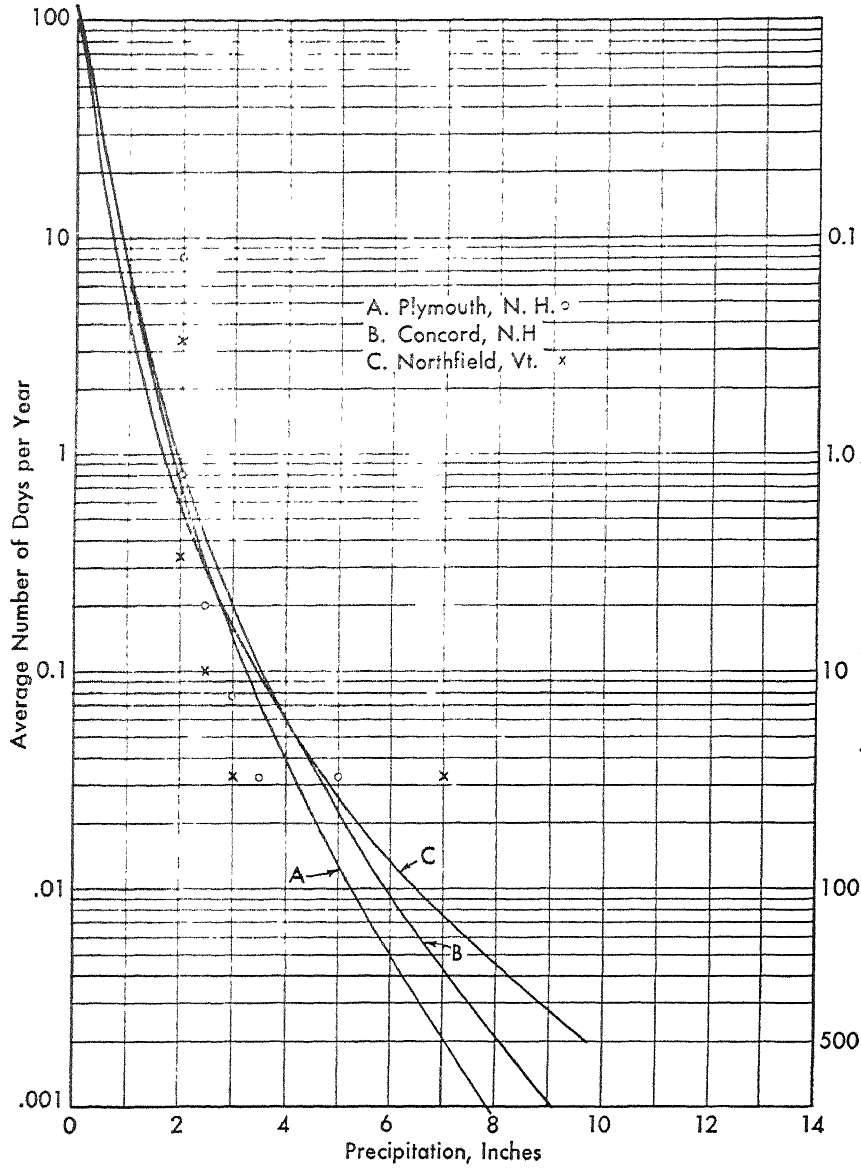



FIGURE 78. Frequency of One-Day Rainfall

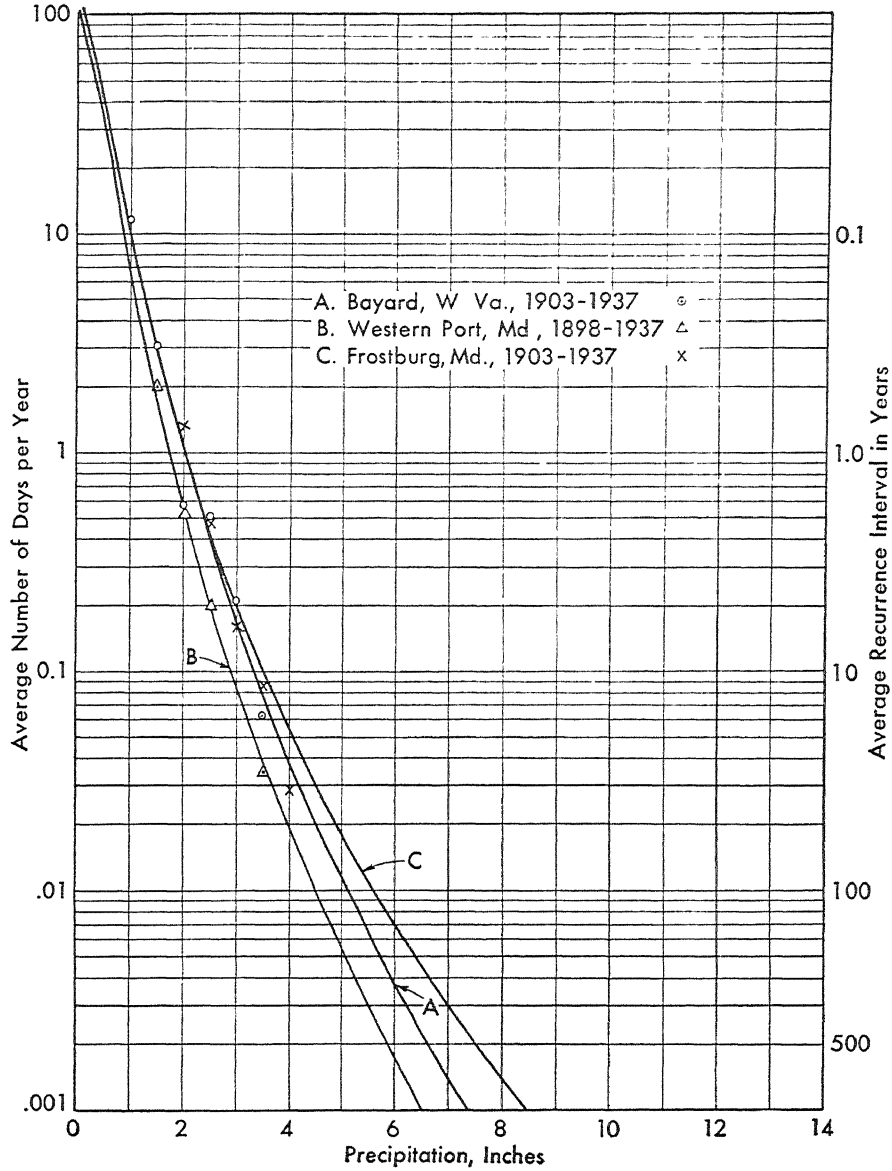



FIGURE 79. Frequency of One-Day Rainfall

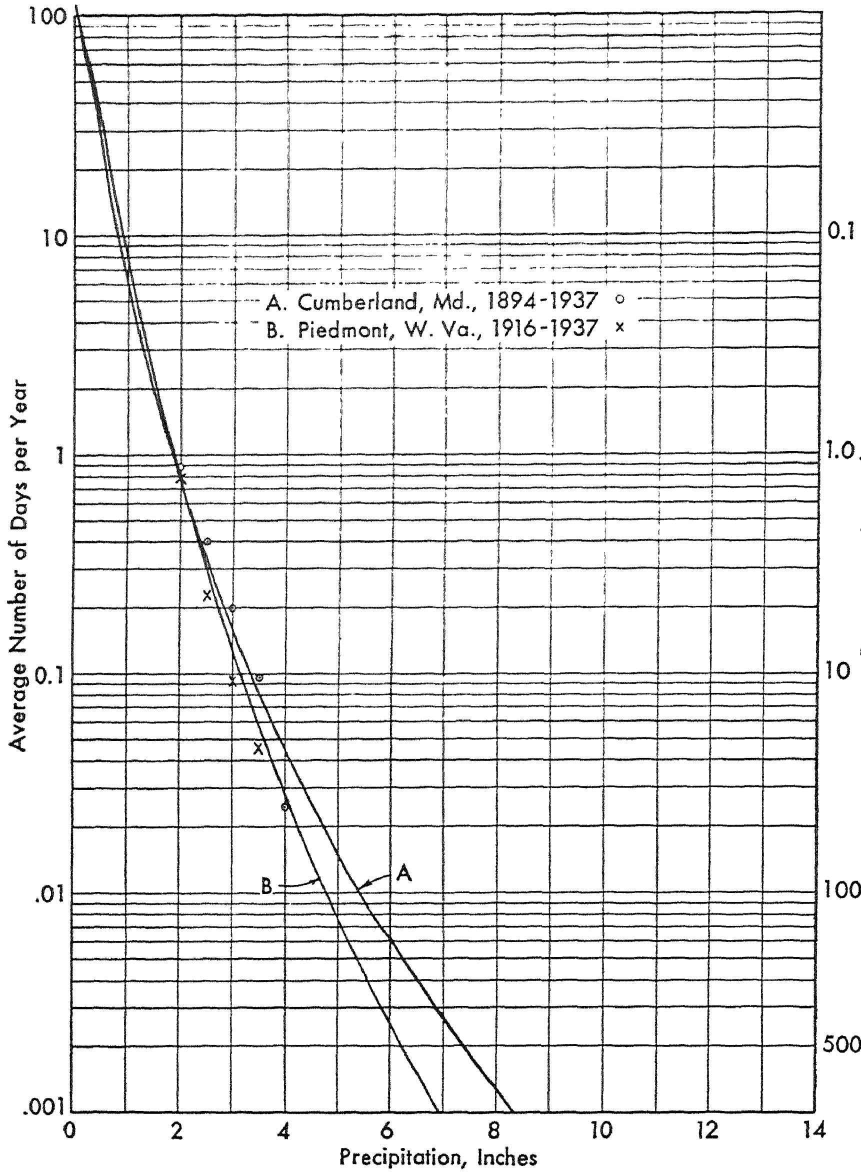



FIGURE 80. Frequency of One-Day Rainfall

The region is subject to invasion by Gulf maritime, polar continental, and modified continental and Pacific air masses; the first named is probably most frequent in summer. It is subject to extratropical cyclones and also to tropical hurricanes which, however, lose much of their violence and a smaller proportion of their potential precipitation before reaching the areas under consideration. Thunderstorms are frequent. As a whole, precipitation is variable, the region being subject at times to heavy and sustained rainfall (except Amarillo, Texas) and at other times to droughts. Amarillo is farther west and somewhat beyond the usual courses of the types of storms mentioned.

TABLE 51. CHARACTERISTIC VALUES OF PRECIPITATION FREQUENCIES IN THE SOUTHERN MIDWEST

| STATION | MEANS, $M$ *Inches* | MOMENTS $U_2$ | $U_3$ | SKEW $D$ | NUMBER OF DAYS PER YEAR |
|---|---|---|---|---|---|
| Amarillo, Texas | 0.39128 | 0.24344 | 0.10183 | 0.84777 | 71.9 |
| Chickasha, Okla. | 0.54796 | 0.33291 | 0.62206 | 3.23851 | 62.0 |
| Fulton, Ark. | 0.66267 | 0.44850 | 0.84695 | 2.81971 | 72.9 |
| Lawton, Okla. | 0.53698 | 0.38082 | 0.91796 | 3.09617 | 62.9 |
| Mangum, Okla. | 0.55695 | 0.30659 | 0.51054 | 3.00739 | 50.5 |
| Pauls Valley, Okla. | 0.58647 | 0.39090 | 0.65468 | 2.67867 | 65.4 |
| Plainview, Texas | 0.40607 | 0.23822 | 0.28457 | 2.44755 | 53.8 |

Curves for the above stations are shown on Figures 81–83.

*Frequencies of Precipitation in Colorado.* Characteristic values of frequency of daily precipitation in Colorado are shown in Table 52 below. Some of the stations listed are situated in the mountains, while others are on the Great Plains at relatively high elevations. The area is subject to invasion by polar continental, Pacific, and Gulf tropical air masses and modifications of each, and is traversed by extratropical cyclones. Precipitation in this region is typically continental; thunder-

TABLE 52. CHARACTERISTIC VALUES OF PRECIPITATION FREQUENCIES IN COLORADO

| STATION | MEANS, $M$ *Inches* | MOMENTS $U_2$ | $U_3$ | SKEW $D$ | NUMBER OF DAYS PER YEAR |
|---|---|---|---|---|---|
| Edgewater | 0.26998 | 0.08824 | 0.07650 | 2.91885 | 41.3 |
| Waterdale | 0.28779 | 0.12019 | 0.20208 | 4.84993 | 34.9 |
| Longmont | 0.25507 | 0.11167 | 0.20500 | 5.49376 | 35.6 |
| Boulder | 0.28817 | 0.12544 | 0.17352 | 3.90558 | . . . |
| Greeley | 0.26966 | 0.10018 | 0.11643 | 3.67204 | 33.7 |
| Kassler | 0.28526 | 0.11106 | 0.12283 | 3.31875 | 38.4 |
| Silver Lake | 0.32473 | 0.12615 | 0.16827 | 3.75582 | 39.2 |
| Longs Peak | 0.27379 | 0.08936 | 0.10820 | 4.05073 | 52.5 |
| Hartsell | 0.22945 | 0.05595 | 0.05105 | 3.85719 | 41.3 |
| Estes Park | 0.24863 | 0.06123 | 0.05484 | 3.59344 | 48.3 |

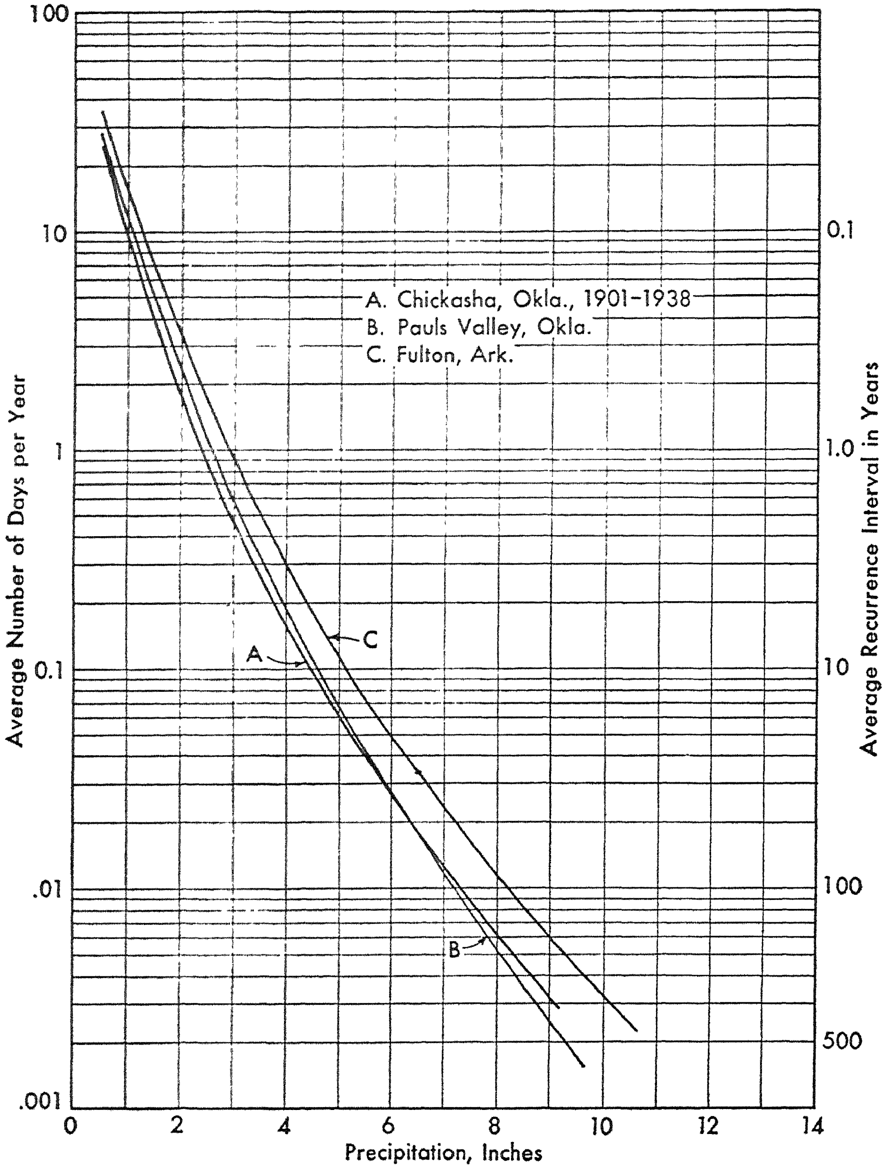



FIGURE 81. Frequency of One-Day Rainfall

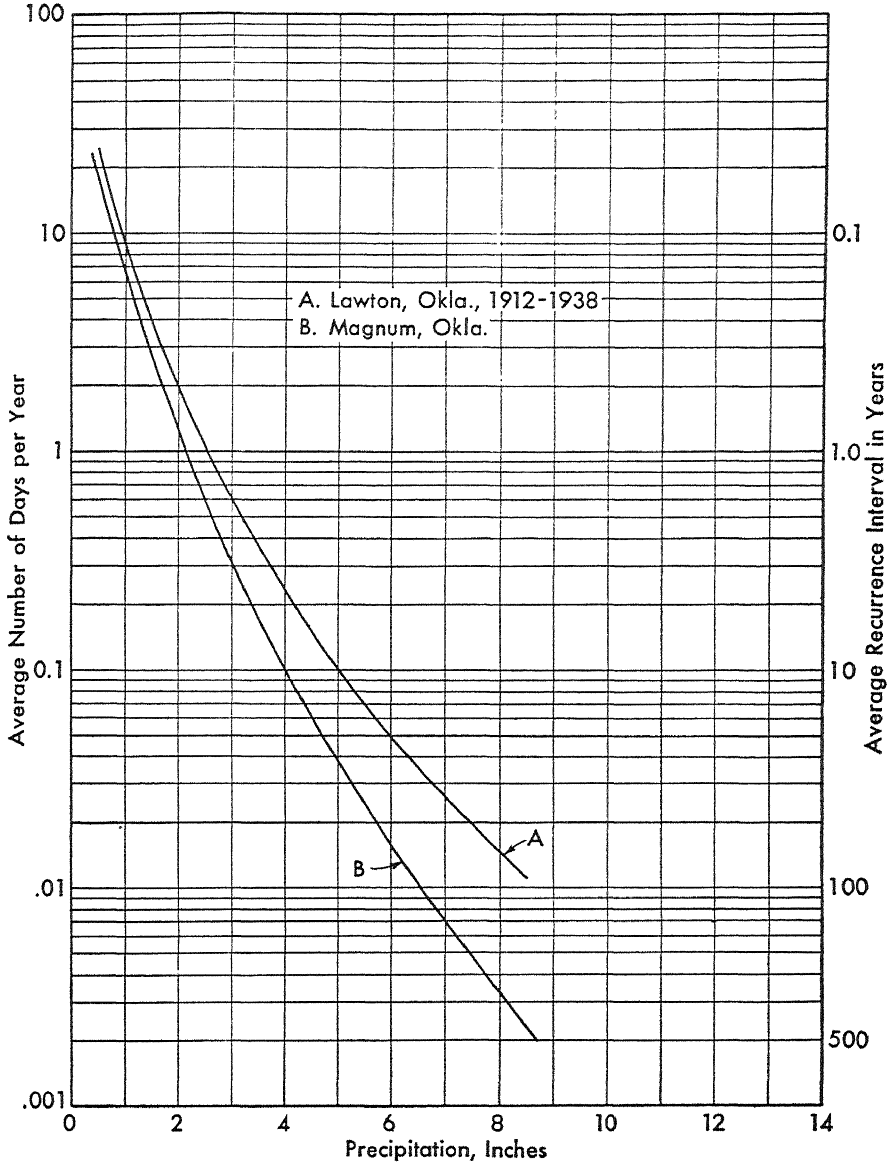



FIGURE 82. Frequency of One-Day Rainfall

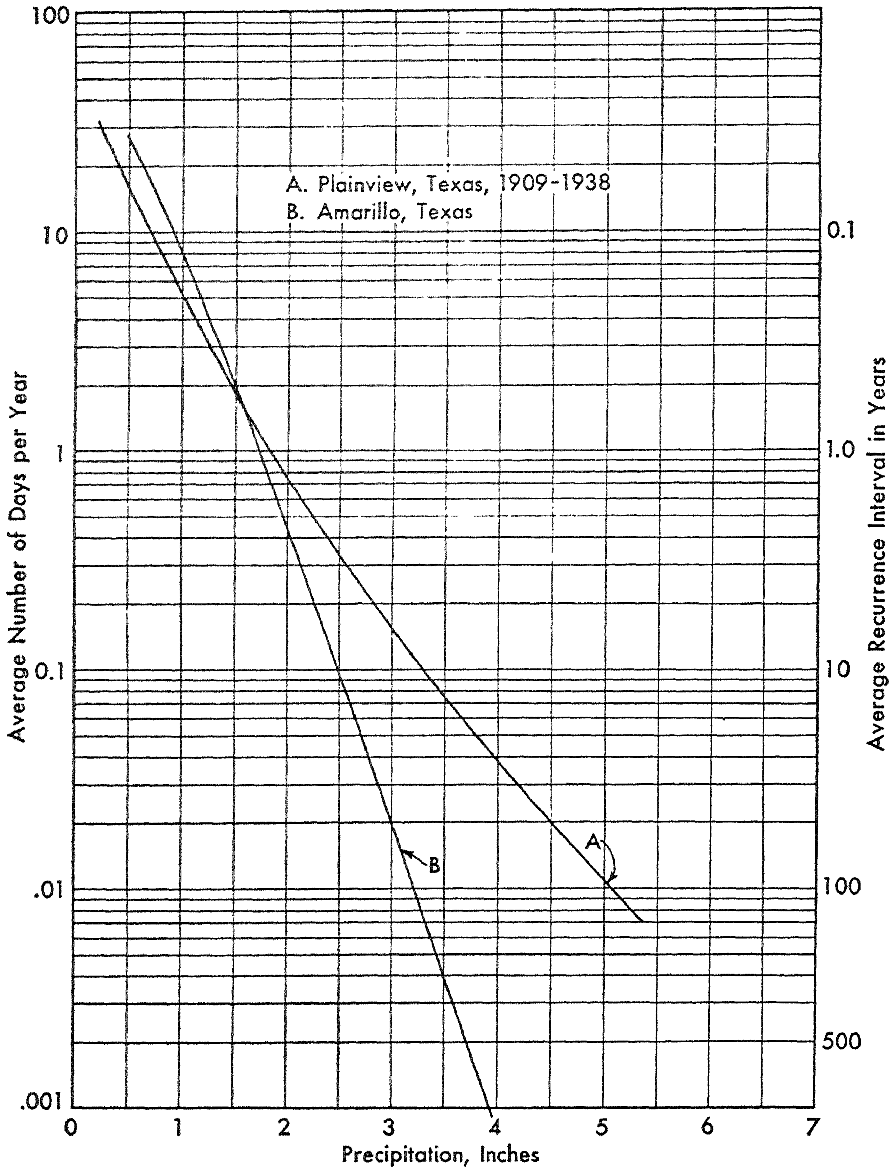



FIGURE 83. Frequency of One-Day Rainfall

storms are frequent, especially in summer, and orographic influence is strong in the mountain areas.

For the stations listed in Table 52, data of the six months warm season, April to September inclusive, are used. The elevations of the first six stations, which are situated on the Great Plains along the eastern toe of the mountains, range from 4637 to 5450 feet above mean sea level, while the last four stations listed range from elevations 8000 to 10,200 feet although they are on the eastern side of the Rocky Mountains.

Frequency curves of the above stations are shown on Figures 84–87.

*Frequency of Precipitation on the Northern Great Plains.* Characteristic values of the frequencies of daily precipitation at four stations in western North Dakota are given in Table 53. The climate, and therefore the summer rainfall, may be taken as typical of the Great Plains north of latitude 45 degrees north, and west of the middle of the states of North and South Dakota. Polar continental air masses are predominant over this area; invasions of polar Pacific air come across the mountains, and in summer Gulf tropical air comes up from the south. Winters are cold, while summers are warm and occasionally hot. Approximately 75 per cent of the annual precipitation falls in summer, as the climate is typically continental. In the table below, the data are for the six warm months, April to September.

TABLE 53. CHARACTERISTIC VALUES OF PRECIPITATION FREQUENCY CURVES, NORTH DAKOTA

| STATION | MEAN, $M$ *Inches* | MOMENTS $U_2$ | $U_3$ | SKEW $D$ | NUMBER OF DAYS PER YEAR |
|---|---|---|---|---|---|
| Bismarck | 0.26866 | 0.10255 | 0.12171 | 3.70672 | 51.0 |
| Dickinson | 0.26549 | 0.10398 | 0.13057 | 3.89392 | 51.2 |
| Dunn Center | 0.33232 | 0.14742 | 0.17672 | 3.12219 | 36.5 |
| Energy | 0.33193 | 0.14802 | 0.16521 | 2.90109 | 37.4 |

The frequency curves are shown on Figures 88 and 89.

*Frequency of Multiple-Day Storms.* In dealing with larger areas such as may be under consideration for flood control, it is desirable to know the frequencies of two- or three-day or longer storms. For this purpose the data of one-day precipitation may be utilized in either of two ways; first the precipitation may be grouped in successive three-day (or other) periods; second, moving totals may be computed by adding to the precipitation of each day that of the two (or more) following days. The following computations from data observed at Concord, N. H., March 9–16, 1936, illustrate both methods.

| Order of Days | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Observed Precipitation | 0.37 | T | 1.39 | 1.82 | 0.06 | 0.01 |
| First Method<br>Combined 3-day data | | | 1.76 | | | 1.89 |
| Second method<br>Moving 3-day totals | | | 1.76 | 3.21 | 3.27 | 1.89 |

It will be noted that except for the two days each included in the first and last totals of the record, the second method will have three times as many data as the first. Statisticians would very likely criticize this second method for the lack of independence of data. This criticism is valid because each three-day value is related to the one or two before and the one or two totals after it, but the relationship does not extend further. Nevertheless, given a reasonably long record in a humid region, it appears that there will be enough independence of data so that the resulting frequency curves are not noticeably impaired.

The second method, however, has one advantage in that it is more likely to secure higher group values by overlapping the one-day periods which may divide a period of intense precipitation. Thus in the above computation the days 3, 4, and 5 combine to produce the highest three-day value of 3.27 inches, whereas in the first method those days are placed in separate groups in which the maximum value attained is only 1.89 inches.

The method of using moving totals was used to construct illustrative frequency curves of two- and three-day periods for Chickasha, Okla. and Concord, N. H., respectively, which are shown in Figures 90 and 91. The full computed number of periods of rain were used for data. The correctness of this procedure is supported by consideration of the difference in precipitation shown for identical frequencies. Take, for example, the frequencies for one to five rains per year on the Concord curves; the difference in one- and three-day precipitation is 0.8 to 1.2 inches while the difference between the maximum one- and three-day rains shown above in the sample computation for multiple-day storm is 1.45 inches. The daily values in the above tabulation are within the range of frequencies mentioned, yet show even greater differences between one- and three-day rainfall than do the curves. The curves, of course, are the result of all data, many of which would be lower than those quoted above.

*Frequency and Intensity of Rainfall.* Problems of intense rainfall often involve frequency also, especially when economic factors are under consideration. In fact many of the studies published have been made on the basis of both intensity and frequency. The studies of Yarnell (200), Bernard (18), and Meyer (133), particularly, were concerned with both aspects of storm precipitation. As was shown

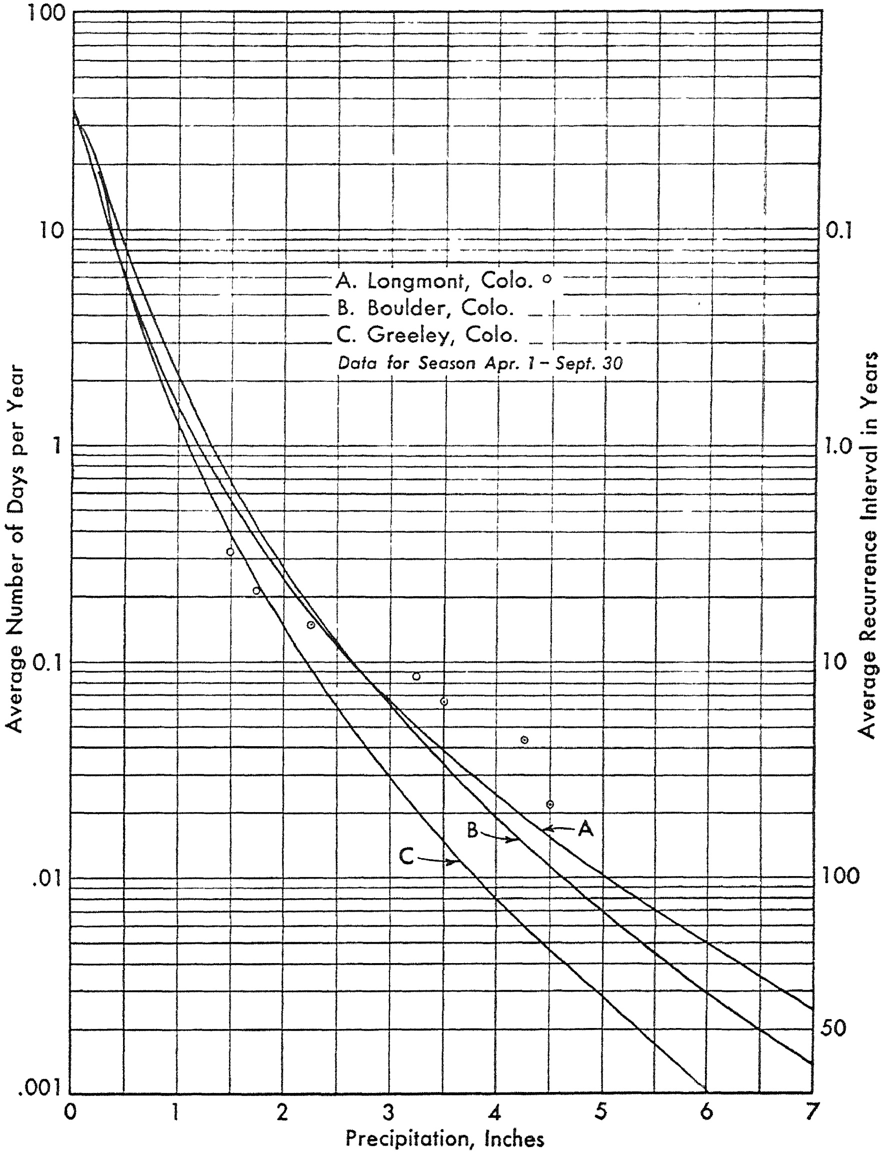



FIGURE 84. Frequency of One-Day Rainfall

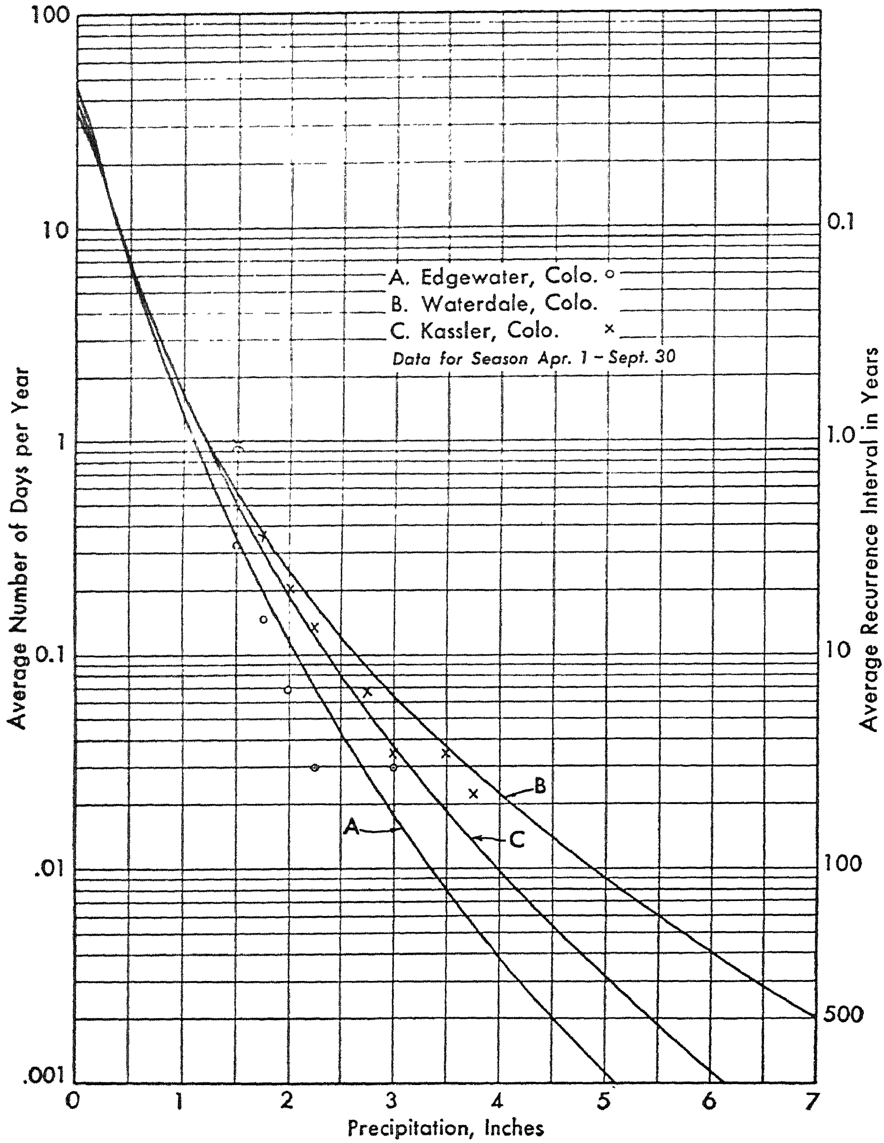



FIGURE 85. Frequency of One-Day Rainfall

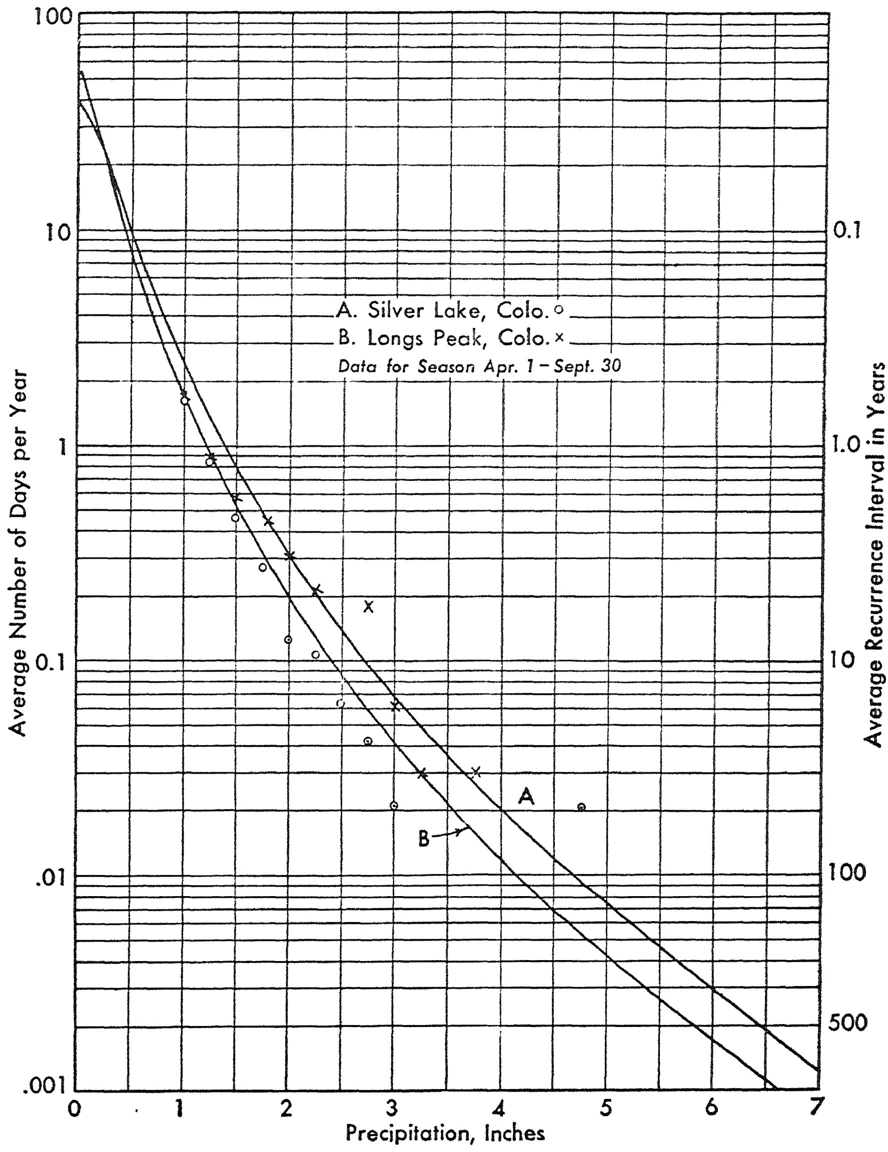



FIGURE 86. Frequency of One-Day Rainfall

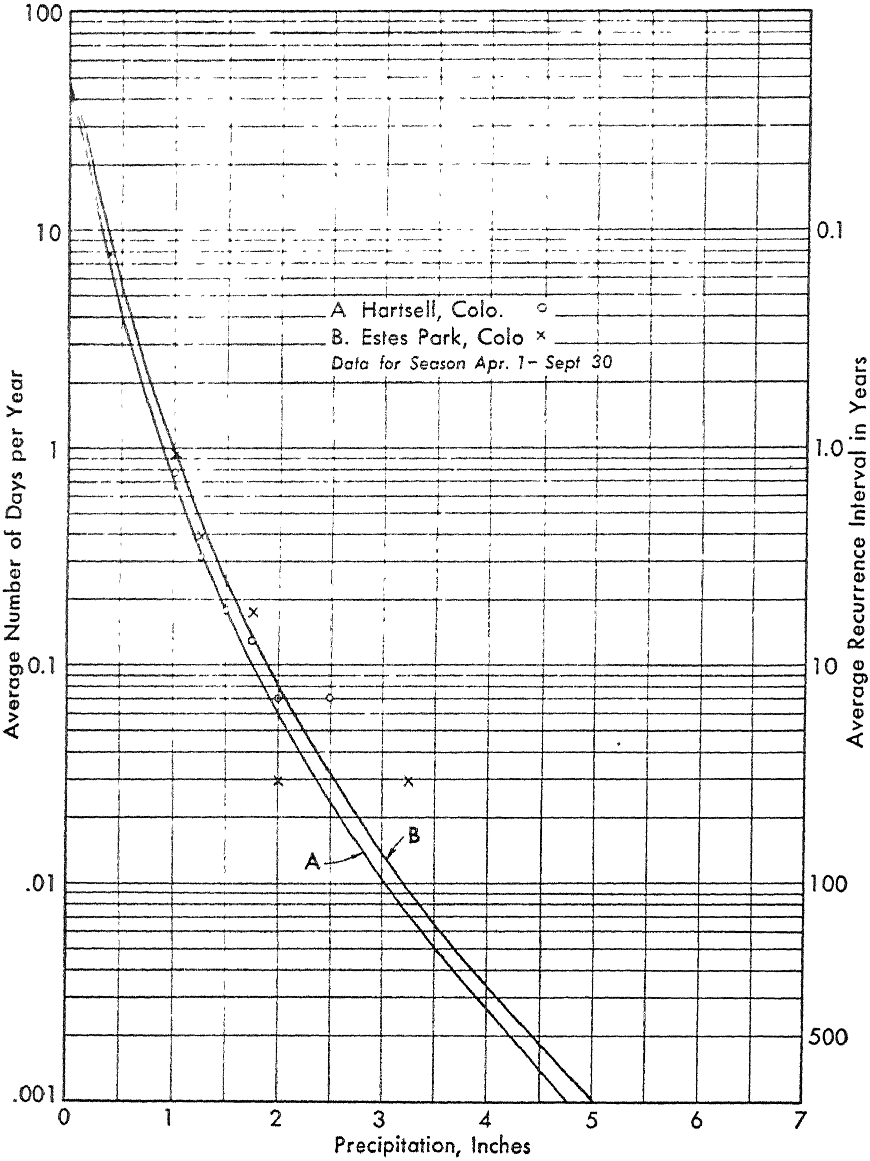



FIGURE 87. Frequency of One-Day Rainfall

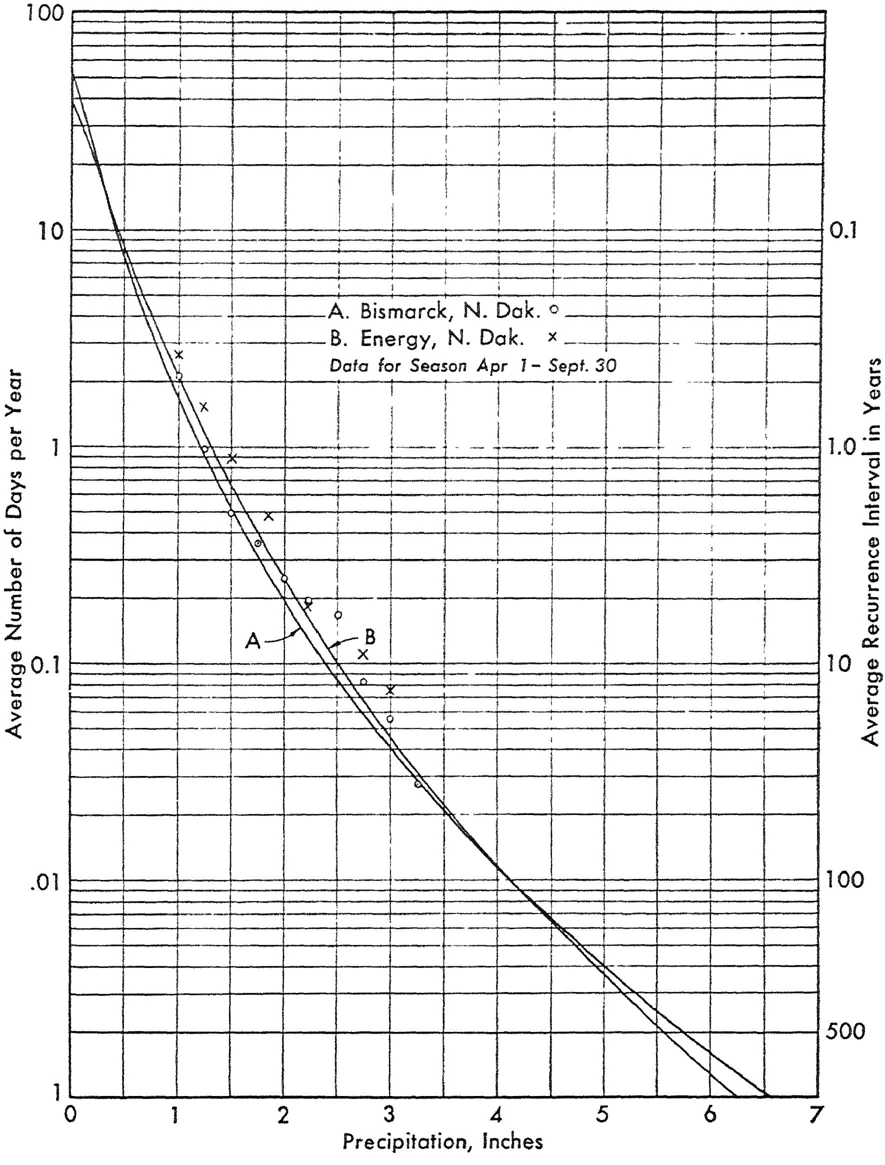



FIGURE 88. Frequency of One-Day Rainfall

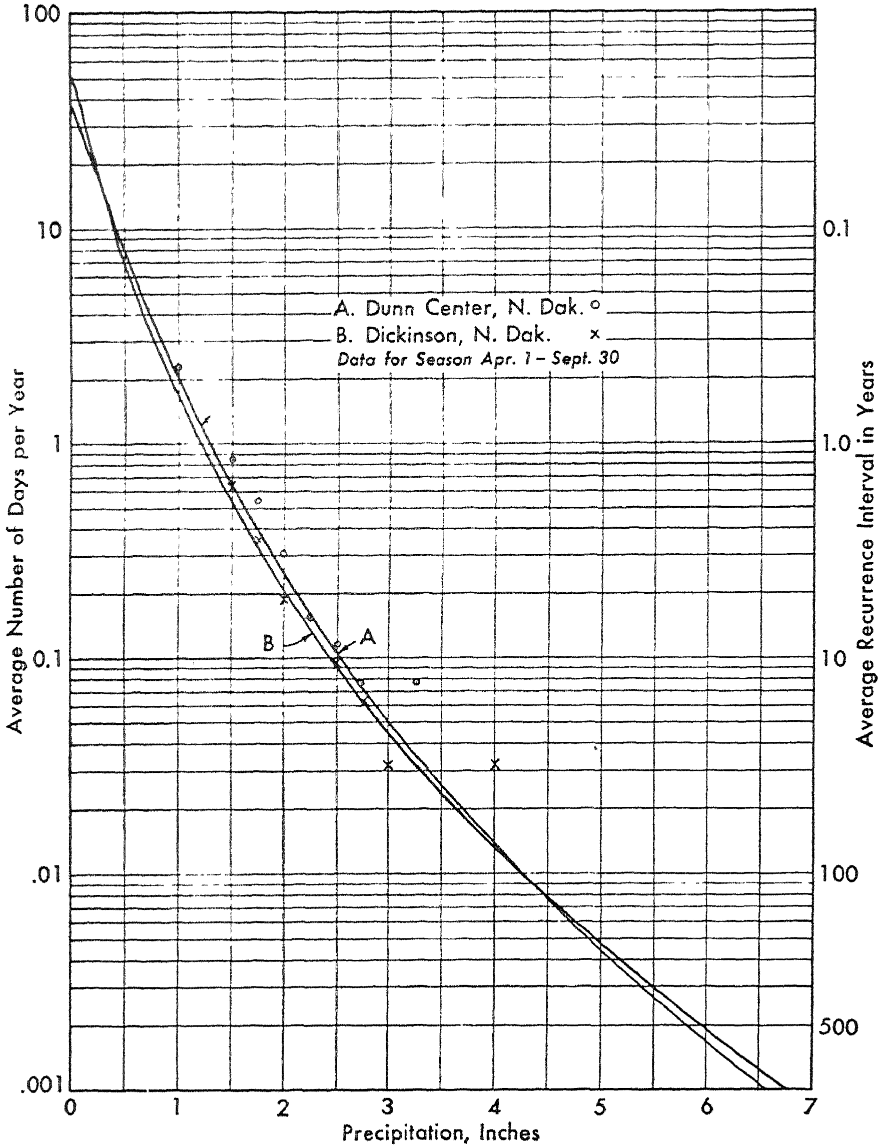



FIGURE 89. Frequency of One-Day Rainfall

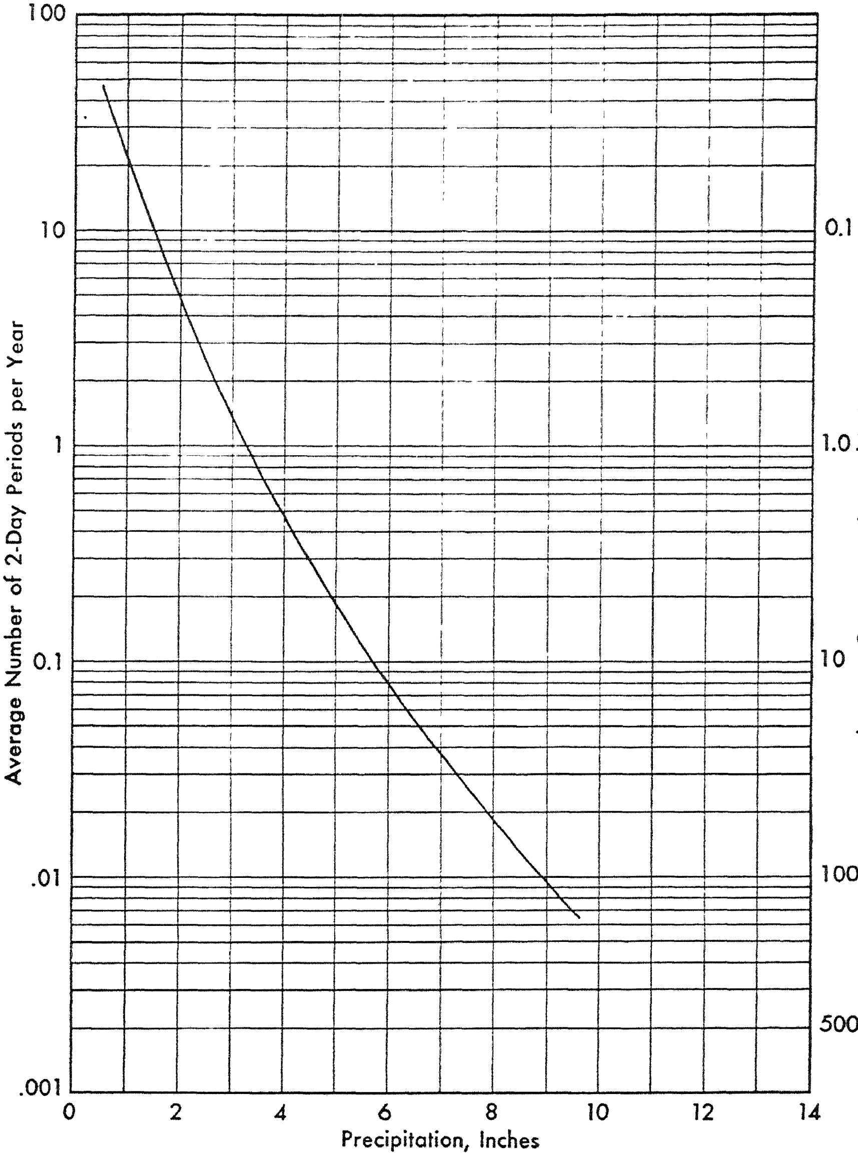



FIGURE 90. Frequency of Rainfall; Two-Day Totals, 1901–1938, Chickasha, Okla.

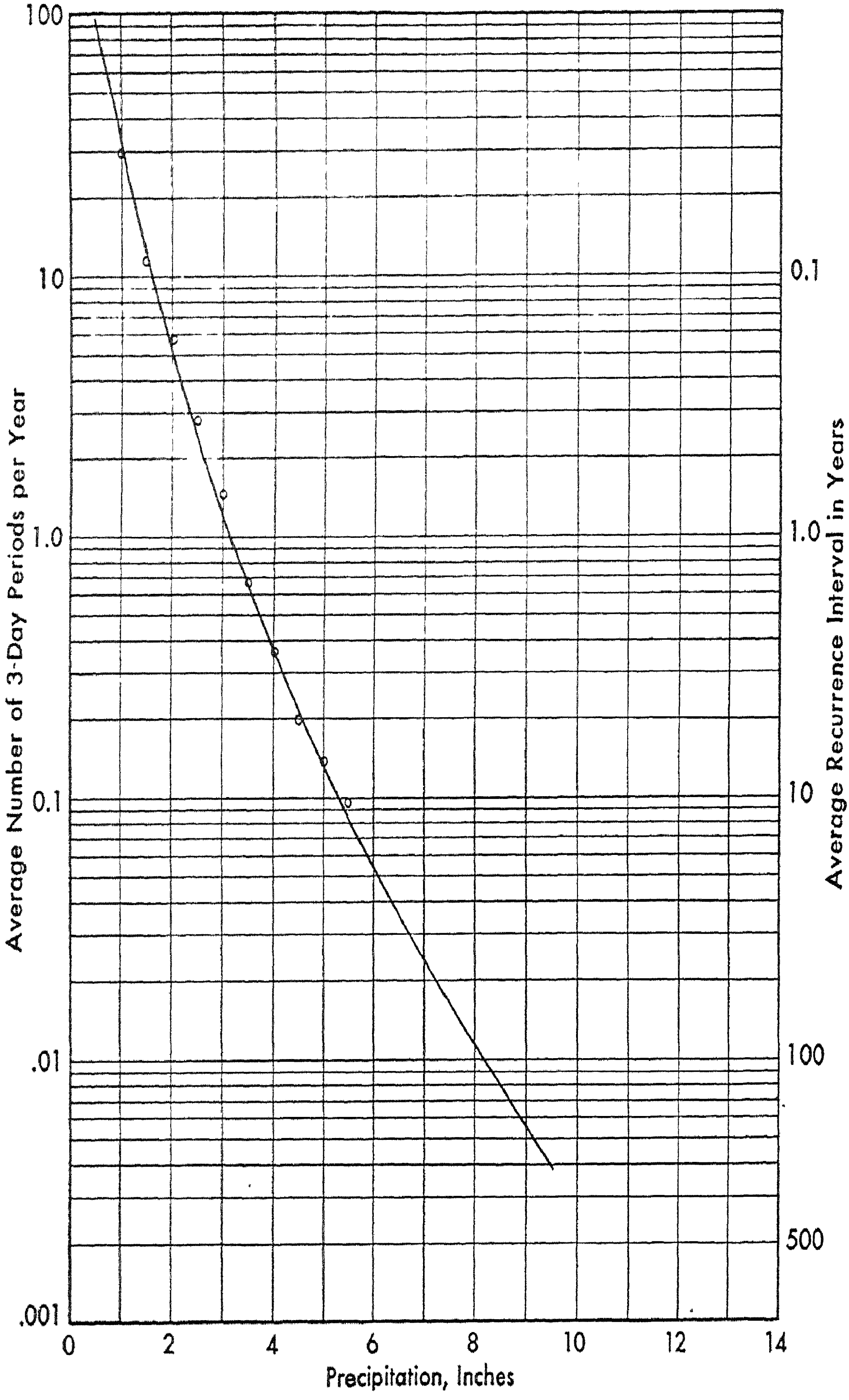



FIGURE 91. Frequency of Rainfall; Three-Day Totals, 1885–1935, Concord, N. H.

previously (Chapter 5) these studies and others of like nature not mentioned were all empirical in the sense that they did not attempt to relate their data by statistical theory.

Since data of precipitation are used for drainage basins of widely varying areas, they must be compiled similarly for such periods as are best suited to the given size of area, ranging from ten minutes to several days. Obviously the computation of frequency curves by Slade's function would involve much work to cover the required range for all purposes, namely, periods ranging from 10 to 30 minutes, periods of one, two, four, eight, or more hours, and periods of one or more days. To obviate this labor of extensive computation, and yet to give the empirical results a reliability closely approaching that attainable through the use of the theory of statistics, some adaptation of the statistical method must be applied. For this purpose it is proposed to plot values for the statistical frequencies of the one- and two-day probabilities as computed for Chickasha, Okla., and the one- and three-day frequency curves of Concord, N. H., with data computed by empirical methods.

The data of frequency obtained by all methods used were plotted on paper having logarithmic scales on both axes, because it is desired to work with straight-line curves. Validity for plotting on such coordinates rests on the observations given in the preceding chapter where it was shown that depth-area curves of rainfall would plot along a straight line on such coordinates as long as an ample supply of warm moist air was available to maintain precipitation. This fact is fortunate because it permits drawing depth-area curves of different frequencies as straight parallel lines and makes it possible to combine frequencies computed by statistical methods for one period with empirically computed frequencies for other units of time.

On Figure 92 the 10-, 50-, and 100-year frequencies of one-day and two-day precipitation at Chickasha, Okla., have been plotted from Figures 81 and 90. Depths of rainfall for the shorter periods of time ranging from 10 minutes to 16 hours have been taken from Yarnell's data (200) for the same frequencies and plotted on the same graph. Likewise, similar data have been computed and plotted from Bernard's (18) exponential formula

$$i = \frac{C}{t^n}$$

in which $i$ is the expected rainfall, $t$ the time in minutes, and $C$ and $n$ are coefficients to be obtained from Bernard's published graphs. A study of these plotted points show Yarnell's data lying on a somewhat

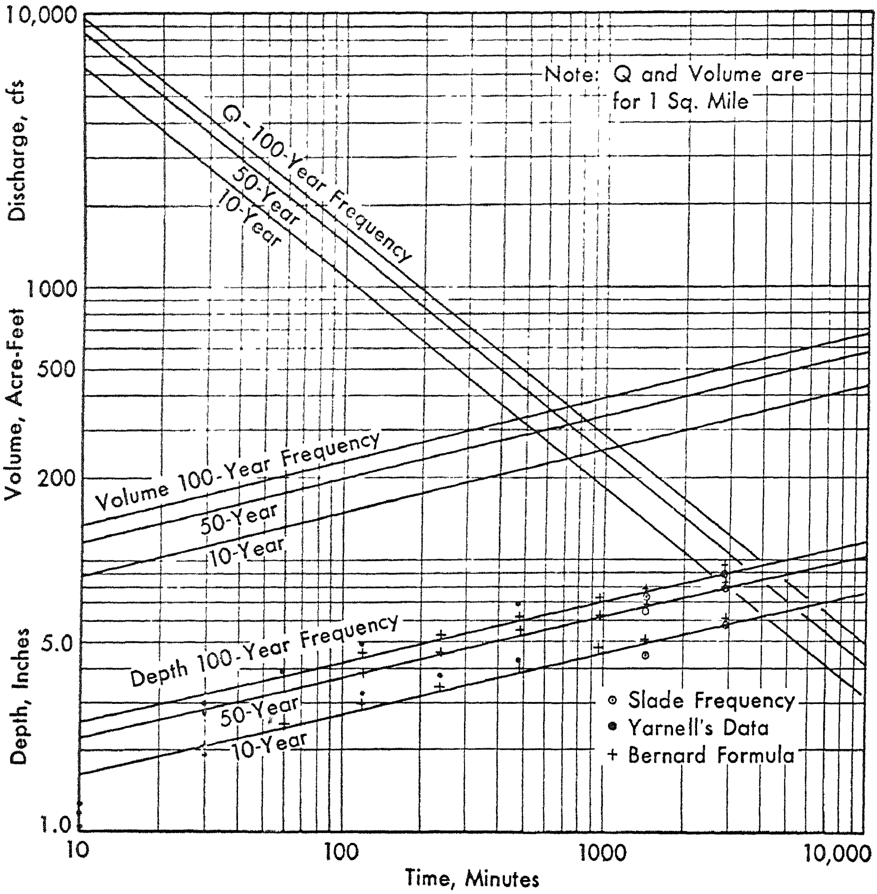



FIGURE 92. Maximum Rainfall Intensity, Chickasha, Okla.

since the coefficients were computed on the basis of a logarithmic function. Both Yarnell's and Bernard's values pass above the depth obtained from the Slade frequency curve for one-day rainfall and Bernard's values also plot above the point for two-day rainfall. Yarnell's values for 24-hour rainfall are apparently the maximum for periods of that length found in the storms studied by him, but his paper (200) is not positive on that point. Bernard's values for one and more days were obtained by interpolation from the charts of the Miami Conservancy District (135) in which the data for calendar days were used, but for two or more days were used in such a way that the maximum rainfall for the period was obtained.

In Figure 92 the value obtained from the Slade frequency curve for one day was averaged with empirical data. The curves were drawn through the points obtained from the Slade frequency for two-day

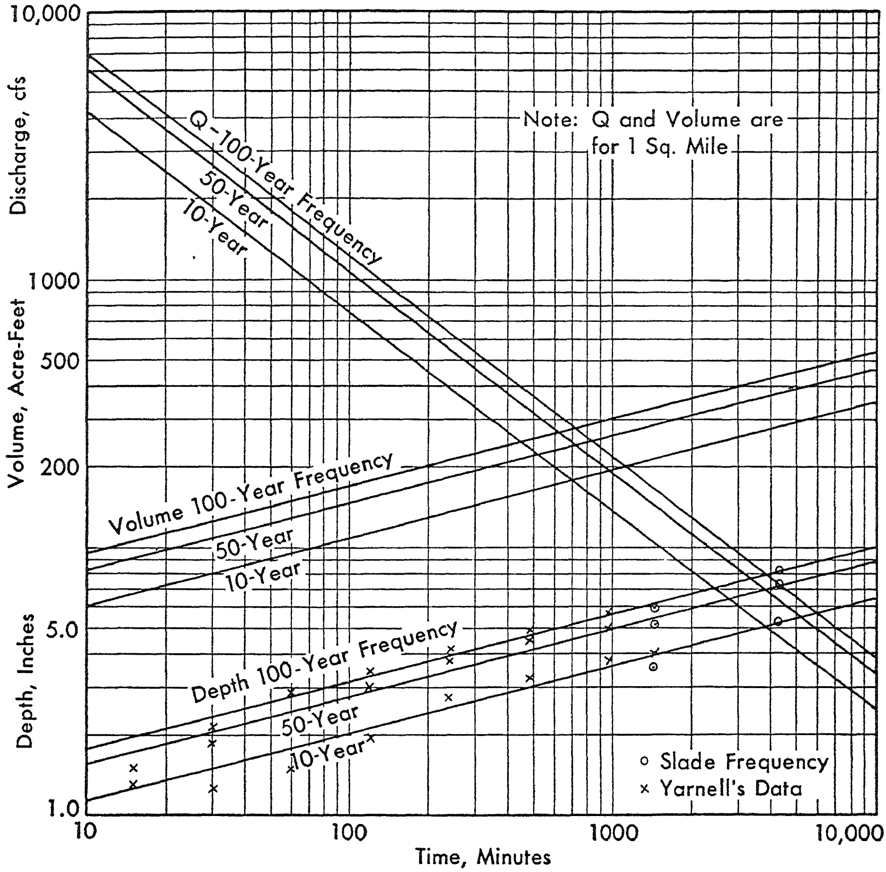



FIGURE 93. Maximum Rainfall Intensity, Concord, N. H.

rainfall, and the direction was fixed by the points from Bernard's values for shorter periods of time. Since all the data appeared to plot more consistently for the higher frequencies, the first line was drawn through the 100-year value and the other lines thereafter were drawn parallel to the first, the depth being fixed by the two-day Slade frequencies.

It should be repeated that these curves represent envelopes of the maximum rainfall to be expected for the given frequencies. The indicated depths for the longer periods of time will be obtained only if the supply of warm moist air is maintained. Furthermore, if different types of storms with diverse moisture-carrying potentialities prevail over the region, these envelope curves may be composed of segments of lines of different slopes instead of being straight as shown on Figures 92 and 93.

The curves for discharge $Q$, and volume were computed on the basis

of an area of one square mile to show the trends of each in relation to the maximum depths of precipitation. No losses were deducted from the rainfall to obtain runoff.

The curves in Figure 93 showing rainfall intensity at Concord, N. H. were constructed in the same manner, except that Yarnell's data only were available to supplement the Slade frequencies. For this station, however, Yarnell's data plot closer to a straight line than in the case of Chickasha, Okla.

*Frequency of Annual Precipitation.* Occasionally it is desired to know the frequency of a given amount of annual precipitation for urban water supply or generation of firm hydroelectric power. For example, a knowledge of the minimum annual precipitation will indicate what may be expected as minimum stream flow. Also, since the effect of drought on agriculture may be serious, the frequency of small amounts of precipitation is desirable information.

As pointed out previously, the data of annual precipitation constitute a time series in which the data are largely independent of each other. That is, they are independent in the sense that the precipitation of one year does not control or appreciably affect the magnitude of the amount falling in another year; as far as can be determined with assurance the precipitation of consecutive years is not the immediate result of the same cause. The only lack of independence is the existence of a general trend or long-time fluctuation which may create special characteristics for the period in question, so that the data could not be considered as a fair sample of the whole. Since the causes of those factors are obscure to the extent that no satisfactory explanation has been found, their effect on the independence of the data of annual precipitation cannot be fully determined, but it does appear to be remote and does not interfere with the variation of the data about the trend. Such data may therefore be considered as properly subject to treatment under the theory of probability. Certain precautions should, however, be observed in such treatment.

The first step in estimating the probability of annual precipitation should be the determination of a trend. This can be done by any of the several ways described in the preceding chapter, or possibly a satisfactory answer can be obtained by merely plotting a graph. However, if there is a marked trend such as that shown by the record at Omaha, Nebr., (Figure 37) the frequency should be determined from the trend. To accomplish this one must first compute the trend line, preferably by the method of least squares; then deduct the variations of annual precipitation from the trend and use the results as the data from which to compute the frequency. Estimates of future probable precipitation

should of course take account of the trend and its possible reversal, and since the trend cannot be projected far into the future, such estimates should be limited to a few years.

Fluctuations present a somewhat different problem. If they are of short periods (a few years) compared with length of the record they may be ignored; consecutive years of similar precipitation can properly be considered as occurring "collectively at random." On the other hand, long-period fluctuations should be rectified in some manner before computing the frequency. The record may be broken into a few shorter records and the variations computed from them. The fluctuations may have a very long period so that the record is contained entirely in the ascending or descending side, in which case only a trend can be detected. In that case the treatment for rectifying a trend is applicable.

The type of frequency curve that may be used for annual precipitation needs some consideration. Inspection of the data of distribution indicates that the normal equation may fit in many or perhaps most cases. In Figure 94 there is shown a number of distributions plotted on arithmetic probability coordinate paper. Two distributions show reasonably straight lines, as they should on such paper if the distributions are normal or nearly so; Omaha, however, shows considerable divergence from a straight line. It can be seen from Figure 37 that precipitation at Omaha has a much stronger downward trend than that at Iowa City, Iowa.

The frequency for annual precipitation at Iowa City, Iowa, was computed by means of the normal probability function

$$P = \frac{N}{\sigma\sqrt{2\pi}} e^{-u^2}.$$

The mean, 1871–1940 = $M$ = 35.26 inches. The standard deviation $\sigma$ = 5.04 inches. The number of years $N$ = 70.

The complete computation of probability is shown in Table 54 on page 231.

$X$ in the first column is taken as the upper class limit, 18 to 52 inches; the computations in the second and third columns are indicated by the headings; the values of $f(z)$ of column 4 are taken from probability tables such as Pearson's or Glover's (71). The computed distribution is given in the sixth column, $f_t(x)$ for comparison with the observed distribution $f_o(x)$.

The last two columns are the deviations of the observed distribution from the theoretical and the divergences as computed for the goodness-of-fit test. The sum of the divergences, $\chi^2$ = 12.3; with $n'$ = 12,

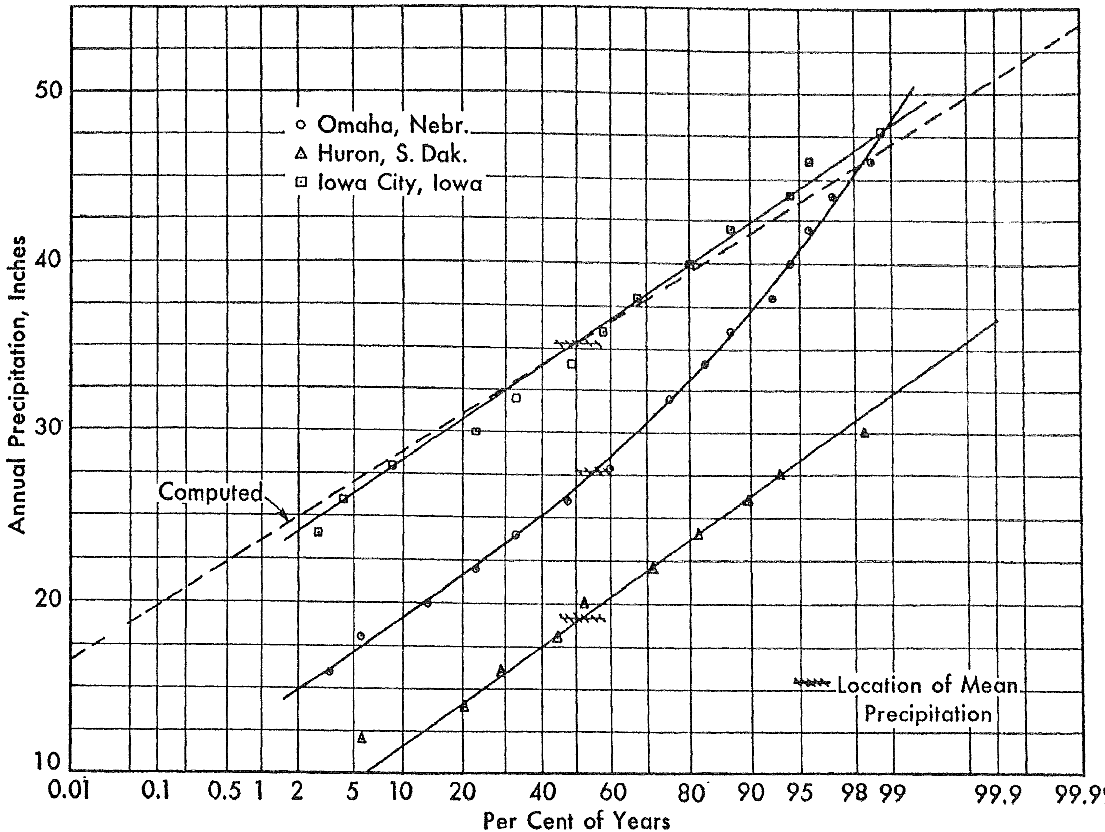



FIGURE 94. Frequency Distribution of Annual Precipitation

TABLE 54. COMPUTATION OF FREQUENCY OF ANNUAL PRECIPITATION, IOWA CITY, IOWA

| CLASSES $X$, *Inches* | $x = X - M$ | $U = \frac{x}{\sigma}$ | $f(z)$ | $Nf(z)$ | $f_t(x)$ | $f_o(x)$ | DEVIATION | DIVERGENCE |
|---|---|---|---|---|---|---|---|---|
| 18–22 | −13.26 | 2.63 | 0.00427 | 0.3 | 0.3 | .. | | |
| 24 | 11.26 | 2.23 | .0129 | 0.8 | 0.5 | 2 | 0.7 | 0.2 |
| 26 | 9.26 | 1.84 | .0329 | 2.3 | 1.5 | 1 | | |
| 28 | 7.26 | 1.44 | .0749 | 5.2 | 2.9 | 3 | −0.1 | 0 |
| 30 | 5.26 | 1.044 | .148 | 10.3 | 5.1 | 10 | −4.9 | 4.7 |
| 32 | 3.26 | 0.647 | .259 | 18.1 | 7.8 | 7 | .8 | .1 |
| 34 | − 1.26 | − .250 | .401 | 28.1 | 10.0 | 11 | −1.0 | .1 |
| 36 | + .74 | + .147 | .558 | 39.1 | 11.0 | 6 | +5.0 | 2.3 |
| 38 | 2.74 | .554 | .707 | 49.5 | 10.4 | 7 | 3.4 | 1.1 |
| 40 | 4.74 | .940 | .826 | 57.7 | 8.2 | 9 | − .8 | .1 |
| 42 | 6.74 | 1.336 | .909 | 63.6 | 5.9 | 5 | .9 | .1 |
| 44 | 8.74 | 1.735 | .959 | 67.1 | 3.5 | 5 | −1.5 | .6 |
| 46 | 10.74 | 2.13 | .983 | 68.8 | 1.7 | 1 | 0.7 | 0.3 |
| 48 | 12.74 | 2.53 | .9943 | 69.6 | 0.8 | | | |
| 50 | 14.74 | 2.82 | .99740 | 69.8 | .2 | 3 | −1.8 | 2.7 |
| 50–52 | +16.74 | 3.32 | 0.99956 | 70.0 | 0.2 | | | |
| | | | | | | | | $\chi^2 = 12.3$ |

$P' = 0.31$, approximately. This is a fair fit and may be accepted as satisfactory and as good as can be expected from the few observations (only 70) together with the trend.

*Compound Frequencies.* In the discussion of the goodness-of-fit test of the frequency curve and observed data from Concord, N. H. the possibility of a compound frequency curve was mentioned. By the term "compound frequency curve" is meant one that is derived from data of two or more elementary distributions of the same type. These elementary distributions would of course necessarily be of the same attribute, such as depth of daily precipitation, but would have different means.

Such shift in means or averages of data can come where two or more distinct causes produce the precipitation. In the case of Concord it could be brought about because the interactions of the polar continental and modified air masses from the west, or the polar Atlantic air most likely bring the lighter and more frequent rains, whereas the upper portion of the curve is based on the data of heavy and rarer rainfall such as could be produced by the invasion of the tropical Atlantic air masses. The dispersions of data obtained by these two causes overlap, however, sufficiently well to make a fairly consistent curve.

Analogous conditions exist in other parts of the country as is shown by the other curves of precipitation on the logarithmic probability paper. It is likely that the contingency is universal. The three stations

in Colorado (Figure 72) are especially noteworthy for the great discordance in a small area between Edgewater and Longmont, which are approximately 36 miles apart. However, if the difference in the means is not too great there will be no serious objections to using all the data in one series for a single curve; in most cases it will very likely be impossible to segregate the data for separate curves.

Before leaving this subject it should also be pointed out again that the use of data collected by arbitrary one-day periods frequently splits the 24-hour period of high precipitation. This would tend to reduce the magnitude of the daily catch for a given frequency in the higher range. This tendency would be a factor in causing the break in the distribution curves of Figure 68.

*The Station-Year Frequency Method.* In the station-year method of determining frequencies of precipitation, data of all selected stations in an area under investigation are combined into a single distribution, the period of which is assumed to be equivalent to total years in all records. This method was used in the investigations of the Miami Conservancy District and later by others, but it has not been entirely acceptable in all quarters.

To illustrate the method and define it more fully, let us assume four stations, $A$, $B$, $C$, and $D$, with records of 36.5, 31.5, 26, and 27 years, respectively. Let these stations have distributions as follows:

$$A:\ f(x_a) = o_1 + o_2 + o_3 + \cdots + o_n.$$

$$B:\ f(x_b) = o_1 + o_2 + o_3 + \cdots + o_n.$$

$$C:\ f(x_c) = o_1 + o_2 + o_3 + \cdots + o_n.$$

$$D:\ f(x_d) = o_1 + o_2 + o_3 + \cdots + o_n.$$

In the above expressions $f(x)$ denotes the distribution and $o$ the observations in each class group. As was noted in Chapter 1, the moments about the means can be computed from the functions of the distribution by multiplying each $f(x)$ by powers of $x$ (the class magnitude) and making the necessary corrections to place the origin at the mean. Thus for $A$:

$$\Sigma x \frac{f(x_A)}{N} = S_1$$

$$\Sigma x^2 \frac{f(x_A)}{N} = S_2$$

$$\Sigma x^3 \frac{f(x_A)}{N} = S_3$$

from which $M = S_1$; $U_2 = S_2 - S_1^2$

and $U_3 = S_3 - 3S_2S_1 + 2S_1^3$.

Now by the station-year procedure the observations in each class for all stations would be added. This may be expressed thus for the first summation

$$\Sigma_A^D xf(x) = \Sigma(o_1 + o_2 + o_3 + \cdots + o_n)$$

where $\Sigma_A^D$ is the total of the observations, class by class, for all four stations.

From these summations another set of $S$ and $M$ values can be derived:

$$\frac{\Sigma_A^D xf(x)}{\Sigma_A^D N} = \Sigma_A^D S_1$$

$$\frac{\Sigma_A^D x^2f(x)}{\Sigma_A^D N} = \Sigma_A^D S_2$$

$$\frac{\Sigma_A^D x^3f(x)}{\Sigma_A^D N} = \Sigma_A^D S_3.$$

Again, after discarding the summation signs as no longer significant,

$$M = S_1$$

$$U_2 = S_2 - S_1^2$$

$$U_3 = S_3 - 3S_2S_1 + 2S_1^3.$$

It can be seen that in using the four sets of data, the power sums $xf(x)$, $x^2f(x)$, and $x^3f(x)$ of the totals were divided by the total number of observations in each case, leaving in effect only the average of the data of the four stations. From consideration of the method of computing the frequency for Concord, N. H. it will be seen that since there is only an average of the individual station moments, the resulting probabilities can be only a weighted average. The number of years does not enter except through the increased number of data. The result would be the same whether one used the data to compute moments as outlined above and then a theoretical curve, or sorted the data by class and divided them by the number of station-years.

Nevertheless, the average data of several stations in an area having a homogeneous climate may be accepted as a distinct improvement over the data of one station, especially if there are included stations with greater depths of precipitation which are relatively rare and which probably are observed at only one or two stations in a given region. These averages, however, should be recognized as contemporaneous

observations, and not a prolongation of record. With the use of the station-year method, however, the greater number of station-years may produce a false sense of accuracy which is undesirable.

One other question may arise from the foregoing discussion, namely, "What is the advantage of a long record if an average can be used?" First, it requires a long record to get sufficient data to serve adequately by any method, including the statistical, and usually it requires a long record to get a typical distribution of the various grades of data, especially those of the higher values. What is needed is a representative sample of data having a typical distribution of low, medium, and high values. If such a distribution is available other records are not essential except as a check.

# 7 SNOW

*Definition of Snow.* Snow is a solid form of precipitation formed by condensation of atmospheric moisture at temperatures below the freezing point. The usual form of snow particles is small thin white flakes of a general hexagonal shape in a multitude of patterns. Occasionally, however, it is formed in soft small spherical particles known as "graupel." Again, at extremely low temperatures fine ice needles are formed instead of snowflakes. As should be expected, these forms are somewhat modified or broken in falling and may even be melted.

After deposition on the ground, snow soon becomes a different substance as a result of the action of the common meteorological agents. Compaction takes place under its own weight, and probably also from the action of the wind; temperatures above freezing cause further consolidation. The snowflakes are broken up and recrystallization occurs under the action of thawing, freezing and compaction. Snow on the ground and under conditions in which it is of economic importance is a complex mass of snowflakes, ice crystals, and also water which may come from melting or rain.

*Areas of Snowfall.* Because the formation of snow is limited to temperatures below freezing, the areas on which it falls and the season of occurrence are limited. The areas of principal occurrence are the north and south temperate, and arctic and antarctic zones. In the low latitudes it is confined to high, mountainous areas. Only in the polar regions and on the higher mountains is it found throughout the year. In the temperate zones it is chiefly a phenomenon of winter with the attendant lowered temperatures.

*Economic Importance.* Snow is a matter of great economic importance in all inhabited regions where any appreciable quantity falls. Its importance varies for many reasons depending upon the density of population, economic development, types of industries, as well as the amount and characteristics of the snow itself. Its affects are many and

varied and they are neither all good nor all bad, for snow may be a blessing or a detriment.

*Direct Effect on Industry, Commerce, and Agriculture.* Generally the effects of snow on industry are detrimental and heavy falls may cause huge damages. Snowfall is a serious hindrance to transportation, especially when heavy enough to disrupt and delay traffic, and it may break or interrupt wire communications. Other forms of industry and commerce suffer from the adverse effects sustained by transportation and communication. Buildings are sometimes wrecked by heavy accumulation of snow on the roofs. In humid regions agricultural operations are handicapped by snow cover. Wild life sometimes suffers severely where deep snow covers its food. On the other hand, agriculture in subhumid and semiarid regions shows a credit to snowfall because it is a source of water for the irrigation of crops.

*Snow Sports.* In certain suitable regions, particularly within reach of populous centers, snow provides the means of well-patronized sports such as skiing and tobogganing. Where winter sports are developed to economic proportions, snow observation is maintained and a detailed nomenclature applicable to these sports has been developed to describe the types and conditions of snow cover (146, 173, 174).

*Value of Accumulation of Snow.* An important effect of snow is the accumulation of precipitation through the winter and its subsequent release during a warm spell in the spring or even early summer at higher elevations. In regions with normally cold winters which permit almost complete accumulation, this feature produces a period of heavy runoff when the snow melts upon the advent of warm weather. The runoff from the spring melt is an important and perhaps almost the sole producer of floods on many streams; in this respect the accumulated winter snow is an economic detriment. On the high lands and mountains in certain dry regions the winter stores the season's catch of precipitation as snow until the crop growing season. This constitutes a valuable economic function because many regions in which irrigation is practised depend primarily upon this water stored as snow. Hydroelectric power projects, navigation improvements, and other stream developments are profoundly affected by the runoff from the melting of accumulated snowfall.

*Snow Measurements.* Since the turn of the century the measurement of snow has been developed into a specialty in those regions where that form of precipitation is important. Some of the methods have been adapted from procedures used in measurement of rain; others have been devised to cope with unique features of accumulated snow. Snowfall

and accumulated snow cover are measured and the water equivalent is determined.

*Measurement of Snowfall.* By the term "measurement of snowfall" is denoted the measurement of snow as it falls, that is to say, the measurement of the catch of each individual storm. This catch may be measured in two fundamentally different ways, by catchment in cylindrical receptacles similar to the standard 8-inch rainfall gage and by catchment on platforms in sheltered spots. To obtain the full story of the snow, it is measured in the liquid and solid states, the former being the water equivalent usually obtained by melting the snow.

When the standard rain gage is used the inner tube and receiver are removed from the 8-inch gage and the snow caught directly in the overflow can. For any acceptable degree of accuracy it is necessary to use this gage in a sheltered spot or to equip it with an effective shield, because the disturbance caused by wind reduces the catch of snow much more than it does that of rain. The depth of the catch is measured in its solid state, after which it is melted usually by adding a known quantity of water to melt the snow. The total quantity is measured and the quantity added for melting the snow is deducted. Recording gages are not often used for snow measurement. The tipping gage commonly used for rainfall does not operate satisfactorily in cold weather. The weighing type such as the Fergusson gage may, however, be used; the snow is caught in an antifreeze, usually a salt solution which is covered by an oil film to prevent evaporation.

For measuring the depth of snowfall a sheltered spot is first located to eliminate or reduce to a negligible degree the effect of wind. The measurement of depth may be made on cleared ground or on a low platform constructed for the purpose of catching the snow. This spot or platform can be swept clean for the next fall or successive snowfalls can be caught on a mat and measured. Another method of measuring depths of snow is use of the snow stake; this is a simple stake approximately 2 by 2 inches in cross section with a scale painted on the side and set vertically in the ground in a suitable spot. Depth of snowfall is usually recorded in inches and tenths, and the water equivalent in inches and hundredths.

*Snow Surveys.* In regions where water from melting snow is important for such purposes as irrigation or water power, data of the accumulated winter snow cover and determination of its water equivalent are needed for efficient operation of reservoirs. In order to obtain data of those factors, systematic snow surveys with special equipment are regularly conducted.

*Instruments for Snow Surveys.* The measuring instruments for snow surveys are a sampler and a weighing scale, usually a spring balance. The sampler consists of one or more sections of jointed tubing with a cutter which may be detached for replacement. A scale in feet is marked on the outside for measuring the depth of snow. The scale used to weigh the sampler with its sample is calibrated so that the weight of snow (with allowance for the weight of the sampler) can be read in inches. The diameter of the tube controls the selection of the scale. The diameter and other details of sampler construction are varied somewhat to fit the conditions under which the instrument is to be used. The Mount Rose sampler developed by the Nevada Agricultural Experiment Station has a diameter of 1.50 inches and is found to be satisfactory for use in the western areas. This same sampler has also been used successfully for snow surveys in parts of New England. The Weather Bureau formerly had a tube with a diameter of 2.655 inches, which was reported in 1940 to be still in use in Maine (15). A sampler was used in Pennsylvania with a diameter of 1.877 inches. This diameter was such that each 0.01 pound of sample was equivalent to 0.10 inch of water (37).

The sample of snow is taken by thrusting the sampler tube vertically through the snow cover to the ground, cutting out a core which is retained on withdrawing the tube. The known tare of the sampler is deducted to give the net weight of the sample, from which its water equivalent can be computed. Several samples, at least four or five, should be and customarily are taken in each course or sampling station to arrive at a reasonably accurate average for the course.

*Operations of Snow Surveys.* Snow surveys consist in periodic measurement of snow depths and water equivalent at predetermined courses that have been selected to furnish a reliable index of water yield for a basin or area. The establishment of a snow survey is a separate problem for each watershed. In the western portion of the United States courses run generally from 500 to 5000 feet long, whereas in New England they are shorter. Snow courses consist of 10 to 12 sampling points or small areas prepared for the reception and measurement of snow; in both regions successive observations are made as close as possible to the same spots. This is important in developing the index value of a snow course. Enough courses or stations are established to permit estimating the runoff from snow water with a small percentage of error. In some areas where conditions are favorable and sufficient experience has been acquired in operation and estimation, estimates are within 5 or 10 per cent of the actual runoff. The distribution of these stations or courses over the basin depends upon the nature of each watershed and should

be made after a preliminary investigation. Topography (particularly elevation), exposure, vegetal cover, state of cultivation if any, prevailing winds, and source of moisture-bearing air should be considered.

The art of snow surveying has become highly developed in the regions of high snowfall and strong demand for stored water. The equipment and protection by means of clothing and shelters for snow surveyors make interesting reading but discussion of them is outside the scope of this book. They have been well described in the extensive literature on the subject, particularly in the *Transactions of the American Geophysical Union.*

*Relationship Between Depth of Snow and Water Equivalent.* The relationship between depth of snow cover and water equivalent is largely a matter of density or specific gravity of snow. Since in sampling the depth of snow and depth of water equivalent are both read directly in inches for columns of equal cross sections, the density can be obtained simply by dividing the latter by the former, that is,

$$\text{Density} = \frac{\text{Water Equivalent}}{\text{Depth of Snow}}.$$

The result is usually expressed as a percentage.

The term "water equivalent" is used to indicate the depth of water obtained by melting a column of snow of the same cross section. The term "water content" is frequently used to express the same concept, but as Wilson (197) points out, a distinction should be made because "water content" can also mean the free or liquid water enmeshed in the snow crystals. Water equivalent can mean only the water obtained from the melted snow.

*Variations in Density.* The density of snow varies widely, as even casual observations on snow conditions through a winter will indicate. A ratio of depth to water equivalent of newly fallen snow is commonly taken to be 0.1, which would mean a density of 10 per cent. Much lower density has been observed. Brooks (23) reported that a light fall at Milton, Mass. had a density of 1/63, or 0.016, and another observer (134) reported densities of 0.008 and 0.004. In these cases, however, the reported depth of snow was small and would have little effect on a winter accumulation. On the other hand, densities of 50 per cent and higher may be found in late spring.

*Correlation of Snow Depth and Density.* The variation in density raises the question of the correlation between depth of snow cover and density. Two investigations were made of data from snow surveys in the Androscoggin River basin up to the year 1936; these data were obtained at the beginnings of March and April. These correlations were

computed by means of the correlation table, similar to that shown in Table 6. The average depths, correlation coefficients, and densities are given in Table 55.

TABLE 55. SUMMARY OF CORRELATION OF DEPTHS AND WATER EQUIVALENT

| MONTH OF DATA | AVERAGE DEPTHS, INCHES *Snow* | *Water Equivalent* | CORRELATION $r$ | DENSITY *Per cent* |
|---|---|---|---|---|
| March 1 | 25.52 | 7.04 | .825 | 25.6 |
| April 1 | 27.83 | 9.20 | .852 | 32.8 |

The data for March were gathered through 26 years, 1911–1936 inclusive; those for April, through 10 years, 1927–1936 inclusive. The data of snow depth and water equivalent are shown graphically in Figure 95. The results of the computation above indicate that correlation between depth of snow cover and density is good for an accumulation of winter snow in spite of the widely varying and low densities of newly fallen snow.

*Depletion of Snow Cover.* Not all snow cover disappears annually, since some is located on high mountains or in polar regions where temperatures never remain above freezing sufficiently long to remove it. But in regions where snow is of economic importance its annual depletion is complete. The term depletion is used to denote all forms of removal — evaporation as well as melting with subsequent runoff.

Although snow is usually light and fluffy when freshly fallen, it soon begins to pack and consolidate. Gravity alone will bring this about if the fall is deep enough; the result is greater density as long as it lies undisturbed. On the other hand, in areas where drifting is possible the wind will pack it, sometimes forming a crust sufficiently strong to support the weight of a man. Light melting may occur through the winter because of radiation, warm air, or even light rain. All these effects reduce the depths of the snow cover and increase the density.

The most potent cause of depletion of snow cover is melting. Although evaporation causes some loss, it is usually small. Compaction by gravity or wind reduces the depth but does not reduce the water equivalent, which is the factor important for runoff.

*Application of Heat on Snow.* The effect of heat on snow varies widely, being dependent upon the means by which it is conveyed to the snow cover. The immediate sources of heat acting on snow cover are as follows:

1. Underlying soil. During the warm season heat will be stored in the soil for some depth; however, some at the surface may be lost before the advent of a winter snow cover. The transmission of heat through

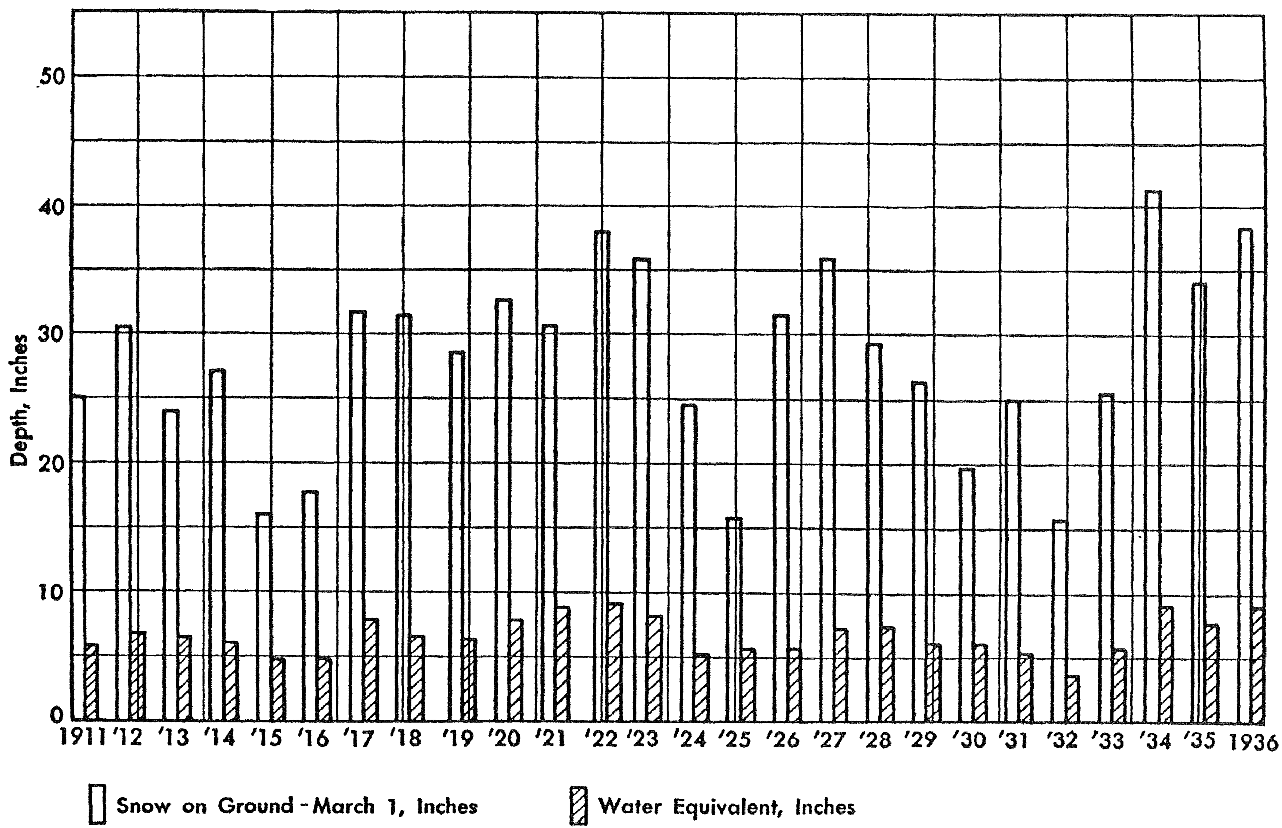



FIGURE 95. March Snow Surveys, 1911–1936, Androscoggin River above Errol, N. H.

soil is small relative to that from other sources, but in so far as it may melt the ground ice thereby making the ground receptive to melt water from snow, it will be a factor in reducing runoff from snow.

2. Conduction from the air. Because of the low conductivity of air this is a minor source of heat in so far as still air is concerned.

3. Rainfall. Rain, being liquid, would have a temperature higher than snow and hence convey heat to it. This source of heat is popularly assumed to be large, but as will be pointed out later, it is not nearly so large as assumed.

4. Radiation. Heat from radiant energy may come from the sun or sky, and on clear days is sufficient to be effective in reducing snow cover.

5. Turbulent exchange of heat from the air. This is one of the most important sources of heat to melt a snow cover and must be considered further in some detail.

6. Condensation of atmospheric moisture. This likewise is an important source of heat for snow cover.

*Effect of Rain.* The effect of rain on snow can be estimated readily from the known facts of heat. It requires 144 Btu to melt 1 pound of ice or dry snow at a temperature of 32 F. Since 1 pound of rain would supply 1 Btu of heat for each degree Fahrenheit above freezing, a rainfall of 14.4 pounds, or 2.76 inches over 1 square foot at a temperature of 42 F would be required to melt 1 pound of snow. The snowmelt that can be obtained by rainfall can be computed by the formula

$$D = P\frac{(T - 32)}{144}$$

where $D$ equals the depth of water in inches from melting snow, $P$ is the depth of rain in inches, and $T$ the temperature of the rain in degrees Fahrenheit, which may be taken as the wet-bulb temperature.

*Action of Radiation.* There are two immediate sources of radiation, that which is direct from the sun and that from the sky, or diffuse radiation. Both supply heat to the snow cover in amounts that vary much from day to day and also vary according to location and elevation of the snow. On cloudy days there is practically no direct radiation and sky radiation may be reduced to nearly nothing. On stormy days with dense cloud cover such as prevails when heavy rainfall occurs, radiation is a small or even a negligible factor in snow melting. On the other hand, on bright sunny days the effect of radiation on a snow cover is great, and even casual observation will show how rapidly snow will waste away under a clear sky. The effect of radiation is greater as

elevation increases, and on the mountains where the air is light and clouds are few, it may be the principal factor in the ablation of snowfields.

Snow also reflects radiant energy at a high rate, the albedo (the ratio of outgoing to incoming radiation) varying from 60 to 90 per cent. The portion of radiant energy that is effective in melting snow is that part which is absorbed and is limited to 10 to 40 per cent, dependent upon the condition of the upper surface of the snow. A dirty surface will absorb more heat and thus melt more of the underlying snow.

Radiation does not penetrate the snow cover to great depths, perhaps less than a foot. Some measurements were made and reported by Hand and Lundquist (81) in which they made observations of radiation at varying depths beneath the surface. As should be expected they found that the radiant energy received diminished rapidly as the depth of snow cover increased; 22.3 per cent of the total received at the surface was measured at a point 0.5 inch below, 7.8 and 16.9 per cent (two measurements) at 1.0 inch, and none at 7.0 inches under the more favorable conditions of transmission. The quality of the snow also effects the transmission of radiation, the lighter and more fluffy snow transmitting the greater percentage.

Accurate measurement of amount of snow melting due to radiation is not practicable at the present time because of the lack of stations observing radiant energy, and because of the large variation of the albedo of snow. Since the albedo changes with the conditions of the surface, a small deposit of dust would greatly reduce the albedo and increase the melting.

*Turbulent Exchange and Condensation.* Turbulent exchange of heat and condensation of water vapor are closely related sources of heat for melting snow. These two sources may be considered together as they are both related to the wet-bulb temperature of the overlying air and the mechanics of atmospheric turbulence operates similarly for both. Furthermore, the heat from condensation may be added to the air before being transferred to the snow cover.

Methods of determining under limited conditions the probable snowmelt from the wet-bulb temperature, wind velocity, and appropriate coefficients of turbulence, have been worked out by Light (115) from investigations of Sverdrup and others. The methods that were worked out by Light were intended for the determination of the maximum possible snowmelt that could be obtained under a storm producing a severe flood. For this purpose he confined his attention to melting from turbulent exchange of heat and condensation, since these are the primary causes of snowmelt under such conditions. Radiation and

rainfall would be negligible sources of heat compared with the heat from exchange and condensation during the passage of a storm involving warm moist air masses. He called the melt from the two primary sources of heat "effective snowmelt," which may be expressed by the relationship,

$$D = \frac{(Q + 600F)}{80} + F = \frac{Q + 680F}{80}$$

in which $D$ is the effective snowmelt and condensation water in centimeters per second, $Q$, the heat transferred by convection in calories per square centimeter, and $F$ is the transfer of water from condensation in centimeters per second.

In order to evaluate the above equation, certain factors must be considered and some assumptions made. The turbulent heat transfer by which the heat of the air is brought in contact with the snow is dependent upon the mechanics of turbulence of the atmosphere and the vertical temperature gradient. The rate of heat transfer from condensation of moisture likewise depends upon turbulent exchange and the vertical gradient of moisture.

*Atmospheric Turbulence.* The motion of a fluid is said to be "turbulent" when its velocity and direction are subject to rapid and irregular fluctuations. This turbulence is found in all fluids, including the atmosphere, and except when no wind is blowing, the atmosphere may be considered as being in a turbulent state for all practical purposes. This atmospheric turbulence is capable of transferring various properties of the air, such as heat or vapor, to adjacent strata at right angles to the direction of the mean velocity. This transfer, also known as "turbulent mass exchange," is much more effective in dispersion or mixing than other types of action, such as molecular conduction of heat or diffusion of gases and water vapor.

The intensity of atmospheric turbulence depends upon the mean horizontal velocity and the roughness of the ground surface. Thornthwaite and Holzman (184) and other investigators have found that the mean velocity of wind at a given point varies (at least under adiabatic conditions) on a vertical plane approximately as a logarithmic function of the height of the point above the ground. Light (115) shows similar vertical distributions for water vapor and heat. Data of these variations by which the distributions can be evaluated, are obtained by anemometers and other suitable instruments placed at different elevations above the ground. Since these observations are taken at elevations on artificial supports, the observed distributions are applicable only to the

very shallow layer of air next to the ground, but this limited applicability is sufficient for the purposes for which the data are used.

*Surface Roughness.* The surface roughness affects any air current flowing over it; the greater the roughness, the greater will be the turbulence for a given velocity. The only practicable approach to measuring the degree of turbulence is observation. Sverdrup (115) has evaluated a coefficient for a smooth level snow field and derived a coefficient of 0.25 for that condition: this value has been accepted for use in computing snowmelt over level fields, and melting from other types of topography is adjusted to that value by means of an empirical coefficient to be discussed later.

*The Empirical Formula for Snow Melting.* The evaluation of the above formula for snowmelt depends upon coefficients of turbulence and other data and some assumptions obtained or deduced from observation. Since it is applied to melting during stormy periods, it is assumed that the wind velocity is sufficient to produce and maintain well-developed temperature and vapor gradients to the snow surface. This assumption could not be made for still air over snow because the cold surface cools and stabilizes the air and stable air tends to inhibit turbulence. Then under the assumption of turbulence made above, the temperature of the surface will be 32 F because of the melting of snow. Likewise under the same conditions, the surface layer of air may be considered saturated, and at a temperature of 32 F would have a vapor pressure of 6.11 millibars, or 0.181 inches of mercury. In order to obtain a second point to establish the gradients of temperature and water vapor, data of those two factors are obtained by instruments at elevations of 10 and 50 feet, respectively, above the surface.

After evaluating the gradients by using 50 and 10 feet as elevations of the higher set of instruments and applying the known coefficients, including Sverdrup's roughness factor of 0.25, the formula for effective snowmelt reduces to

$$D = U[0.00184(T - 32)10^{-0.0000156h} + 0.00578(e - 6.11)]$$

in which $D$ is the effective snowmelt in inches obtained in 6 hours, $U$ the average wind velocity in miles per hour, $T$ the air temperature in degrees Fahrenheit, $h$ the elevation of the surface above mean sea level in feet, and $e$ the vapor pressure in millibars. The factor $10^{-0.0000156h}$ is included to correct for changes in elevation, but it may be omitted for basins near sea level.

Since Sverdrup's value of roughness of 0.25 was used in the constants of the above formula, it is limited in use to clear, level open fields. Only the smallest basins can be expected to approximate that

condition so that for most cases it is necessary to find another coefficient to adjust the equation to a given basin. This can be done by estimating the snowmelt runoff from the basin for a past period and using the ratio of the observed to the theoretical snowmelt as a coefficient. A method of making such an estimate will be given in Chapter 9.

The Hydrometeorological Section of the Weather Bureau has used the above formula to determine the snowmelt on a number of river basins. The basin coefficient, designated $K$, was derived by estimating the snowmelt from the runoff; these values are shown in Table 56.

TABLE 56. BASIN COEFFICIENT FOR SNOWMELT

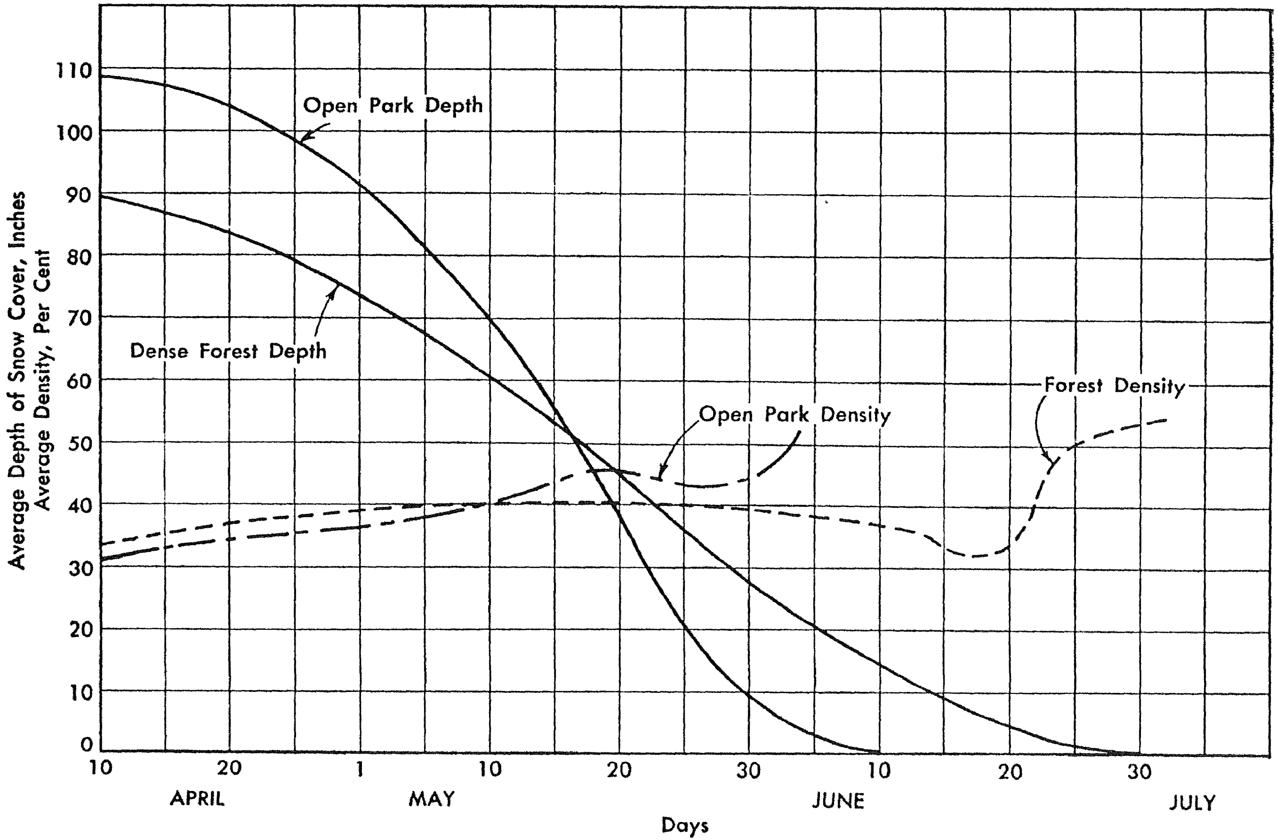
| BASIN | | COEFFICIENT, $K$ |
|---|---|---|
| Ohio River, above Pittsburgh, Penn. | Average | 0.65 |
| | Range | 0.60 to 0.72 |
| Middle Fork, Yuba River, above Milton, Calif. | | 0.75 |
| Middle Fork, Yuba River, above N. San Juan, Calif. | | 0.67 |
| North Fork, Yuba River, above Sierra City, Calif. | | 0.51 |

The foregoing formula and coefficients will determine the expected snowmelt. The runoff may not follow immediately, as the initiation of runoff is dependent upon the condition of the snow cover. As pointed out previously, the snow will hold considerable water so that the density must be high before runoff can start.

*Snow Runoff.* The runoff phenomena presented by the melting of snow are distinguished from those of rainfall. The first application of heat to snow raises its temperature to 32 F, if it is below that point, and further application will cause melting. In a widespread snow cover, melting starts on the surface and the water percolates downward. There is no immediate runoff, but instead the melt water serves to consolidate the snow and increase its density. Light rains will function in the same manner until the capacity of the snow to hold water is filled. When that state is reached, runoff will begin.

In Figure 96 taken from Clyde's paper (34) are shown some curves which illustrate the consolidation of a snow field and the changes in density.

*Daily Temperatures and Melting.* The close dependency of snow melting upon temperature as shown by the formula for effective snowmelt indicates that there may be some empirical relationship between daily temperature and snow runoff. As has been pointed out by a number of investigators, such a relationship does exist. This relationship, of course, exists wherever there is a snow cover to be acted upon by warm air, with or without rainfall. In various reports of studies




*After Griffin, Mon. Wea. Rev., vol. 46*

FIGURE 96. Melting Changes of Snow, Yakima Watershed, 1917

on this subject the temperature is usually expressed by the term "degree-day" which is a unit consisting of one degree of temperature for one day; the datum of the temperature is commonly taken within a few degrees of freezing.

In their investigations of storm flow prediction on index areas, Leach, Cook, and Horton (112) found a good agreement between accumulated degree-days above 32 F and total runoff. Collins (36) made use of the same relationship in order to predict annual runoff for power generation in the state of Washington; he plotted the monthly summation of degree-days above freezing, by areas at different elevations, against the percentage of total annual runoff, and obtained a very satisfactory agreement. Collins' curves are reproduced in Figure 97. Wilson (198) has likewise pointed out the same relationship and its utility for prediction of spring runoff in Yellowstone Park. This relationship is most effectively used for prediction of runoff in regions where most of the discharge of the season of prediction comes from snowmelt. Rainfall preceding and following the snow season complicates the prediction and must be included in the runoff, frequently with some adjustment.

Collins' methods are particularly interesting, as they were devised for the very practical purpose of predicting the volume of water that would be available for power generation. The studies extended over seven climatic years, 1920–1927, beginning November 1. The curves in Collins' paper (36) are means drawn through all the results of observation in those years; the plotted points show some scattering but all are close enough to the curves (reproduced in Figure 97) to indicate high correlation.

Wilson (198) presented a similar curve for the Gardiner River, Yellowstone Park, from the observational data of one year, 1939. This curve, which is reproduced in Figure 98, is plotted as accumulated runoff against accumulated degree-days from March 1. It has a very flat *S*-shape, similar to that shown by Collins' curves. This reversed curvature denotes a changing relationship between degree-days and runoff and indicates that the melting in the basin, as evidenced by snow condition, depth, elevation, and area of contribution, is progressing and shifting during the melting season.

Before leaving these curves a few mathematical properties may be noted. They may be expressed in a differential form by the equation

$$dT = K\,dQ$$

from which by integration $\qquad T_t = kQ_t$

and $$k = \frac{Q_t}{T_t}.$$

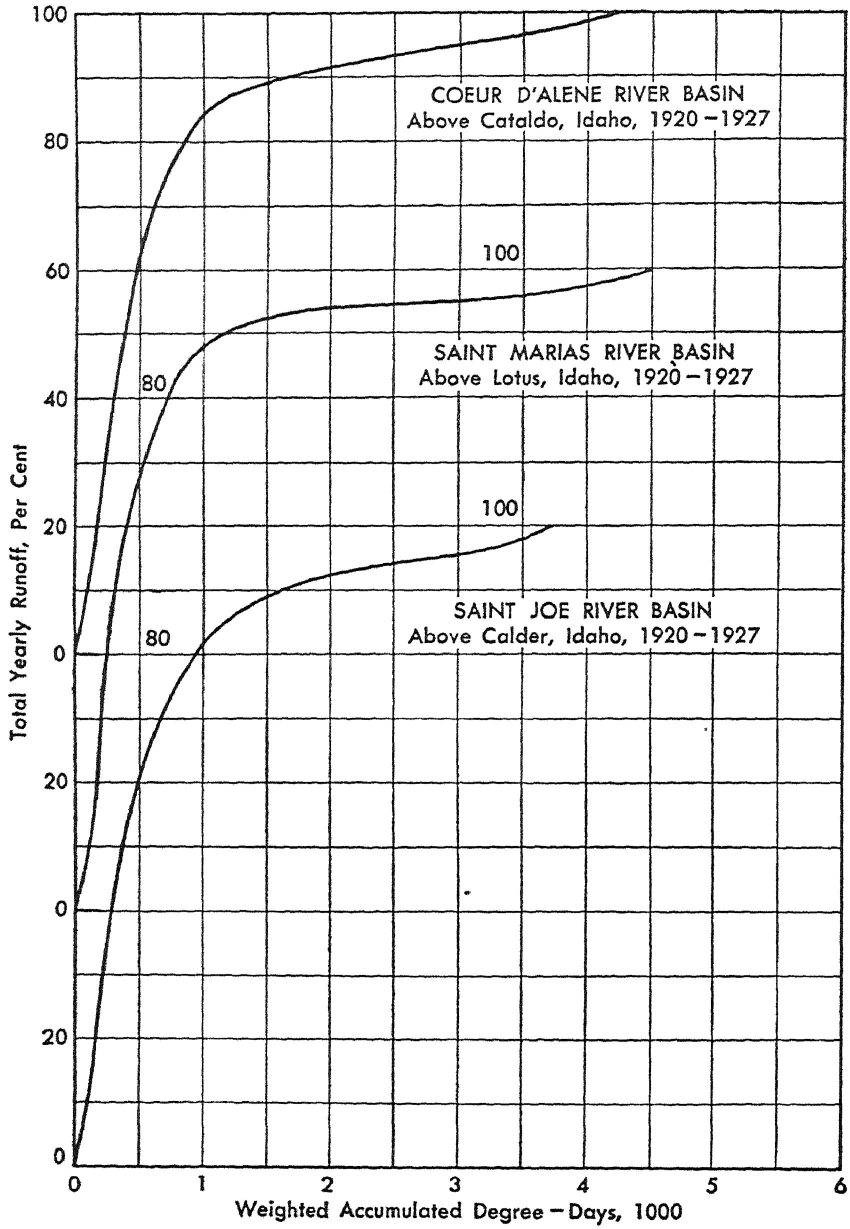
The slope at any point equals $$\frac{dT}{dQ} = k.$$

In the above equations $T$ equals temperatures; $Q$, flow; $t$ time; and $k$, a constant which may be taken as the rate of snowmelt per unit of time.

*The Snowmelt Constant.* The snowmelt constant $k$, in the preceding paragraph, is in present-day usage commonly taken as the depth of water in inches produced by the heat available from air for each degree of temperature above 32 F for one day. Although the heat required to melt ice or snow is 144 Btu per pound (or 80 calories per gram), this fact cannot be utilized under general conditions at present because of a number of difficulties. It depends upon the transfer of heat from the atmosphere to the snow, which, except under the very limited conditions specified by Light (115), cannot be evaluated at present. Under normal conditions the air is not saturated, so that no water or heat from condensation would act on the snow. It cannot be expected that all available heat above 32 F of the air will be used for melting. Furthermore, radiation will add an indeterminate amount of heat for melting. Therefore the snowmelt constant under usual conditions remains a value to be determined experimentally for each basin and season.

*Forecast of Annual Water Supply.* The principal reason for developing snow surveys was the need for knowing how much water would be available in the spring runoff. There are several methods for predicting the potential spring runoff from snow, each dependent upon climatic and topographic conditions and uses to which the water would be put. The complete story of runoff will be given later, but at this time it is appropriate to discuss the factors relating to snow.

The simplest method of forecasting runoff was developed in the semiarid mountainous western states where the total annual runoff was obtained preponderately from snow, and it was devised for the purpose of predicting the supply of irrigation water for the following season. The object of the forecast was simple, and consequently the methods need not be elaborate. The rugged topography and great variation in other factors prevented laying out snow courses that would permit computing the total water available in the snow cover. Recourse was had to determining the average water equivalent found on the snow courses for the given year to compare with the mean of past records, and predicting similar deviations of runoff from the mean of the same




*After Collins*

FIGURE 97. Relation of Temperature to Runoff from Snow, Idaho

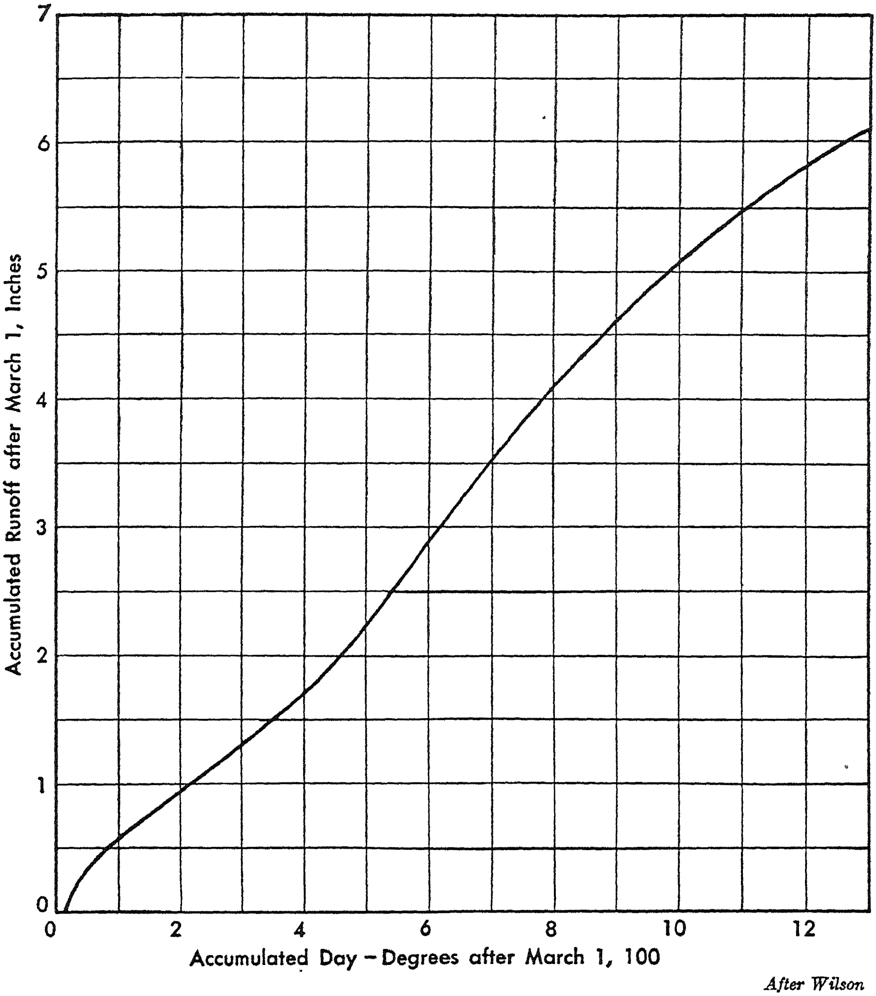
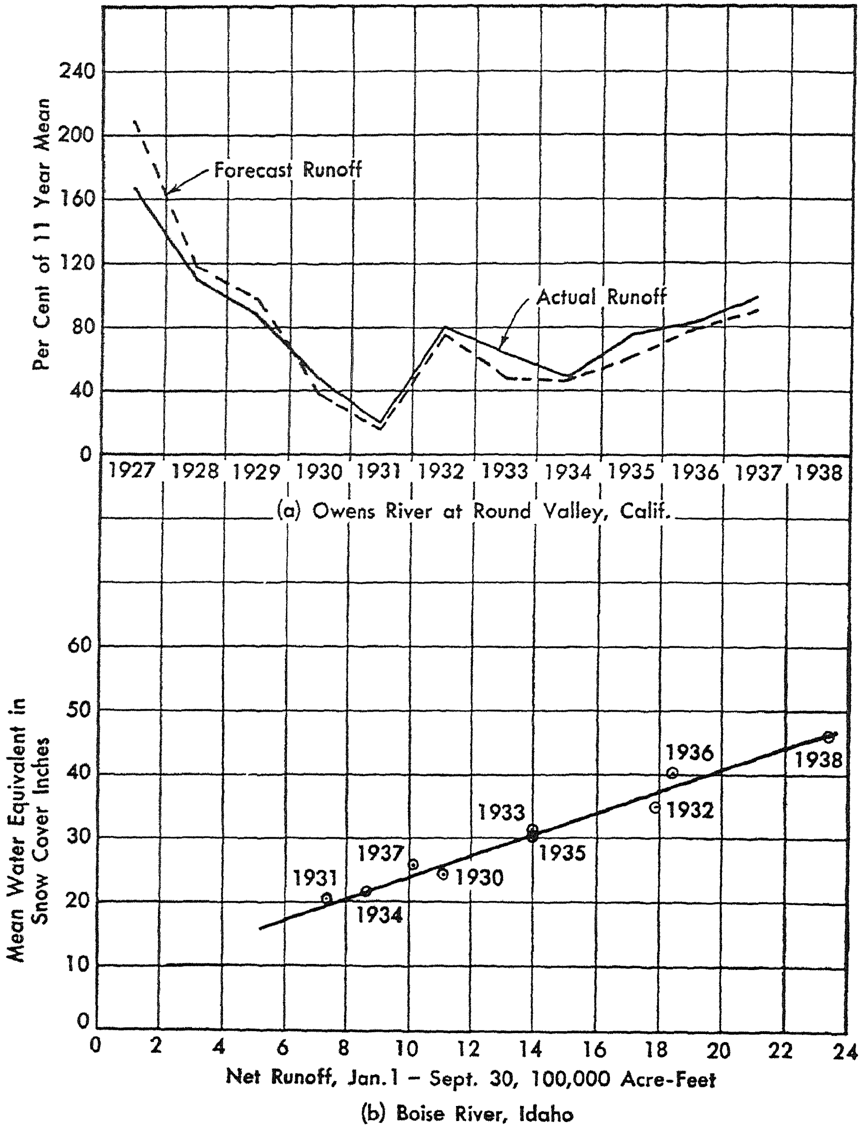



FIGURE 98. Relation of Temperature to Snow Runoff, Yellowstone Park

period. This is known as the percentage method of forecast because the predicted annual runoff was given in terms of percentage of the mean. For this purpose the snow courses are meant to give an index of the water equivalent of the snow cover on the watershed rather than data from which the annual yield of water can be computed directly. The courses should be located so that the data obtained from them form reliable indicators of the mean on the basin as evidenced by the runoff produced.

Figure 99(a), from a paper by Elges (55), shows a comparison between actual runoff and that predicted from data of snow surveys from the Owens River basin, California. Figure 99(b), likewise from




*After Elges*

FIGURE 99. Comparisons of Predicted versus Actual Snow Runoff

Elges' paper, shows the relationship between the water equivalent of the snow cover and the runoff of the Boise River, Idaho.

The reliability of the foregoing method of prediction is based on the premise that there exists a stable relationship over the entire basin between the data obtained from the snow courses and the runoff produced by the snow cover. Such a relationship is not easy to obtain and not infrequently some adjustment of the courses is necessary to

yield satisfactory correlation. The suitability of the data can be determined only after sufficient data have been obtained to make a reasonable comparison and then methods of statistics can be used to measure the correlation. While snow surveys on one watershed may be used to predict runoff on an adjacent basin, the reliability may be small and becomes less as the point of measurement of runoff is moved away from the area of the snow courses. In his study of correlation among headwater streams, Monson (139) points out that reliability is best on high watersheds where a season of precipitation in the form of snow is followed by a season of runoff with little rain.

*Correlation between Data of Snow Surveys and Runoff.* Monson used the formula

$$r = \frac{\Sigma xy}{N\sigma_x\sigma_y}$$

to determine the correlation between the data of water equivalent from snow surveys on the headwater of the Snake River in the vicinity of Yellowstone Park, Wyo., and the runoff observed on the Snake and Yellowstone Rivers. He applied the same method for correlation between data of water equivalent from snow surveys on the Swift Current Creek, a tributary of the Milk River with headwaters adjacent to Glacier Park, Mont., and various gaging stations on the Milk River. The notation is the same as used heretofore for the correlation equation, the small letters $x$ and $y$ being the deviations.

In the Snake and Yellowstone River study he found the correlation between the water equivalent and the discharge of 12 stations on the Snake and Yellowstone Rivers. These stations gaged the flow on the large tributaries of the Yellowstone, Big Horn, Clark's Fork, as well as smaller tributaries, and one on Madison River, a headwater tributary of the Missouri River. All these streams have headwaters in the mountains surrounding Yellowstone Park and mountains in the upper portion of the Snake River drainage basin. In all streams the snowmelt is the largest portion of the headwater runoff. The period of record ranged from 14 to 18 years. The correlation coefficients found on the Snake-Yellowstone study are given in Table 57.

The low coefficient for Lamar River is attributed to greatly differing runoff characteristics, although the stream is mainly within Yellowstone Park; the low coefficient for the Big Horn River is due to the fact that it drains much territory at considerable distance from the location of the Snake River snow courses; and for Glendive, because that station is far distant downstream from the headwaters of the Yellowstone River. The other stations are located adjacent to Yellowstone

TABLE 57. CORRELATION COEFFICIENTS, SNAKE-YELLOWSTONE STUDY

| STREAM | LOCATION | PERIOD OF RECORD, *Years* | CORRELATION COEFFICIENTS $r$ |
|---|---|---|---|
| Snake River | Moran, Wyo. | 18 | 0.81 |
| Henry's Fork, of Snake River | Warm River, Idaho | 14 | 0.73 |
| Warm River | Warm River, Idaho | 14 | 0.74 |
| Tower Creek | Tower Falls, Yellowstone Park | 14 | 0.66 |
| Lamar River | Tower Falls, Yellowstone Park | 14 | 0.47 |
| Gardiner River | Mammoth, Yellowstone Park | 14 | 0.78 |
| Yellowstone River | Canyon, Yellowstone Park | 16 | 0.68 |
| Yellowstone River | Corwin Falls, Mont. | 14 | 0.76 |
| Yellowstone River | Glendive, Mont. | 14 | 0.31 |
| Clark's Fork, of Yellowstone River | Chance, Wyo. | 16 | 0.62 |
| Big Horn River | Thermopolis, Wyo. | 18 | 0.28 |
| Madison River | West Yellowstone Park | 18 | 0.60 |

Park and the correlation is not bad when compared with that of the Snake River itself.

Similar correlation is found between the data of water equivalent observed by snow surveys on the Swift Current Creek and the runoff below on surrounding basins. These coefficients are given in Table 58.

TABLE 58. CORRELATION COEFFICIENTS, SWIFT CURRENT CREEK

| STREAM | LOCATION | PERIOD OF RECORD, *Years* | CORRELATION COEFFICIENTS $r$ |
|---|---|---|---|
| Swift Current Creek | Many Glaciers, Mont. | 15 | 0.85 |
| Flathead River | Polson, Mont. | 15 | 0.87 |
| Marias River | Shelby, Mont. | 15 | 0.75 |
| Marias River | Brinkman, Mont. | 12 | 0.59 |
| South Fork, Milk River | U. S. – Canada Boundary | 15 | 0.39 |
| North Fork, Milk River | U. S. – Canada Boundary | 15 | 0.20 |

These stations again show close correlation of runoff with water equivalent on the basin of Swift Current Creek, if located where the snow cover and runoff conditions are similar. The two lowest coefficients are those for the Milk River which drains different areas from those of the snow courses.

*Forecast of Discharge*. The annual runoff forecasted by the percentage method does not indicate the rates of flow or discharge to be expected during the ensuing season. Studies of discharge hydrographs of streams in mountainous areas showed that the recession curves of all streams were very similar in shape from year to year regardless of variations in the annual runoff. This fact was then used to predict the rates of discharge through spring and summer. It was necessary only

to adjust the total runoff to a recession curve of similar shape to obtain the prediction of flow.

Another method, described by Wilson (198), of predicting annual runoff and seasonal flow was developed under conditions similar to the foregoing methods and follows the general procedure adopted by Collins and discussed above. Essentially this method consists in computing the degree-days above 32 F and plotting these data against the runoff. Prediction of season flow was made on the basis of the shape of the curves of accumulated degree-days versus runoff, since it was found that the shape of these curves was similar from year to year. This method is fundamentally an extension and refinement of the forecasting method discussed previously and would therefore have comparable accuracy and correlation.

*Runoff Prediction for Reservoir Operation.* In cold regions the winter season may very likely be the critical one with respect to water supply for power generation, and where there is a strong demand for hydroelectric power it is desirable to draw down the reservoirs as much as possible in winter to maintain the output of energy. Such conditions of water supply and power prevail in New England and therefore another method of prediction of runoff from snow was developed to control reservoir operation during winter.

This plan of forecasting as developed by the Union Light and Power Company and presented by Bean (14) relies on the data of the water equivalent of the snow cover for estimates of the volume of water held in storage in the snow cover. Snow measuring stations are located for this purpose. Satisfactory accuracy has been obtained by the use of one station on every 50 square miles. A daily check of the conditions of the snow cover is obtained by a weather station established on the upper portion of the basin where daily records of temperature, depth, and water equivalent of the snow cover are obtained.

The data of weather records and snow cover are used graphically to depict the conditions of the basins, the volume of water stored in the basins, and the space available in the reservoirs for storage of the spring runoff. The data are plotted as they become available and thereupon furnish a current record of conditions from which operation of the reservoirs can be regulated. The operation and results are illustrated in Figure 100.

*The Part of Snow in Floods.* Enough has been said heretofore to indicate the importance of the part that snow plays in the creation of floods. In all regions where appreciable snow remains on the ground during the winter, spring is the season of floods. This is illustrated by the graphs of seasonal runoff in Chapter 9 and Figure 101 which shows

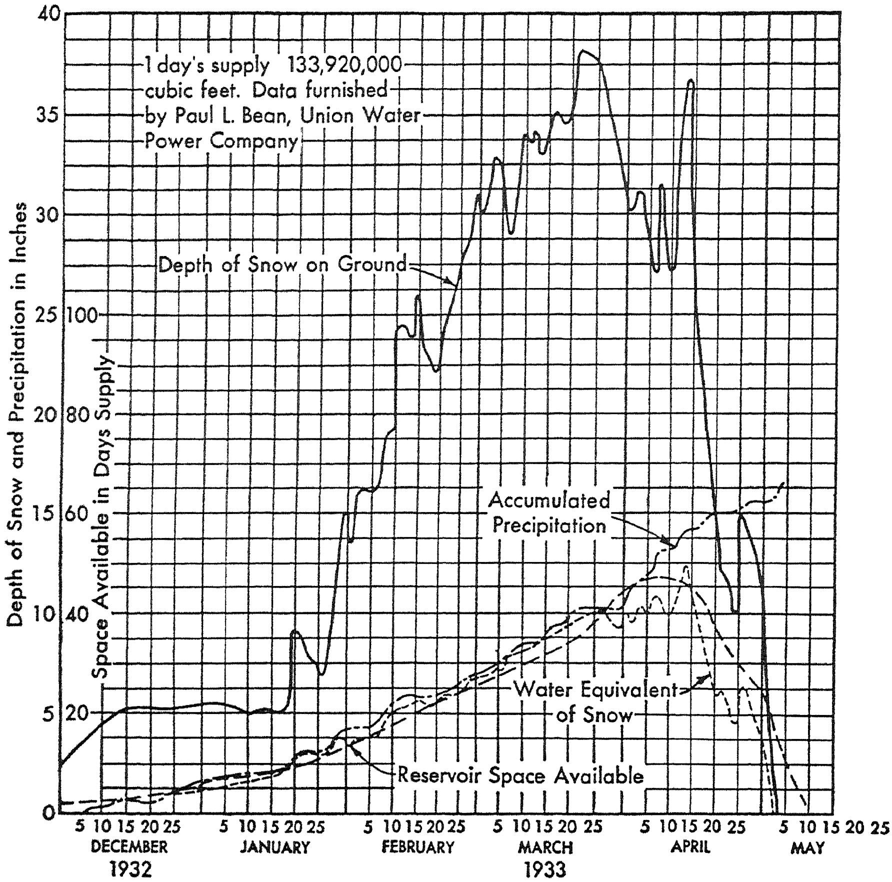



FIGURE 100. Effect of Snow Runoff on Reservoir Operations

the proportion of the annual runoff occurring in spring for two rivers in New England. A watershed with a heavy snow cover and subject to an influx of warm moist air, either with or without rainfall, is an excellent situation to produce a serious flood; rainfall, of course, can add considerably to the runoff.

Both floods and runoff will be discussed in some detail in later chapters so that no elaboration will be made here of the general statements in preceding paragraphs. There will, however, be given below brief descriptions of two floods caused largely or almost wholly by snowmelt.

*The Floods of March, 1936.* The second flood of March 1936 was the largest of a long record on many rivers in New England. A study of the antecedent conditions indicates that in so far as the Connecticut River was concerned it was caused primarily by heavy snow runoff. The isohyetal maps for these storms are shown in Figures 46 and 47. It is

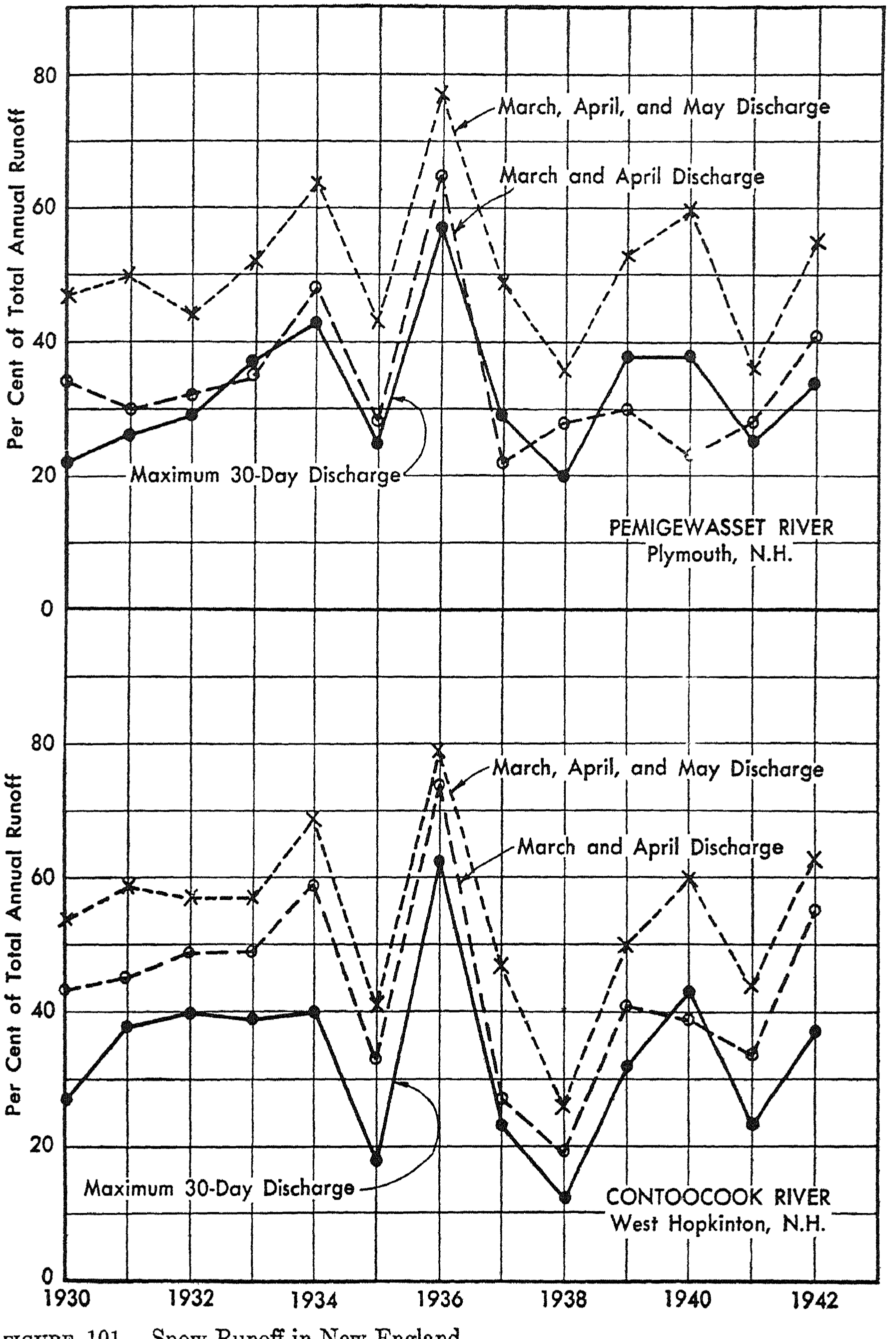



FIGURE 101. Snow Runoff in New England

seen from these maps that while the rainfall was moderately heavy it was not sufficient to have caused the greatest flood of record.

During the winter of 1935–36, most of New England, including the basin of the Connecticut River, was covered with snow to somewhat greater than average depth. Furthermore the temperature through the winter had been below average; it remained continuously colder than usual so that the snow cover was not depleted by the customary winter warm spell and retained a rather light density until spring. Apparently for this reason it retained much of the rainfall of the first storm, March 11–13, 1936, which was only slightly over 3 inches. The next storm, March 17–19, showed a peak of somewhat over 4 inches on the Connecticut River basin. Both of these storms were caused by an invasion of tropical Atlantic masses of warm moist air. It is evident therefore that the flood on the Connecticut River was caused primarily by the rapid melting of the ripe heavy snow during the second storm, aided by the moderate quantities of rainfall.

*The Flood on the Middle Missouri River, 1943.* Prior to March 14, 1943, western North Dakota had been practically cleared of snow except some of the remnants of drifts that remained in sheltered spots. Following that date a storm broke and when it cleared an unusually heavy snowfall was left on the frozen ground. Unofficial observations gave as much as 30 inches in the vicinity of Minot, N. Dak.; one to two feet covered the western and central portion of the state. This snow remained for nearly a week. Then occurred an invasion of moist warm air from March 20 to 22. Strong, warm south winds were very much in evidence at Omaha, Nebr., for two days previously, but there was no more precipitation at Omaha or in North Dakota. When this warm air covered the snow, melting was rapid; the snow cover was practically depleted in the two warm days. The rapid snowmelt on the frozen ground produced record floods on most of the tributaries of the region. The Missouri River, which drains the entire area, rose rapidly although the ice had not left the channel. Some ice jams were reported above Bismarck, but they were not the prime cause of the flood. Below Bismarck as far downstream as Omaha, the Missouri River rose to stages not exceeded since the great flood of 1881.

This flood was entirely the result of rapid melting of snow, except for some ponding caused by the ice jams mentioned above. Surface conditions were probably most conducive to heaviest runoff, since except that retained in ponds and depressions in the glacial moraine substantially all melted snow water ran off.

These two examples provide adequate proof that snow alone can cause a major flood if sufficient snow cover exists, melting conditions

are favorable, and infiltration is reduced or stopped by frozen ground. The two primary factors required to produce a flood from snowmelt are sufficient snow cover on the ground and a period of high temperature. Ground conditions are, as in any flood, an important auxiliary factor since the amount of runoff is a residual of the total snowmelt less the quantity of water that is absorbed or percolates into the earth. The quantity of snow on the ground need not be excessive, as a moderate amount may be sufficient. The condition of the snow is important since snow must be packed or consolidated before runoff begins. In order to produce a flood the melting must be rapid, and this means high atmospheric temperature either from an invasion of warm air or a period of intense insolation. Since such high temperatures cannot be predicted many days in advance, the magnitude of flood peak cannot be forecasted in the present development of technique as can the volume of seasonal runoff.

# 8 EVAPORATION

*Definition of Evaporation.* The process of "evaporation" may be defined as a change from the liquid or solid to the gaseous state. Most liquids are subject to loss by evaporation but this discussion is confined to the loss sustained by water through the process of evaporation under natural conditions. In common usage evaporation means net evaporation, that is, the term is confined to the actual loss of mass from the liquid body; the same understanding is retained here. Evaporation is commonly measured by depth of liquid that has been evaporated.

*The Importance of Evaporation.* Evaporation is a universal process and takes place under all conditions of nature throughout the world; it supplies all the moisture for precipitation. Losses of water by evaporation on land are not felt greatly in humid or wet climates, since there is generally an excess of moisture over the needs of life. However, in subhumid or arid regions the losses are an important economic factor in the storage of water. In investigation for and operation of power plants and irrigation projects in dry regions, the loss of water caused by evaporation from reservoirs must be carefully and accurately evaluated.

*Kinetic Theory of Matter as an Explanation of Evaporation.* The kinetic theory of matter depends upon two hypotheses: the molecular theory and the theory that heat is a manifestation of ceaseless motion of the molecules about a mean path or position. According to the latter theory, if there were no motion, the temperature would be at absolute zero, which is −273 C. Above that point the temperature increases as the motion of the molecules becomes faster and their kinetic energy becomes proportionately greater. The velocity of individual molecules is conceived to vary and from time to time one of those at the surface of a liquid will acquire sufficient momentum to carry it beyond the restraining forces of the liquid and escape from the body of the fluid into the overlying atmosphere. This loss of the liquid constitutes the total evaporation. The rate of loss increases as the temperature of the fluid rises, particularly above normal atmospheric temperatures.

In order to analyze the areal aspects of evaporation, assume a very small area of water surface, sufficiently small to be considered a differential, $dA$, in comparison with an area of an evaporation pan or larger body. Assume also, that $dA$ has a side of 1.0 mm or an area of 0.01 sq cm. The diameter of a molecule of water is of the order of $(3 \times 10^{-8})$ to $(4 \times 10^{-8})$ cm, and the corresponding projected area is $(7.1 \times 10^{-16})$ to $(12.6 \times 10^{-16})$ sq cm. On this differential area there would be space for $[0.01/(7.1 \times 10^{-16})]$ to $[0.01/(12.6 \times 10^{-16})]$, or $(1.4 \times 10^{13})$ to $(0.8 \times 10^{13})$ molecules. In 1.0 cc of water there are $(6.2 \times 10^{23}/18.02)$ or $(0.334 \times 10^{23})$ molecules; in a column 1.0 cm long under $dA$ there would be $(0.334 \times 10^{21})$ molecules. In the Colorado experiments (to be discussed later) evaporation at rates ranging from approximately zero to 0.35 inch or 0.762 cm per day was observed. The higher figure is equal to an average of about $(0.29 \times 10^{16})$ molecules per second leaving through the differential $dA$, or about 8 to 10 per second from each area of one square molecular diameter. This is indeed a very small area, and 10 per second is a rather steady stream. It may therefore be concluded that evaporation is steady over any differential area and that the total mass of liquid changing state varies as the area. This does not mean that the total evaporated mass will be lost from the liquid, because other factors enter the process.

*Difference Between Total and Net Evaporation.* After escaping from the fluid free molecules retain or accelerate their motion, and occasionally some of them move close enough to the surface of the fluid to be recaptured, or condensed, into the liquid again, so that loss from evaporation is thereby diminished. The difference between the material escaping from the surface and that being returned is designated the "net evaporation." This net evaporation is what actually becomes manifest and is commonly known as "evaporation." Hereinafter the term "evaporation" will be understood to mean the difference between the material escaping from the liquid and that being returned by condensation.

*Factors Promoting Evaporation.* Like all other hydrologic phenomena the net evaporation of water is the result of several causes. The factors causing evaporation are temperature, (particularly temperature of the water), the humidity of the surrounding atmosphere, wind, insolation, and atmospheric pressure. These factors will be studied in more detail in the following paragraphs.

It is to be emphasized that the complete change of state by evaporation through which the vapor is lost to the liquid consists of two processes; first, escape of molecules from the surface of the water to form

vapor, and second, removal of the vapor from the area of the surface so that it cannot be condensed again. These two processes respond largely to separate causes and are therefore substantially independent of each other except in so far as they may be actuated by a common source of heat.

*Operation of Temperature on Evaporation.* The temperature that directly promotes evaporation is that of the water itself as may be seen from a brief consideration of the kinetic theory of matter. Since increasing the heat of a fluid accelerates the velocities of the molecules, thus resulting in greater numbers leaving the free surface, it is evident that evaporation must increase with the temperature of the liquid. The relation between temperature and vapor pressure is shown in Figure 102. From the curve of natural values it is apparent that the rate of increase in vapor pressure is nearly constant from 0 C to 35 C which, although a small proportion of the total range, encompasses that of primary interest in hydrology. The curve departs somewhat from a straight line, but errors resulting from the use of the lineal relationship would be negligible for most purposes, and would probably be of a degree that would be obscured by the greater errors inherent in most hydrologic data. Somewhat less curvature is shown by plotting the logarithm of vapor pressure against the absolute temperature, so that if accurate data warranted the refinement, computations could be based on logarithms in lieu of natural values.

In studies utilizing available meteorological data for determining losses by evaporation it is usually necessary to use data of air temperature instead of water temperature, since the former are practically all that are available. However, temperature is of much importance of itself since much of the heat of inland bodies of water is received through conduction from the atmosphere. This fact justifies use of air temperatures in lieu of those for water. This procedure involves wider approximations in values of estimated evaporation because of the fluctuating air temperatures, which vary with each type and cycle of air mass as well as with other conditions. An influx of a cold polar mass will bring relatively low air temperatures, while the water will retain its stable thermal conditions; likewise an influx of warm tropical air will bring high temperatures as compared to water. These fluctuations of air temperatures are independent of the thermal status of a given body of water, although if the latter be very small, such as an evaporating pan, its temperature will follow that of the air fairly closely. Over relatively long periods of time such as a month, the temperature of small bodies of water will approach the average of that of air, lagging more as the size of body of water increases. Large bodies of water such

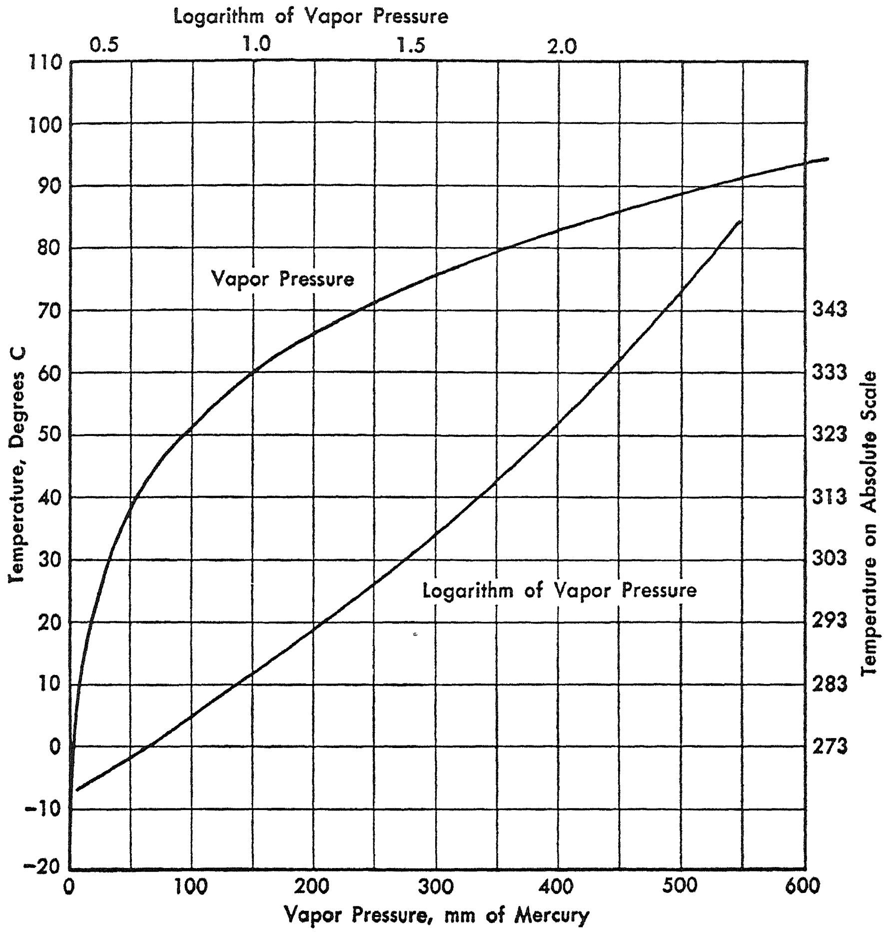



FIGURE 102. Relationship of Temperature and Vapor Pressure

as oceans vary little in temperature despite large fluctuations of air temperature.

In some investigations the observed water temperatures in summer were found to be higher than air temperatures. In Figure 108 are shown some comparisons made by plotting data obtained in the Fort Collins (159) experiments. It may be seen that at most points the water temperature is above the average air temperature and that the difference becomes greater as the size of the body of water increases. How far this tendency would continue with still larger bodies of water is not known at present.

The effect of temperature on evaporation has not been clearly evaluated experimentally as yet, although some fairly definite but apparently conflicting conclusions have been reached by experimenters,

Rohwer (159) stated that under still air conditions in the laboratory at Fort Collins there was no relation between evaporation and the temperature of either air or water, but that there was a definite relation between evaporation and the difference between the temperature of the air and that of water. Under conditions of controlled wind he found no relation between evaporation and temperature of the air or water, or evaporation and the difference in air and water temperatures. Cox (41), who made his experiments out of doors, stated that the difference in temperature of the air and water governed the rate of evaporation to a considerable extent; he added that high water temperatures are conducive to high rates of evaporation. (If the latter were not true it would be difficult indeed to get steam by stoking coal under a boiler.)

The foregoing may be called the direct operations of temperature. It has also an indirect effect on evaporation because of the dependence of the latter upon humidity.

*Operation of Humidity on Evaporation.* Although the rate at which molecules escape depends upon the temperature of the water, the number of molecules lost from the liquid surface depends greatly upon humidity for the reason that when the atmosphere becomes saturated, the number of molecules returning to the water surface equals the number escaping. Under this condition there is no loss by evaporation. The usually accepted theory of evaporation and actual observations show that net evaporation varies proportionately to the difference between the actual humidity (if less than saturation) and saturation at the prevailing temperatures. The difference then between the actual humidity and the humidity at saturation constitutes a control of evaporation and provides the most convenient means of calculating it.

The humidity of saturated air is a function of the air temperature as shown by Figure 102, and through humidity evaporation is again related to atmospheric thermal conditions. In cold weather evaporation is much less than in warm, other conditions being the same, simply because of the smaller capacity of the adjacent air space to receive and retain molecules from the liquid. Air temperature fluctuates widely. The air is cooled by loss of heat through radiation at night and is heated during the day by absorbed radiant energy from the sun and from the earth itself. The coming and passing of air masses of different thermal properties bring changes in air temperatures and resultant variations in moisture capacity, and consequently in evaporation.

The action of net evaporation under various temperature conditions, all other factors being considered constant, may be summarized as in Table 59.

TABLE 59. ACTION OF EVAPORATION UNDER CONDITIONS OF HEAT

| Temperature | | | Action of Net Evaporation |
| --- | --- | --- | --- |
| *Water* | *Air* | *Dew Point* | |
| Assumed to be at a fixed point | Same as water | Same as air (Saturated) | None |
| | Same as water | Lower than air | Evaporation till saturation, then none |
| | Higher than water | Same as air | None due to saturation |
| | Higher than water | Lower than air | Evaporation till saturation, then none |
| | Lower than water | Same as air | None due to saturation |
| | Lower than water | Lower than air | Evaporation till saturation, then none |

*Evaporation in Still Air.* Evaporation into still air merits some consideration before discussing the effect of wind. Humphreys (96) presents an analysis developed by the Austrian scientist Stefan, which is stated here. This analysis was limited to evaporation from a circular tank with a diameter $2r$, filled to the brim and evaporating into still air. The general equation is

$$e = -\frac{K}{P - p}\frac{dp}{dz}$$

in which $e$ is the rate of evaporation per unit of area of the tank; $K$, the coefficient of diffusivity determined experimentally; $P$, the total atmospheric pressure; $p$, the vapor pressure; and $dp/dz$, the pressure gradient away from the surface of the water in the tank. This equation may be stated thus,

$$e = -K\frac{d}{dz}\log\frac{P - P_o}{P - p}.$$

By making the logarithmic expression equal to $U$, the second formula may be reduced to an equation identical with that of force in an electrostatic field, and by integrating over the entire surface there is obtained upon the substitution of the proper coefficients and reductions

$$E = -4rK\frac{p_1 - p_o}{P}$$

in which $E$ is the rate of evaporation from the entire surface of the tank, and $p_1$ and $p_o$ are the vapor pressures at the water surface and in free air, respectively.

It can be noted that this equation states that evaporation varies as the lineal dimension of the tank, instead of the area as was found previously. But it should be noted that the controlling factor is the pressure gradient $dp/dz$; this control follows throughout the analogy with the electrostatic field into the final equation. It is therefore evident that the equation for $E$ does not give the rate of change of state from liquid to vapor but instead gives only the rate of escape of the molecules from the space above the water into the outer atmosphere.

This analysis does emphasize the fact that evaporation into still air is a different matter from evaporation under conditions controlled only by the change of state. Evaporation into still air occurs seldom under natural conditions, and hence in the following paragraphs evaporation is considered as occurring under conditions of turbulent atmosphere unless stated otherwise.

*Wind.* In view of the kinetic theory of heat, wind cannot be said to be a direct cause of evaporation, but observation and the above consideration of evaporation into still air show conclusively that it is an important auxiliary. Its most important operation is without doubt the removal from the water surface of the moisture-laden air and replacing it by some that is drier. This is achieved in two ways, first, by a movement of translation whereby the more moist air is removed bodily from the water surface, and second by turbulence, by which action the more moist air is lifted and mixed with the relatively drier upper atmosphere. The discussion of atmospheric turbulence in Chapter 7 is as applicable to evaporation as to snow melting. Folse (63) mentions other aspects in which wind may promote evaporation. At moderate or higher velocities it causes waves which in effect increase the area of the free water surface from which evaporation may take place. Again, the higher velocities produce whitecaps and pick up drops of water which are carried bodily into the atmosphere. In addition to these aspects it is likely that friction of rapidly moving air on the water surface also promotes evaporation.

Generally, the relationship between wind velocity and evaporation has been found to be linear, that is, the relationship may be expressed in the form $(a + bw)$ where $a$ and $b$ are constants and $w$ the velocity of the wind. Rohwer (159) found the relationship to be $(0.44 + 0.118W)$ in which $W$ was the velocity in miles per hour. Cox (41) produced two formulas dependent upon the type of pan, but both were linear in so far as the wind was concerned. Folse (63) found the relationship to be $(w/100 - 2.6)$ in which $w$ was the velocity in miles per day. Negative values are discarded.

*Insolation.* "Insolation" is defined as the reception of energy or radiant heat from the sun. Its effect on evaporation comes through the raising of the temperature of the water, since radiation from the sun is the primary source of heat for all purposes on the earth. However, the atmosphere transmits the radiant energy with relatively small loss so that the air is not heated greatly by direct radiation. The greater portion of solar radiation passes through the atmosphere to the earth, from which it is later emitted as radiant energy of greater wave length. These longer waves are more effective than the shorter kind in raising atmospheric temperatures. Radiant heat received by a water surface but not reflected is absorbed directly by the water and operates to raise its temperature without affecting the air temperatures.

The average radiation received at the outer edge of the atmosphere is 1.94 gram calories per minute per square centimeter. The value fluctuates from day to day but the range of variation is not great, at most being about 2 per cent. The radiation received at the surface of the earth is much less and varies through a much wider range. The amount received at a given point on the earth varies with the latitude of the place, the season of the year, the moisture in the air, dust, and cloudiness. These factors produce a wide variation in the daily receipt of radiation.

In order to gain a concrete idea of insolation and its variation it is desirable to study it as observed at one station. The mean monthly distribution of insolation is illustrated by the data obtained at Lincoln, Nebr., for the years 1923–1942. These values in gram calories per day are given in Table 60.

TABLE 60. MEAN MONTHLY INSOLATION, LINCOLN, NEBR.

| | | | |
|---|---|---|---|
| January | 191.4 | July | 587.4 |
| February | 266.6 | August | 490.7 |
| March | 359.3 | September | 397.4 |
| April | 444.3 | October | 303.4 |
| May | 512.4 | November | 204.1 |
| June | 559.1 | December | 155.8 |

These data are shown graphically in Figure 103.

Because insolation is the ultimate source of atmospheric heat, a study was made to determine statistically the degree of relationship between insolation and evaporation from data observed at Lincoln, Nebr., for the years 1924, 1938, and 1939; the months April to October only were used. During these months the insolation averaged 497.2 gram calories per day, the range of daily receipts varied from 33 to 801, and the standard deviation was 171.8.

The correlation coefficient between evaporation and insolation was

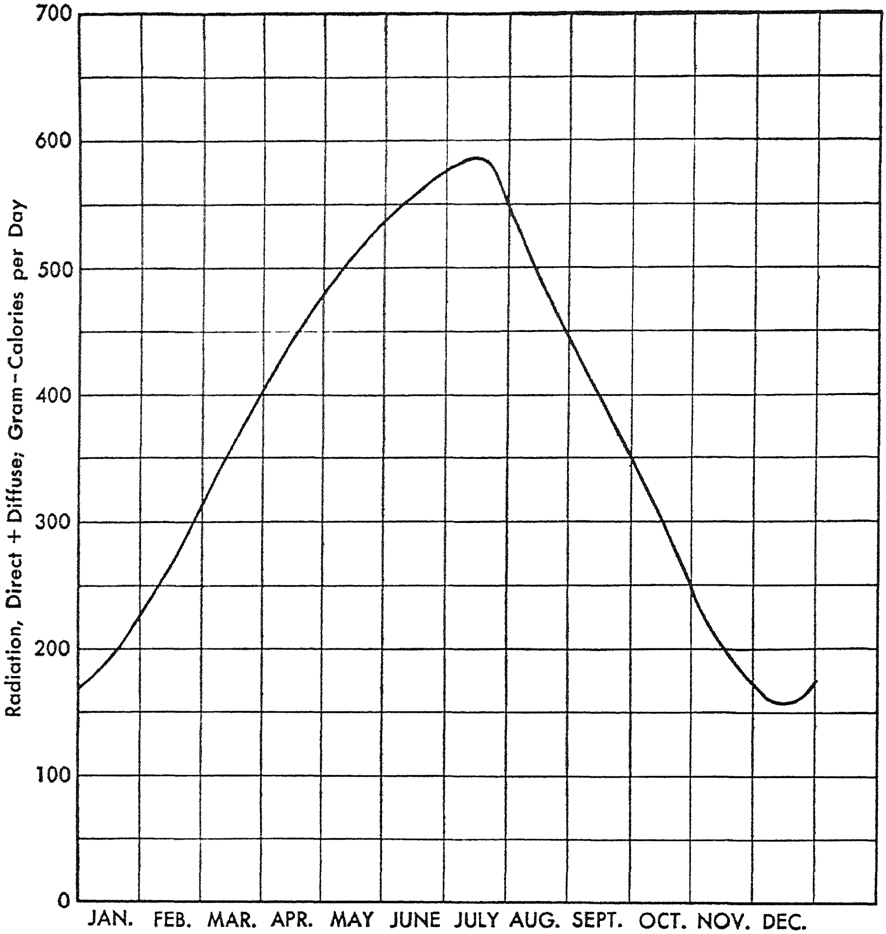



FIGURE 103. Mean Insolation, 1923–1941, Lincoln, Nebr.

computed by means of a table similar to Table 6; the coefficient was found to be 0.507. This does not show a high degree of correlation between insolation and evaporation but does indicate some relationship, especially when one considers the variation in other factors which promote or affect evaporation.

It is conceivable that insolation and its variations should have appropriate effects on evaporation from lakes, reservoirs, pans, or other water surfaces. Cummings and Richardson (44) assumed that insolation was one of the principal factors in causing evaporation. Cox who used several different pans reported that values computed by a formula derived from observations on a copper pan were about 10 per cent more than the data observed with galvanized pans. He also stated that a coating of asphaltum paint on the galvanized pan caused

an increase of about 10 per cent in the observed evaporation. These findings indicate the effect of insolation on various types of the interior surfaces of pans.

*Effect of Pressure.* Again in accordance with the kinetic theory of matter, it may be seen that molecules can escape from the free surface of a liquid more freely as the density of the air is less. This principle is used in industrial evaporation which is carried on in a partial vacuum. Under these conditions the variation in density of the vapor or gases above the liquid is relatively great as compared to variations in atmospheric pressure. Except where there are great changes in altitude with corresponding differences in barometric pressure, the density of the atmosphere has little effect on evaporation under natural conditions. It has been confirmed by observation, however, that evaporation, with other conditions the same, is greater at high elevations.

*Measurement of Evaporation.* Evaporation has been measured by a diverse assortment of instruments and methods. Some of the smaller instruments that have been used are the Piche evaporimeter, the evaporation scale of H. Wild, and porous clay bulbs of various design. All these instruments measured evaporation from very small volumes of water, microscopic when compared to volumes considered in engineering hydrology.

For many years past, chief reliance has been placed on pans of various sorts, dimensions, and conditions of exposure, to secure information on evaporation for engineering purposes. In a few notable instances attempts have been made to determine rates of evaporation from natural bodies of water by measuring and estimating closely the inflow and outflow, and attributing the difference to losses by subterranean seepage and evaporation.

*The Weather Bureau Pan.* For measuring evaporation from a free water surface, the Weather Bureau has commonly used a pan 4 feet in diameter and 10 inches deep, water being maintained at a level 2 or 3 inches below the top. This pan is mounted on supports 6 inches above the ground in order to eliminate difficulties from drifting soil or snow, and is thus exposed to the air on all sides. Evaporation is measured by means of a hook gage located in a stilling well. The equipment of a station for measuring evaporation includes instruments to observe concomitant meteorological conditions, including temperature, precipitation, wind, and humidity.

*The Pan of the Bureau of Plant Industry.* The United States Bureau of Plant Industry has made observations in many localities by using a circular pan 6 feet in diameter and 2 feet deep. This pan is set in the ground to a depth of 20 inches and the water is maintained at

approximately ground level. Evaporation is measured by means of a point gage in a well outside of the pan. Meteorological conditions are observed by a set of standard instruments, including rain gage, anemometer, thermometers, and psychrometer for determination of humidity.

*The Colorado Pan.* The Colorado Experiment Station, Fort Collins, Colo., utilizes a square pan, 3 feet on the side and 1½ to 3 feet deep. It is also set in the ground so that 2 to 6 inches extend above the surface. The water surface in the pan is maintained at approximately the elevation of the ground surface; a deviation of only one inch upward or downward is permitted. Evaporation is measured by means of a hook gage.

*The Floating Pan.* The floating pan, widely known as the U. S. Geological Survey pan, has been employed by that agency and others for the determination of evaporation from extensive water surfaces with a view to obtaining results under conditions identical with large bodies of water. The pan is 3 feet square, 18 inches deep, and is mounted on a raft floating on a relatively large body of water. Baffles are provided within the pan to prevent surging. Evaporation is measured by replacing the water lost by means of a special cup holding a volume equal to that formed by a depth of 0.01 inch over the area of the pan.

*Relation of Evaporation between Pans and Reservoirs.* The use of pans to measure evaporation raises some important problems of correlation of data, first, between the various types of exposure and sizes of pans, and second, the relation between pans and lake or reservoir surfaces. It has been found by observation that rates of evaporation differ among the several types of pans that have been used by various experimenters and that it varies also with different dimensions of pan, the diameter of the water surface being the most potent factor. There is, for example, considerable difference in evaporation from the Weather Bureau pan and the floating pan of the U. S. Geological Survey. The sunken land pan yields results differing from both, because of differences in exposure. The causative factors of evaporation are appreciably affected by the dimensions and probably the shape of the pan, and by the location with respect to surroundings and surface of the earth, full or partial exposure in the air or submergence in water.

The pans described above each have their individual peculiarities in relation to the meteorological elements. Since the Weather Bureau pan is fully exposed on all sides the wind acts most effectively on this pan. Heat is received from the atmosphere on all sides and from the sun by way of the top and exposed sides. The sunken land pan used by the

Bureau of Plant Industries or Colorado Experiment Station is less fully exposed to the elements than the preceding pan: the sides and bottom are not exposed to the atmosphere or sunshine, and receive or lose heat only from or to the earth, and likewise it is less affected by the winds. The floating pan is similar to the land pan in its exposure to the weather but is subject to other disturbing factors: waves rock the pan and may splash water into it or spill it out. The rocking washes water in the pan up the sides where it is quickly evaporated.

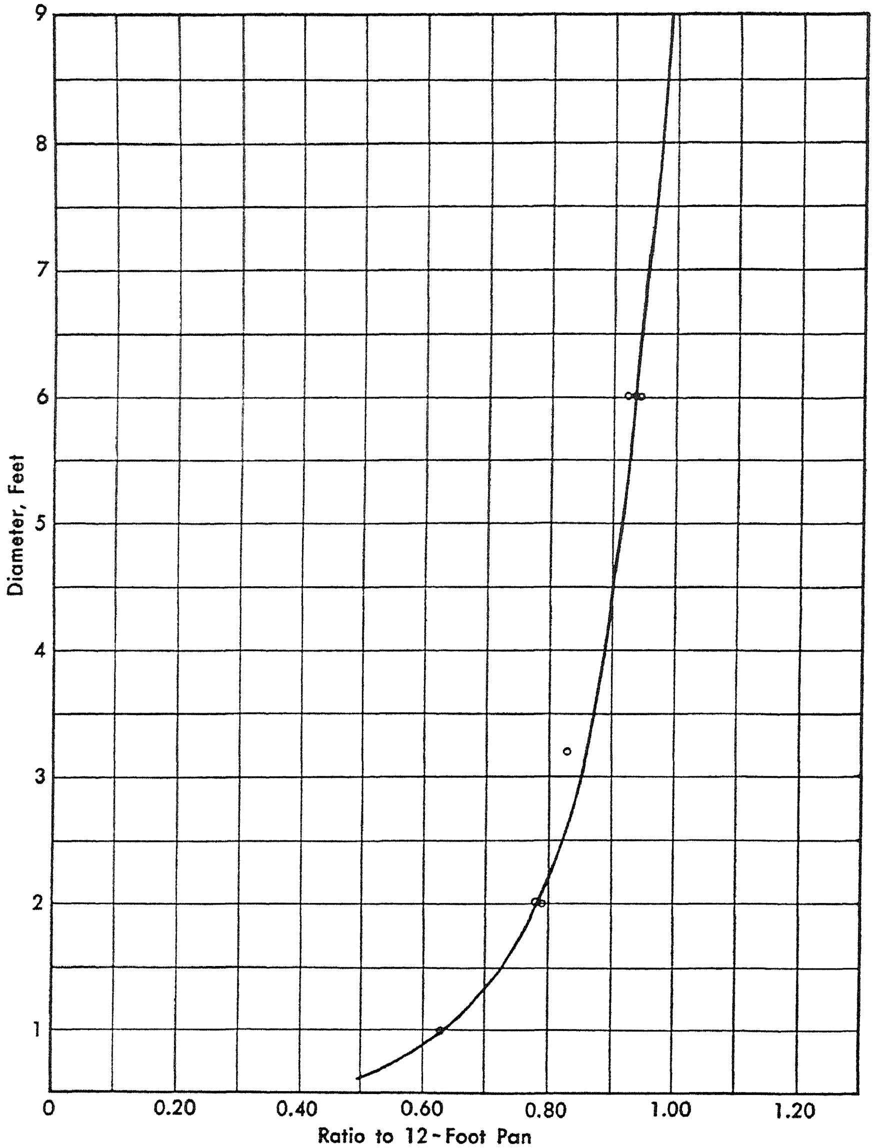
The size of a pan affects the rate of evaporation, apparently because of the greater influence of wind on those with smaller diameters. The results of observations on this as on other aspects of evaporation show a wide variation. Taking the results for a 12-foot sunken land pan as a basis, Rohwer (159) has made a comparison of evaporation between pans of the various types and diameters. Each comparison was expressed by a ratio of the evaporation in the 12-foot pan to that of the compared pan. The results for sunken pans of various diameters are used to show the variation in Figure 104.

Since the determination of loss of water from lakes and reservoirs by evaporation is one important objective of the hydrologist, the ratio of evaporation from the various types of pans and that from large bodies of water is of utmost importance. Many investigators have considered this problem. Their results have been compiled and analyzed by Rohwer (158) and Follansbee (61) under the sponsorship of the Special Committee on Irrigation Hydraulics, ASCE (7); the results were expressed as coefficients to be used as multipliers for reducing the observed evaporation from pans to what could be reasonably expected from reservoir surfaces. These coefficients are given in the following table.

TABLE 61. COEFFICIENTS OF PAN EVAPORATION

| TYPE OF PAN | COEFFICIENT | RANGE OF COEFFICIENT |
|---|---|---|
| Class A land pan, Weather Bureau | 0.70 | 0.60–0.82 |
| Colorado sunken land pan | 0.78 | 0.75–0.86 |
| U. S. Geological Survey floating pan | 0.80 | 0.70–0.82 |

*Need for Theoretical Study.* The lack of evaporation stations and particularly the fact that existing stations are seldom located so that observed data may be used directly for a study of a given reservoir, make it imperative that the theoretical aspects of evaporation be thoroughly understood. Furthermore, very large reservoirs may extend beyond the area to which the observed data of a suitably located station may be applicable. A complete investigation of the hydrology of any reservoir requires a thorough study of all factors




*After Follansbee*

FIGURE 104. Effect of Diameter of Pan on Evaporation

promoting or affecting evaporation, and the paucity of data make it necessary that all be utilized.

Before full utilization can be made of observational data in any field, knowledge must be had of the laws governing the observed phenomena. Urged on then by need, and by perhaps purely scientific curiosity, many persons have conducted investigations and studies and have deduced many formulas for computing evaporation.

*Experiments to Determine Laws of Evaporations.* In order to determine the laws governing the rates of evaporation, many experiments have been made by various observers. Some experiments were made primarily on the factors governing evaporation, while others were conducted as a part of a general investigation of the hydrology of a selected locality; in other cases long observations were conducted on the hydrological elements of a region and evaporation was studied as a part of the general investigation. One of the most notable recent series of experiments was conducted at Fort Collins, Colo., by the U. S. Department of Agriculture in cooperation with the Colorado Agricultural Experiment Station. The experiments conducted by the Engineering Experiment Station and the U. S. Geological Survey at Baton Rouge, La., included the study of evaporation as a part of the general hydrology of Bayou Duplantier. An example of the third type is found in the work of Folse (63) who investigated evaporation in order to determine the runoff of the Great Lakes.

*Formulas to Express the Laws of Evaporation.* The first statement of the laws of evaporation was made in 1802 by Dalton, who discovered that the rate of evaporation from a free water surface, other factors being constant, depended upon the difference between the vapor pressure at saturation for the temperature of the water, and the vapor pressure actually existing in the air above the water. This principle has in general been accepted in subsequent investigations.

Starting with this premise many other investigators have derived formulas to express mathematically the rate of evaporation in terms of the meteorological elements causing or affecting it. A few of these formulas are discussed below to illustrate the results of the investigations.

One of the earlier investigators who studied evaporation for engineering purposes was Fitzgerald. In the period 1876–87 he made a careful and complete series of observations under controlled laboratory conditions and under natural conditions. As a result of his investigations and studies he proposed the formula

$$E = (0.40 + 0.199W)(e_s - e_d)$$

in which $E$ = evaporation in inches per 24 hours
$W$ = velocity in miles per hour
$e_s$ = vapor pressure of saturated air in inches of mercury at the temperature of the water surface
$e_d$ = mean vapor pressure of air in inches of mercury above the water surface.

No account was taken of the effect of altitude.

Russel in 1888 made a study of evaporation measured by Piche evaporimeters at 18 stations of the Weather Bureau over the United States. He prepared the formula

$$E = \frac{(1.96e_w + 43.88)(e_w - e_d)}{B}$$

where $e_w$ = the vapor pressure in inches of mercury for the mean wet bulb temperature

$B$ = mean barometer reading in inches of mercury at 32 F and the other symbols are as given above.

In 1907–10 Bigelow made an extensive series of experiments for the Weather Bureau. He proposed the formula

$$E = 0.138 \frac{e_s}{e_d} \frac{de}{ds} (1 + 0.07W)$$

in which the derivative, $de/ds$ is the rate of change of the maximum vapor pressure with temperature and the other symbols as designated above.

Another formula was developed independently by A. F. Meyer and J. R. Freeman as follows:

$$E = (0.5 + 0.05W)(e_s - e_d).$$

Approaching the problem of evaporation from a considerably different angle, Cummings and Richardson proposed the formula

$$E = \frac{(H - S - C)}{2.54L(1 + R)}$$

in which $H$ = the net radiation received in calories per square centimeter

$S$ = the heat stored in a unit column of water in calories

$C$ = a correction factor for the interchange of heat between the walls and the water

$L$ = the latent heat of the water in calories

$R$ = Bowen's ratio between the heat carried away by convection and that carried off by vapor.

This approach seeks to determine the evaporation by considering the heat utilized to vaporize water; the difference between the heat received and the heat lost by radiation and conduction to surrounding media should be the amount of heat required to supply the latent heat of vaporization. This formula takes into consideration insolation which

is not included in other formulas. It does not consider wind but this approach does not require that its effect be included. The influence of altitude can be accounted for through the latent heat. The theoretical approach of this method is sound but its utilization is handicapped by difficulties of obtaining reliable data of the factors entering the formula, and for this reason the method has not been widely accepted.

*The Fort Collins Experiments.* This series of experiments included observations under many conditions; observations were made principally on Colorado sunken pan but observations were also made on the Weather Bureau land pan, the floating pan of the U. S. Geological Survey, and 85-foot tank, and a small reservoir. The work was initiated by R. L. Parshall and was transferred later to Carl Rohwer who is the author of the publication (159) describing the results. The experiments were continued under L. R. Brooks.

Experiments were made to determine the rate of evaporation in still air, under conditions of wind blowing at various controlled speeds, and under open air and fully exposed conditions. Other experiments were made to determine the rates of evaporation from various sizes of pans and under different types of exposure and from small reservoirs, as well as observations on pans over a wide range of altitude.

Out of these experiments there was developed a complete formula for evaporation:

$$E = 0.771(1.465 - 0.0186B)(0.44 + 0.118W)(e_s - e_d)$$

in which the nomenclature is the same as given above. The factor 0.771 was introduced to reduce the evaporation of the Colorado land pan to that of the reservoir used in the experiments. The factor $(1.465 - 0.0186B)$ was used to compute the evaporation at various elevations and it was derived empirically from observation. The factor $(0.44 + 0.118W)$ was designed to account for the effect of the wind: it also was derived empirically from the observations of controlled wind.

In summarizing the work of these experiments, Rohwer reported the following general conclusions:

1. For still air conditions in the laboratory, there was no relation between the evaporation and temperatures of the air and water, but there was a definite relationship between the difference in temperature and evaporation, as there was also between the difference in the vapor pressure and the evaporation.

2. Under conditions of controlled wind in the laboratory, evaporation bore no relation to the temperature of the air or of the water or to the difference in temperature of the air and the water. However, a definite relation was found

to exist between evaporation and wind velocity and between evaporation and difference in vapor pressure.

3. Observations at Fort Collins, Colo., on the evaporation from a heated water surface in winter indicated that Dalton's law holds for these conditions. The mean values showed that the evaporation from the heated water surface was about equal to the mean summer evaporation at Fort Collins.

4. Comparison of the observed and computed evaporation from ice in a pan 17½ inches in diameter, under controlled conditions in the laboratory, showed that when there was no perceptible air movement the observed evaporation loss was considerably less than the computed evaporation, but that when the air was agitated slightly by an electric fan the observed and computed evaporation agreed fairly well.

5. A comparison of the evaporation as computed from the meteorological data by formula 10 with evaporation at various points in the United States observed by other agencies showed that the formula has a general application. (Equation 10 referred to in the quotation is the same as given above, except that the factor 0.771 is omitted.)

Rohwer's first conclusion confirms the statements made previously concerning evaporation into still air. The controlling factor of evaporation under such conditions is the removal of the evaporated moisture and not the temperature of the water, so that agreement could not logically be expected between the rate of evaporation and the temperature of the air or the water, and less agreement should be expected between evaporation and the difference between saturated and actual vapor pressures than would be observed under conditions of turbulent atmosphere.

*The Baton Rouge Experiments.* A series of experimental observations were made on the hydrology, including evaporation, of Bayou Duplantier by the Engineering Experiment Station, Louisiana State University, in cooperation with the U. S. Geological Survey; the results were published in a bulletin under the authorship of Dr. Glen N. Cox (41). As a result of these observations the following equation evaporation was derived:

$$E = (e_a - e_d + 0.0016T.D.)\left(0.564 + 0.051T.D. + \frac{W}{300}\right)$$

in which $E$ = evaporation in inches per day

$e_a$ = saturated vapor pressure at air temperature

$e_d$ = actual vapor pressure

$T.D.$ = difference between mean temperature of the air and that of the water

$W$ = velocity of the wind in miles per day.

This formula differs in two respects from those given above and using the same meteorological elements: it uses the saturated vapor pressure at air temperature instead of water temperature, and it introduces the factor of the difference between water and air temperatures.

As a result of these investigations, Cox drew the following conclusions:

1. The evaporation from a floating pan can be computed with greater accuracy than that from a land pan.
2. The difference between the temperature of the air and the temperature of the water in a pan governs the rate of evaporation to a considerable extent. High water temperatures are inducive to high rates of evaporation.
3. A low relative humidity is accompanied by a high rate of evaporation.
4. High rates of evaporation accompany high wind movements.

The similarity of conclusion (2) and the conclusion reached by Rohwer with respect to evaporation into still air is noteworthy. The winds that Cox observed were rather light with velocities usually of only a few miles per hour. The result found by Cox seems to indicate that an appreciable proportion of the evaporation observed by him took place under a condition of still or nearly quiescent air.

*The Great Lake Studies by Folse.* Folse (63) published the results of detailed investigations made by him on evaporation and runoff of the Great Lakes. Although his studies of evaporation were only a part of the investigations for runoff, they are the primary interest here because of the evaporation formula which he derived.

The approach to the problem in these studies differs greatly from those previously described, since Folse used the large bodies of water of Lakes Superior, Michigan, and Huron for his observations. Observations were made for elevations of the water surfaces, vapor pressure, wind velocities, and precipitation on the lakes. Precipitation on the adjacent land draining into the lakes was also taken into account. An observation formula was set up, thus,

$$eE_1 + e\left[\left(\frac{W}{100} - x\right)\right]E_2 + I = v$$

where $e$ = mean differences for the lake between (1) the saturation vapor pressure corresponding to the mean air temperature for 2 days ending at midnight on the day to which the observation pertains, and (2) the mean actual vapor pressure for the 24 hours ending at 12:30 P.M. on the day to which the observation relates: (1)–(2) is designated the vapor pressure potential.

$E_1$ = the portion of the evaporation which is proportional to the vapor pressure potential.

$W$ = the average speed in miles per 24 hours of the wind for the 2 days ending at midnight of the day of observation.

$x$ = the speed of the wind below which it has an inappreciable effect on the evaporation.

$E_2$ = the portion of the evaporation proportional to the product of the vapor pressure potential and the wind velocity.

$I$ = the net change in the surface elevation of the lake due to net inflow from the lake above, the runoff from the adjacent land, precipitation on the lake and outflow into the next lake below, and evaporation.

$v$ = the residual due to discrepancies between the observed $I$ and the fall in surface elevation of the lake due to evaporation.

From observed data of the vapor pressure potential, wind, and changes in elevation of the lake surface, the factors $E_1$, $E_2$, and $x$ were evaluated as constants by the method of least squares. The final equation is as follows:

$$Ew = 0.319e + 1.49\left[e\left(\frac{W}{100} - 2.6\right)\right]$$

where $Ew$ is the total evaporation expressed in hundredths of inch per day and $w$ is wind in miles per day.

Although the author of the above equation claims that it is of universal application, it has not received general acceptance. As the author states, it was derived directly from observations of evaporation under natural conditions. However, the Great Lakes are so much greater than reservoirs for which losses by evaporation are to be determined that the conditions of comparison are at as great a variance because of excessive magnitude as such conditions are because of the usual evaporation pans being too small. Furthermore, the constants $E_1$, $E_2$, and $x$ would have to be determined for each new situation for which it may be impossible to obtain the requisite data.

Folse's formula has been criticized because wind at velocities of less than 10.8 miles per hour is eliminated. This elimination, however, is undoubtedly proper for the large bodies of water with which Folse was working. On the Great Lakes the wind would cover long distances over an area with a very smooth surface as compared with land areas. For this reason turbulence should be diminished and be less effective in carrying the saturated air from the surface. Furthermore, in the dis-

tance traversed over the lakes the air would acquire considerable moisture so that its turbulent action would tend to replace saturated surface air with upper air approaching saturation and the difference in vapor pressures would be reduced.

*Observations on Evaporation.* Observations on evaporation have been made in many parts of the world, but more frequently in arid or semiarid regions where stored water has a high economic value. In the United States, extensive observations have been made by the U. S. Weather Bureau, the U. S. Geological Survey, the U. S. Bureau of Plant Industry, the U. S. Bureau of Reclamation, the Department of Agriculture in cooperation with various state agricultural schools, and some state agencies; private corporations and individuals have likewise conducted series of observations.

Nevertheless, data of evaporation are still inadequate. There are many fewer evaporation stations than even regular meteorological observations. Real long-time records of evaporation are virtually unknown; the longest known record is that conducted by the Colorado Agricultural Experiment Station at Fort Collins, Colo., which began observations in 1887.

All data from carefully made observations in any given locality are valuable in forming an estimate of evaporation for that locality. A record may be short, yet it may be extremely valuable because of the absence of other records.

Data obtained by the Weather Bureau are usually published monthly in *Climatological Data.* Horton and Cole (92) have compiled and published a series of records obtained by the Bureau of Plant Industries. Karper (105) has published extensive records of evaporation in Texas. The publication of data of evaporation is not limited to these few authors; others have published data from time to time. Usually, however, such data must be sought from the people or agencies who make the observations.

To illustrate the annual distribution, Figures 105–107 are drawn from published sources mentioned above or the experiments previously described. Figures 105–106 are constructed from data of mean monthly evaporation in Texas as given by Karper (105). Since the data are rather complete, the graphs show the relationship or lack thereof between mean monthly evaporation and nearly all other factors affecting it. These graphs show a rather close normal relationship between evaporation, temperature, vapor pressure, and vapor pressure difference, in that these four elements generally rise and fall during the same seasons of the year. There are, however, some minor variations in this relationship that denote the effect of the wind and precipitation;

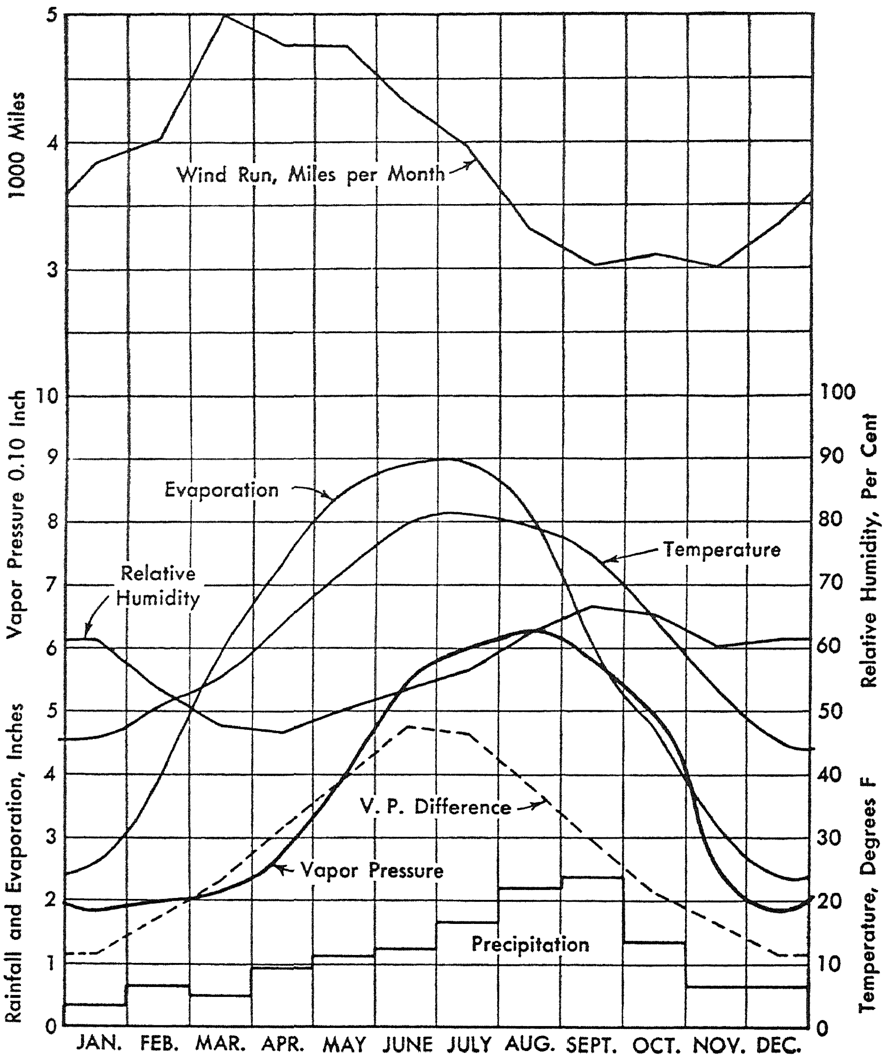



FIGURE 105. Evaporation and Concomitant Meteorological Phenomena, Balmorhea, Texas

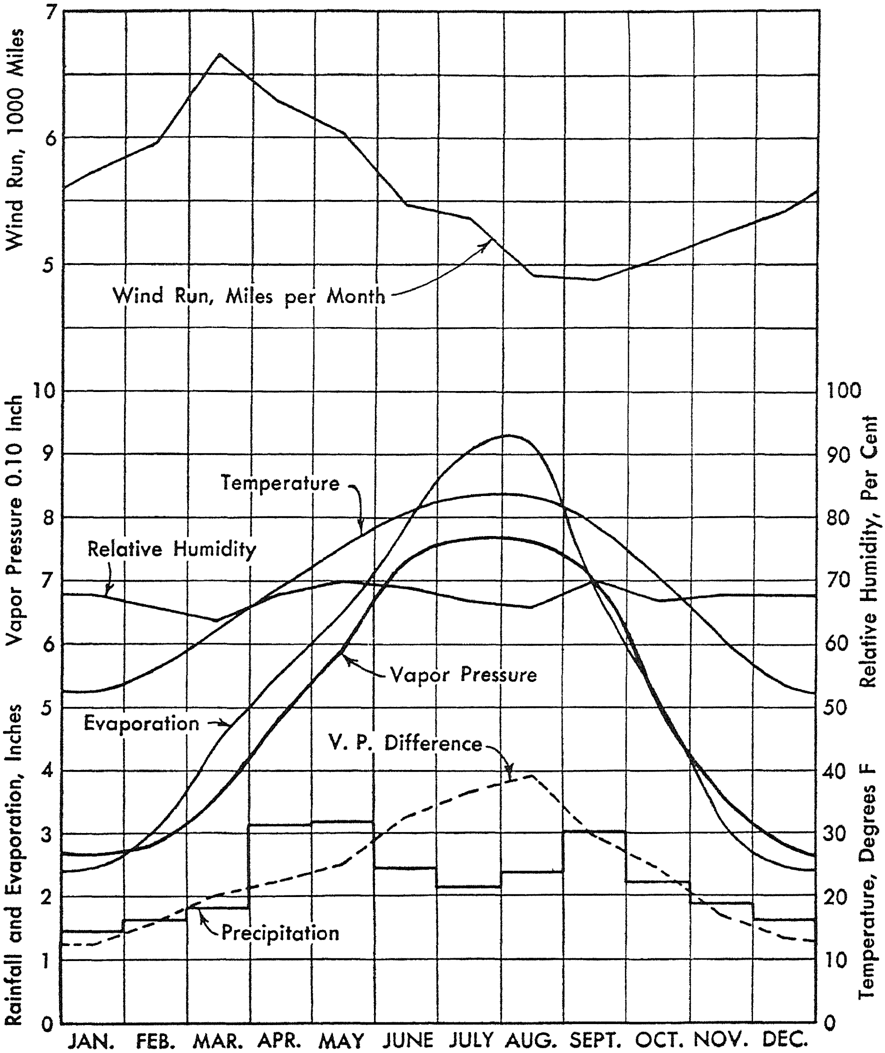



FIGURE 106. Evaporation and Concomitant Meteorological Phenomena, San Antonio, Texas

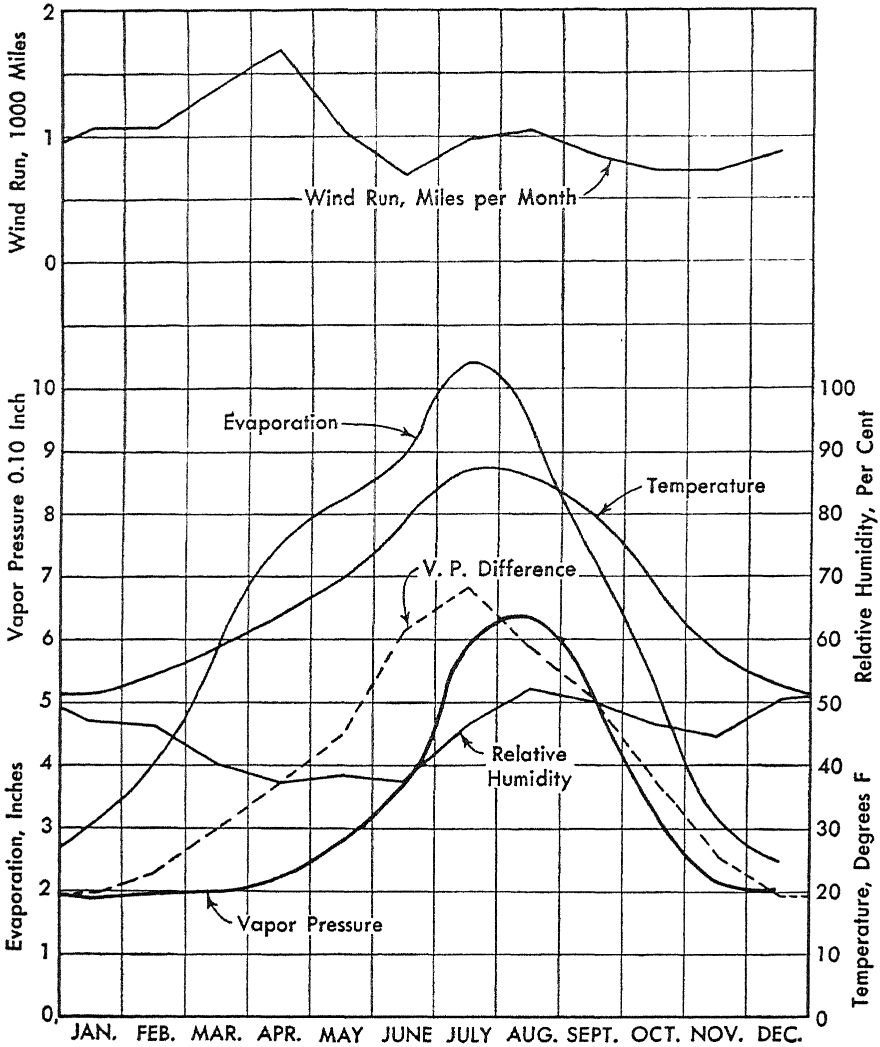



FIGURE 107. Evaporation and Concomitant Meteorological Phenomena, Yuma, Ariz.

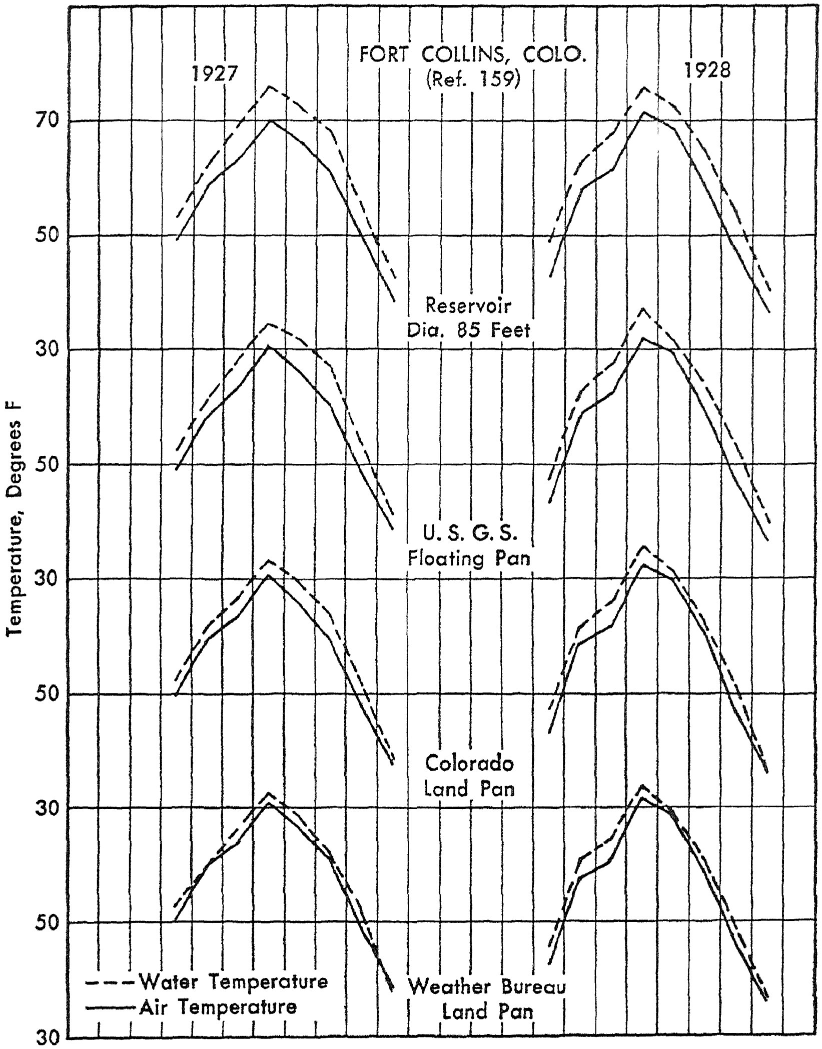



FIGURE 108. Comparison of Air and Water Temperatures

especially the graph for Balmorhea shows the earlier peaking of the evaporation curve, which is probably due to the spring winds. Figure 107, depicting evaporative conditions at Yuma, Ariz., shows also the relationship between evaporation and its causative factors.

*Air and Water Temperatures.* Some formulas for computing evaporation, Rohwer's for example, are based on the water temperature; others, as Cox's, utilized air temperatures. Usually in computing a record of evaporation it is necessary to use air temperatures exclusively because of the lack of data of water temperature. What then is the relationship between air and water temperatures? Data of mean

monthly temperatures taken from Rohwer have been plotted to form the graphs of Figure 108. These comparisons cover the reservoir with a diameter of 85 feet, the U. S. Geological Survey floating pan, and the Colorado and Weather Bureau land pans. In nearly every case water temperatures are above those of the air; only in one month for the Weather Bureau land pan does the water have lower temperature than air. The differences in general decrease in the order of reservoir or pan given above. The reason for the prevailing higher temperature of water may be ascribed to insolation which is absorbed by the water after passing through the atmosphere.

*Correlation Between Elements of Evaporation.* Since there is so much variation in the observed amounts of evaporation, the elements which cause it, and the divergence in the data of evaporation with respect to the causative factors, the question arises as to how close a correlation exists between the various elements and resulting evaporation. Correlations between a few factors have been studied very briefly and the results are presented below. The question as to how close is the correlation cannot be answered by a few studies the results of which are given below, but the few may point a way in which the answer can be reached. Much work remains to be done in studying such correlations and thereby increasing the usefulness of limited local data by correlation with complete data available at other localities.

From Rohwer (159) data are taken to compute the correlation between the daily evaporation observed on a small reservoir (East Park Reservoir at Stoneyford, Calif.) and that observed in regular pans. The results were as follows:

| | |
|---|---|
| Reservoir compared with the U. S. Geological Survey pan | 0.50 |
| Reservoir as compared with the U. S. Weather Bureau land pan | 0.13 |

These correlation coefficients are rather small; the coefficient of the reservoir as compared with the land pan, in particular, is so low that it indicates practically no correlation. This coefficient is probably a result of rapid changes in meteorological elements, which in the small units of time would affect the pan much more readily than the larger body of water in the reservoir.

Correlation between daily evaporation and the difference between the vapor pressure at the water temperature and the actual vapor pressure of the atmosphere is illustrated by means of data taken from Rohwer (159) and Cox (41). In Cox's data, however, the saturated vapor pressure for the atmospheric temperature was used. The correlation coefficients are given in Table 62.

It will be noted that the tendency toward higher correlation is

TABLE 62. CORRELATION COEFFICIENTS BETWEEN EVAPORATION AND VAPOR PRESSURE DIFFERENCE

| SOURCE OF DATA | TIME OF OBSERVATIONS | TYPE OF PAN | COEFFICIENT |
|---|---|---|---|
| Rohwer | 1927 | U. S. W. B. | 0.94 |
| Rohwer | 1928 | U. S. W. B. | 0.83 |
| Rohwer | 1927 | U. S. G. S. | 0.78 |
| Rohwer | 1928 | U. S. G. S. | 0.85 |
| Cox | Oct. 1936 | U. S. G. S. | 0.68 |
| Cox | Mar. 1939 | U. S. W. B. | 0.45 |

opposite for the two groups of data, the coefficients for the U. S. Weather Bureau land pan being higher than those for the U. S. Geological Survey floating pan in Rohwer's experiments, while the reverse is true of Cox's data. This may tentatively be ascribed to the different methods of computing the vapor pressure difference but the final answer must await more complete information and extended research.

*Evaporation From Other Surfaces.* Heretofore only evaporation from water surfaces has been considered but evaporation takes place wherever there is liquid moisture. The rate of evaporation follows the laws expressed above only to the extent that there is ample liquid available from a water surface. The rate of evaporation from other surfaces depends upon what has aptly been called the "evaporation opportunity." This evaporation opportunity varies with the type of surface and supply of water.

*Evaporation from Soil.* Evaporation abstracts large quantities of moisture from the soil wherever there is any available moisture. The rate of evaporation from saturated soil may be as high as from a water surface, or it may be higher under suitable conditions because soil temperatures become higher under insolation. Although evaporation from soils is limited to the water suspended in the interstitial spaces, it may also take water drawn from considerable depths by capillary action.

The extent, variation, and rates as compared with evaporation from water surfaces can be obtained only by observation. Blaney and Morin (13) reported some observations on evaporation at Carlsbad, New Mexico. Evaporation from a U. S. Weather Bureau pan was 92.64 inches and from a water table 2 feet below the surface of the bare soil it was 31.87 inches, being a ratio between the ground water and water surface evaporation of approximately 34 per cent. Parshall (145) reported on a number of experiments made by himself and some made by others to determine the rates of evaporation from saturated soils and river sands. In one set of experiments he reported the ratio of evaporation from soils to that from a free water surface to range from

14 to 93 per cent for depths to water below the soil surface of 1 to 12 inches in various types of soil. In a later set of experiments he reported ratios ranging from 4 to 109 per cent. These experiments show that the determination of evaporation from soils is further complicated by the considerable variation in soil types and conditions. The experiments have also shown that loss of soil water decreases as the depth from the surface of the soil to the available water increases.

*Transpiration.* Transpiration is the loss of water from living plants which occurs as an essential function of their normal growth. Since plants grow only in the warmer seasons, this water is nearly always evaporated as it leaves the pores of the plant. Huge quantities of water are thus disposed of: in temperate zones a large part of the summer precipitation is returned to the atmosphere by transpiration.

The quantity of water lost by transpiration varies widely with different species of plants, in addition to all the factors that operate to promote evaporation. The use of water by plants is best shown by stating the quantity of water consumed to produce a unit weight of dry plant matter. This quantity is designated the "transpiration ratio." C. H. Lee (123) gives a list of transpiration ratios which he has compiled from various sources. A few representative samples are given in Table 63.

TABLE 63. REPRESENTATIVE TRANSPIRATION RATIOS

| PLANTS | RATIO<br>*Lb of Water per Lb of Dry Matter* |
|---|---|
| Corn (average of 3 values in U. S. A.) | 345 |
| Wheat (average of 9 values) | 375 |
| Potato (average of 3 values) | 426 |
| Cotton | 568 |
| Alfalfa (average of 3 values in U. S. A.) | 829 |
| Buffalo grass | 308 |
| Sunflower | 577 |
| Larch | 1165 |
| Beach | 1043 |
| Elm | 738 |
| Norway spruce | 242 |
| Fir | 86 |

*Measurement of Evaporation from Land Surfaces.* The measurement of evaporation from bare soil and through plants has been accomplished generally by means of pans containing the samples of soil or plant growth through which it is desired to measure the losses. Various types of soil, either bare or with vegetation, are set in the pans upon a sand and gravel subsoil. The water is admitted to the gravel stratum at the bottom of the pan, and is measured as it is delivered to the pan to replace losses by evaporation or transpiration.

This method of measuring evaporation from soils and by transpiration is analogous to the evaporating pan with free water surface, with added uncertainties of the behavior of various types of soils and plants. Its greatest defect is probably the lack of similarity between conditions in the pan and those under natural conditions in the field. In spite of the best care that can be given to pan experiments the comparison between evaporation in them can be considered only approximate in view of the uncertainty of the relationship between evaporation in experimental pans and lakes and reservoirs.

*Measurement of Evaporation by the Moisture Gradient.* For measurement of evaporation from land and water surfaces Thornthwaite and Holzman (184, 185) have developed a method that promises to be of great usefulness and wide application over any surface from which evaporation is taking place. This method depends upon the vertical gradient of water vapor in the atmosphere and the coefficient of turbulent exchange, which were discussed in Chapter 7 in connection with the melting of snow.

In this approach to evaporation, the rate of change of state is assumed to equal the movement of moisture vapor from the surface. The differential equation for the rate of evaporation on this basis is

$$E = -A\frac{dq}{dh}$$

in which $A$ is the coefficient of turbulent exchange and $dq/dh$ is the gradient of vapor concentration with respect to altitude. Researches by von Karman and Rossby have provided means of evaluating the coefficients and arriving at a formula that can be used with observational data. Von Karman's constant of 0.38 enters the final equation, and a coefficient of surface roughness, designated $Z_o$, is included. Then the equation becomes

$$E = \frac{K^2\rho(q_1 - q_2)u_2}{\log_e\left(\frac{h_2}{h_1}\right)\log_e\left(\frac{h_2}{Z_o}\right)}$$

in which $K$ is von Karman's constant; $\rho$, the density of the air; $q_1$ and $q_2$, the vapor concentration at levels 1 and 2; $u_2$, the wind velocity at the upper elevation; $h_1$ and $h_2$, the elevations of the lower and higher sets of instruments; and $Z_o$, the roughness coefficient. The logarithms are to the base $e$.

The roughness coefficient can be determined from data of wind

velocities at the two elevations, $h_1$ and $h_2$, by the following formula:

$$Z_o = \frac{u_2 \log_e h_1 - u_1 \log_e h_2}{u_2 - u_1}.$$

The evaporation formula can then be simplified to

$$E = \frac{K^2 \rho (q_1 - q_2)(u_2 - u_1)}{\left(\log_e \frac{h_2}{h_1}\right)^2}.$$

The above evaporation formula can be reduced still further when the elevations of the instruments are known and when vapor pressures are substituted for concentrations. The elevations used by Thornthwaite and Holzman were $h_2 = 28.6$ feet and $h_1 = 2$ feet so that their formula was reduced to

$$E = \frac{17.1(e_1 - e_2)(u_2 - u_1)}{T + 459.4}$$

where $T$ is the temperature in degrees Fahrenheit, $e_1$ and $e_2$ are the vapor pressures in inches of mercury at the lower and upper levels, respectively; and $E$ is the evaporation in inches per hour. Placed in a form which can be used with the instruments at elevations other than those used by Thornthwaite, the equation becomes

$$E = \frac{0.0227(e_1 - e_2)(u_2 - u_1)}{(T + 459.4)[\log_{10}(h_2/h_1)]^2}.$$

This approach to the measurement of evaporation differs markedly from previous widely accepted methods. It is in fact only since 1930 that the problems involved in atmospheric turbulence have been analyzed sufficiently well to permit the development of this method of measuring evaporation. Many elements must be studied in different regions and over some time to create confidence in the results.

However, the results presented by Thornthwaite and Holzman indicate great promise for this method of determining evaporation. The gradients of moisture found by them appear to be well established and changes are promptly recorded by the instruments. The published results list hourly evaporation and condensation in quantities of the order of $1 \times 10^{-4}$ to $10 \times 10^{-4}$ inch, which indicates that the method is sensitive enough to obtain accurate results. However, the data available have not been sufficient to permit statistical analysis, nor have they been of such a nature as to permit correlation with data

obtained by other means of the total evaporation from soil and vegetation.

*Consumptive Use of Water.* The term "consumptive use" has been adopted from usage in the western portion of the United States in connection with irrigation and was originally intended to mean water lost by evaporation from the cultivated soil and by transpiration from crops. Its meaning has been extended to include similar losses from land and natural vegetation. It would in fact be the evaporation observed over a field by the apparatus and methods of Thornthwaite which have been described previously. Other terms sometimes used synonymously with consumptive use are "water loss," or "evapo-transpiration losses."

Methods of direct measurement of consumptive use are the same as those used to measure evaporation from land surface and transpiration from plants. Lee (123) enumerates six methods that are used in present practise. The methods may be grouped into four general classes, namely, volumetric, lysimeter, changes in ground water, and the relationship of heat or temperature. Some of these methods are as reliable as any such determinations can be, while others are relatively undeveloped.

The volumetric methods are applied to a watertight tank, field plot, or similar area, so situated that the inflow and outflow and any changes in storage can be measured with reasonable accuracy. The tank such as used to measure evaporation from soil is probably the most practicable and the most widely applicable. In other applications, field plots are utilized and even irrigated farms and irrigation projects. Another type of volumetric determination consists in measuring the inflow and outflow of a ground water reservoir, with proper correction for change in storage. This type of volumetric determination of consumptive use involves the application of the ground water equation which will be described in detail in Chapter 11.

Diurnal changes in ground water have been utilized by White (192) to determine consumptive use in Escalante Valley, Utah. This method consisted in measuring the daily fluctuations of ground water caused by withdrawals by plant transpiration, and from those data computing the volume of water consumed. In this valley the upper surface of the ground water was found to be within 2 to 10 feet of the ground surface, so that roots of vegetation tapped the ground water. The fluctuations of the water surface in wells is obtained by means of recording gages, some of the charts from which are reproduced in Figure 170 (Chapter 11). It was found that the rise during the four hours from midnight to 4:00 A.M. represented closely the average rate of refill, since surface

evaporation and transpiration were practically negligible then. One-fourth of the total refill of that period, marked $4r$ in Figure 170, was taken as the hourly rate for the full day. Then, after making an allowance for the net daily change, $s$, the consumptive use was expressed thus,

$$q = y(24r + s)$$

where $q$ is the consumptive use in inches; $y$, the specific yield in per cent; $r$, the mean refill in the well in inches per hour; and $s$, the net daily change of the water surface in the well, in inches. The use of this method depends upon finding suitable conditions of ground water.

More pertinent to the subject of this chapter is the method of determining consumptive use by its relationship to heat or air temperature. This method is still in the experimental stage and it has not been developed so that entirely satisfactory application can be made for all purposes. Briefly stated, the method equates the consumptive use of water to the heat available for plant growth. This method was developed in 1924 by C. R. Hedke and reported briefly by the Committee on Duty of Water, American Society of Civil Engineers (8). Hedke's equation was

$$U = KQ,$$

in which $U$ is the consumptive use in feet; $Q$, the effective heat of the area expressed in day-degrees and taken as the difference between the mean monthly temperature and the germinating temperature for each crop, the difference being multiplied by the number of days in the growing period; and $K$, the proportionality coefficient.

Lowry and Johnson (118) presented a similar method of computing the consumptive use of water, differing from Hedke's only in detail. They published their data of annual consumptive use and effective heat so that it was possible to estimate the accuracy of the method. Using the method of correlation heretofore given and applying it to all data of annual consumptive use and effective heat from ten river basins scattered over the United States, the over-all correlation coefficient was found to be +0.82, which value indicated good correlation. However, the application of the same methods to the data of the individual basins did not produce such satisfactory results; using the annual data of effective heat and consumptive use for the same ten basins in separate computations, it was found that the correlation coefficients ranged from −0.254 to +0.282, the average being +0.012, which is definitely not good. These discordant results merit some explanation. It is apparent that the high correlation found when the data for all ten

basins were used in one computation was the result of the close relationship between the average consumptive use and the average insolation which is the primary factor in effective summer heat received by the basins. The correlation in this case was controlled in reality by the average differences in the regional heat of the individual basins, rather than the relationship of annual heat and consumptive use, while the correlations for annual heat versus consumptive use on the individual basins were controlled by chance variations of the correlated elements. It may be concluded from this result that average consumptive use can be determined from average annual heat but that year-to-year records of consumptive use cannot as yet be obtained by the same method.

*Equation of Regression Line for Consumptive Use.* Since this method of estimating consumptive use may be used to obtain the average annual consumptive use of water, the study of correlation is extended to include the equation of the regression line of consumptive use on heat. The type equation is

$$Y = \bar{Y} + r\frac{\sigma_y}{\sigma_x}(X - \bar{X}).$$

Let $Y$ = the consumptive use of water in acre-feet per acre and $X$, the heat in degree-days. The other symbols are as used heretofore. From the computation to obtain the value of correlation given above, the following values are obtained:

$$\begin{aligned} \bar{Y} &= 2.382 \text{ acre-feet} \\ \sigma_y &= 0.568 \text{ acre-feet} \\ \bar{X} &= 10{,}320 \text{ degree-days} \\ \sigma_x &= 3{,}170 \text{ degree-days} \\ r &= 0.817. \end{aligned}$$

Upon substituting in the above type equation there is

$$Y = 2.382 + 0.817\frac{0.568}{3{,}170}(X - 10{,}320)$$

which reduces to

$$Y = 0.870 + 0.000146X.$$

This equation may be used to compute the average annual consumptive use of water when the quantity of heat in degree-days is known.

In a study comparable to the above for individual basins a determination was made of the correlation between average annual temperature and water losses (taken as equivalent to consumptive use for this purpose) for the basin of the Cedar River above Cedar Rapids, Iowa.

The data covering the years 1904–1939, inclusive, were compiled, computed, and furnished by Mr. L. C. Crawford, U. S. Geological Survey. The correlation coefficient, computed by the same methods as used above, was determined to be $+0.34 \pm 0.099$. This result supports the conclusion that year-to-year records of consumptive use cannot yet be synthesized from data of temperature. Instead of using average annual temperatures, it would have been more logical, had such data been available, to have used temperatures of the growing season only, and it is possible that better correlation could have been obtained.

In closing this chapter a warning against a misapplication of the methods of correlation should be given. In using statistical correlation between series of data such as those just discussed with reference to consumptive use of water and heat, care must be taken not to attempt to correlate average values of one variable with those of the other for the same period. For example, the yearly data from each of ten stations above were correlated in one computation and the yearly data from all ten areas were likewise correlated in a second computation. But the average annual consumptive use should not be correlated with average heat because averaging reduces the variations of the data and produces better correlation than actually exists. This danger has been pointed out by several writers on statistics (54, 64).

# 9 RUNOFF

*Definition of Runoff.* The term "runoff" designates all water that drains from land areas by surface channels into which the water collects from overland flow or subterranean passages. Runoff is, therefore, the water remaining from precipitation after the losses from evaporation, transpiration, and seepage into the ground water. The principal losses occur before the water reaches the streams, so that runoff is a residual quantity dependent upon the action of the consuming elements on the ground surface.

Runoff constitutes the last phase of the hydrologic cycle. Not all water reaches or passes through this phase, however, since large quantities are evaporated directly from the ground surface or through vegetation to pass through an abbreviated cycle. The water of precipitation that goes into deep underground seepage may complete the cycle by returning to the ocean by subterranean channels or by percolation. But permanent losses by deep seepage must be comparatively small since the ground water must be considered a substantially stable quantity, that is, the underground capacity of the earth must be presumed to be filled except for fluctuations resulting from outflow. Some ground water will be lost by percolation directly to the ocean, but it is not likely that this loss can be large because favorable conditions for general percolation are lacking. The two aspects of runoff to be carefully studied for hydrologic purposes are the surface runoff and ground water outflow.

The present discussion is confined primarily to the hydrologic aspects of surface runoff and does not enter into the hydraulics of channel flow which is an extensive subject in itself and beyond the scope of this book. There will be given very briefly, however, a few of the better known equations useful for determining stream flow, and a brief outline of methods of measuring stream discharge. These discussions will be given only to the extent needed to understand the

hydrologic aspects of runoff. An extensive summary of American and European formulas was compiled and published by the Miami Conservancy District (93), and other details can be found in any standard textbook on hydraulics.

*Hydraulic Formulas.* Hydraulic formulas in common use state the laws of both discharge and velocity. Discharge is a function of the cross-sectional area and of the velocity, that is,

$$Q = AV$$

where $Q$ is the discharge in cubic feet per second; $A$, the area in square feet; and $V$, the average velocity in feet per second. (These units are of the so-called foot-pound-second system used predominantly in English-speaking countries.)

The formulas most used for computing open-channel stream flow make velocity a function of the slope of the water surface and the hydraulic radius, which is defined as the ratio between the area of a cross section of the stream and the portion of the channel covered or wetted by the water, called the "wetted perimeter." An experimentally determined coefficient is included for evaluating energy losses due to friction and turbulence caused by the roughness of the channel.

*Chezy's Formula.* The first well-known formula devised for determination of velocities in streams was that of Chezy, a French engineer of the eighteenth century,

$$V = C\sqrt{RS}$$

in which $V$ is the velocity; $C$, a coefficient; $S$, the slope of the water surface; and $R$, the hydraulic radius. This formula has been widely adopted and is still used with coefficients derived by Ganguillet and Kutter's formula.

*Ganguillet and Kutter's Formula.* Ganguillet and Kutter, two Swiss engineers, devised a formula based on data of stream gagings in America and Europe to determine values for the $C$ in the Chezy formula. Their formula, commonly referred to as Kutter's, evaluates $C$ in terms of slope, hydraulic radius, and a roughness coefficient designated $n$; thus,

$$C = \frac{41.65 + \dfrac{1.811}{n} + \dfrac{0.00281}{S}}{1 + \left(41.65 + \dfrac{0.00281}{S}\right)\dfrac{n}{\sqrt{R}}}.$$

There are available tables of values of $n$ which are derived from stream measurements in various types of channels and under specified condi-

tions of flow. While the velocity of a channel may be computed by combining Kutter's formula with Chezy's, the latter is commonly used with values of $C$ that have been computed from Kutter's and made available in published tables.

*Manning's Formula.* Manning, a British engineer, introduced another formula which is comparable in simplicity to Chezy's; thus,

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}$$

in which the notation is the same as above. The formula was derived on the basis that the values of $n$ would represent the same degree of roughness as the $n$ in Kutter's formula. The Manning and the Chezy formulas are generally used in the United States for computing stream flow because of their simplicity and the availability of tables of values of the roughness coefficient. As long as judgment and experience are still primary requisites for application of these formulas, particularly in the selection of the values of $n$ or $C$, a more involved formula can add little to the accuracy of the result.

*Data of Runoff.* Most data of runoff are obtained at special gaging stations established solely for this purpose at carefully selected points on streams. In the United States the agency charged with nationwide responsibility of securing and publishing data of stream flow is the U. S. Geological Survey, Water Resources Branch. This organization has an extensive network of about 5,000 gaging stations in operation under United States jurisdiction. The major portion of these stations are operated in cooperation with other public and private agencies.

Data of discharge are also obtained from a number of projects utilizing or controlling streams, such as hydroelectric plants, and to some extent such data are obtained by other Federal agencies and many states, much of which work is done in cooperation with the U. S. Geological Survey. Data of river stages are obtained by the U. S. Weather Bureau from an extensive system of stations. River stages are also obtained at many points as a part of the operation of improvements for navigation on rivers.

For runoff studies the data most generally used and published by the U. S. Geological Survey are the mean daily discharges. Except for some flood periods, the day is usually the smallest unit of time for which discharge is computed and published, since that period forms the basis of much of the study concerned with runoff. These data, with summaries of monthly and annual volumes, are prepared and published by the U. S. Geological Survey in a series of publications called *Water Supply Papers*.

For studies of water supply, power, irrigation, and flood control the quantity of water or runoff must be obtained. In studies for flood control, discharge values for shorter intervals of time are usually required for defining hydrographs of individual floods which are essential in the development of a flood control plan. Such values, together with many related data of floods, are published for major floods in the United States in special *Water Supply Papers* of the U. S. Geological Survey. Daily stages are useful for many purposes but they do not give the necessary values of discharge.

*Gaging Stations.* The collection of runoff data requires installation and maintenance of regular gaging stations. It is not proposed to describe here such stations in detail since this has been done by others well qualified for such a task (40)(76), but a brief outline of the methods of obtaining data of runoff will be given.

The acquisition of data of stream flow, such as mean daily discharge, is based on a continuous record of stages taken at frequent intervals and the evaluation of these recorded stages in terms of stream discharge. Therefore a gaging station requires two principal components: first, a gage for reading the river stage; second, means for readily and accurately measuring the actual river flow at required intervals. The location of the station is usually determined: first, by the part of the stream under study and the use to which the data will be put; second, by the existence of the local physical conditions of the river producing an adequate or rateable control; third, convenient access and the availability of existing structures such as bridges for making satisfactory discharge measurements. The control is an important item; it is the section or reach of the channel downstream from the gage that controls the stage-discharge relation of the flow past the station. It is highly desirable that the control be permanent or stable to give as constant a stage-discharge relation as possible since this is the primary factor affecting the accuracy of continuous discharge records.

The gage in its simplest form is a staff with a scale by which the stage may be read and is fastened to a suitable support in the water, such as a bridge pier. Other forms of non-recording gages are chain or wire weight gages; such a gage is located on a bridge or other structure of convenient access over the stream and the weight is lowered to touch the water surface. Depending upon the length of chain or wire let out, the stage is then read on a scale fastened to a bridge rail or on a graduated reel. This type of gage is read intermittently, once or twice a day by a regular observer. At stations with rapidly fluctuating discharge, or of sufficient importance to warrant the expense, a continuous water stage recorder is usually installed. There are several suitable types

having time-stage coordinates, but each utilizes a float which records the stage through suitable connections on a chart which is mounted on a drum revolved by a clock mechanism to measure the time. Records from this type of station are particularly important in the study of many problems of runoff from medium and small drainage basins.

Discharge measurements are usually made at a regular measuring section provided by an existing bridge or with the installation of a cableway. The measuring section is taken as close as practicable to the gage. In the usual procedure the velocity of the stream is measured by a current meter at two points in each vertical located at regular intervals along the measuring section, together with the depths of water at these verticals. From the velocities, depths, and spacing of the verticals the discharge is computed section by section across the stream and the sectional discharges added to give the total stream discharge.

*Rating a Gaging Station.* The rating of a station consists in making a series of discharge measurements as opportunity is presented, to cover the range of stages caused by the varying flow of the stream at the station. The data obtained by each discharge measurement are plotted as made, with the value of the discharge as abscissa and stage as ordinate, to form the discharge rating curve. The area of the cross section of the stream at the measuring station and the mean velocity of the stream frequently are plotted against stage on the same sheet. An illustration of such a curve, together with the area and velocity curves at a measuring section for the station is shown in Figure 109. When a satisfactory rating curve is established a rating table can be computed and applied to the daily gage readings to give the appropriate discharge. Since few stations have a fixed discharge relationship, periodical measurements for checking are needed to discover and evaluate the changes that are continually occurring in the stage-discharge relationship.

A rating table is useful only as long as the relationship between the stage and discharge remains constant. There are unfortunately many things that upset that relationship. Ice in the winter is probably the most common cause of change in northern climates and this is an annual occurrence of considerable duration. Ice usually forms on the control causing backwater, but it has been known to form on the control in such a way as to act like a siphon and produce the opposite effect. Surface ice reduces the cross section of a stream and diminishes the velocity of approach above the control, and where a reach of channel is the control, produces varying degrees of backwater. Other forms of ice, such as anchor and frazil, obstruct the channel. Another common cause of disturbance to the stage-discharge relationship met in all

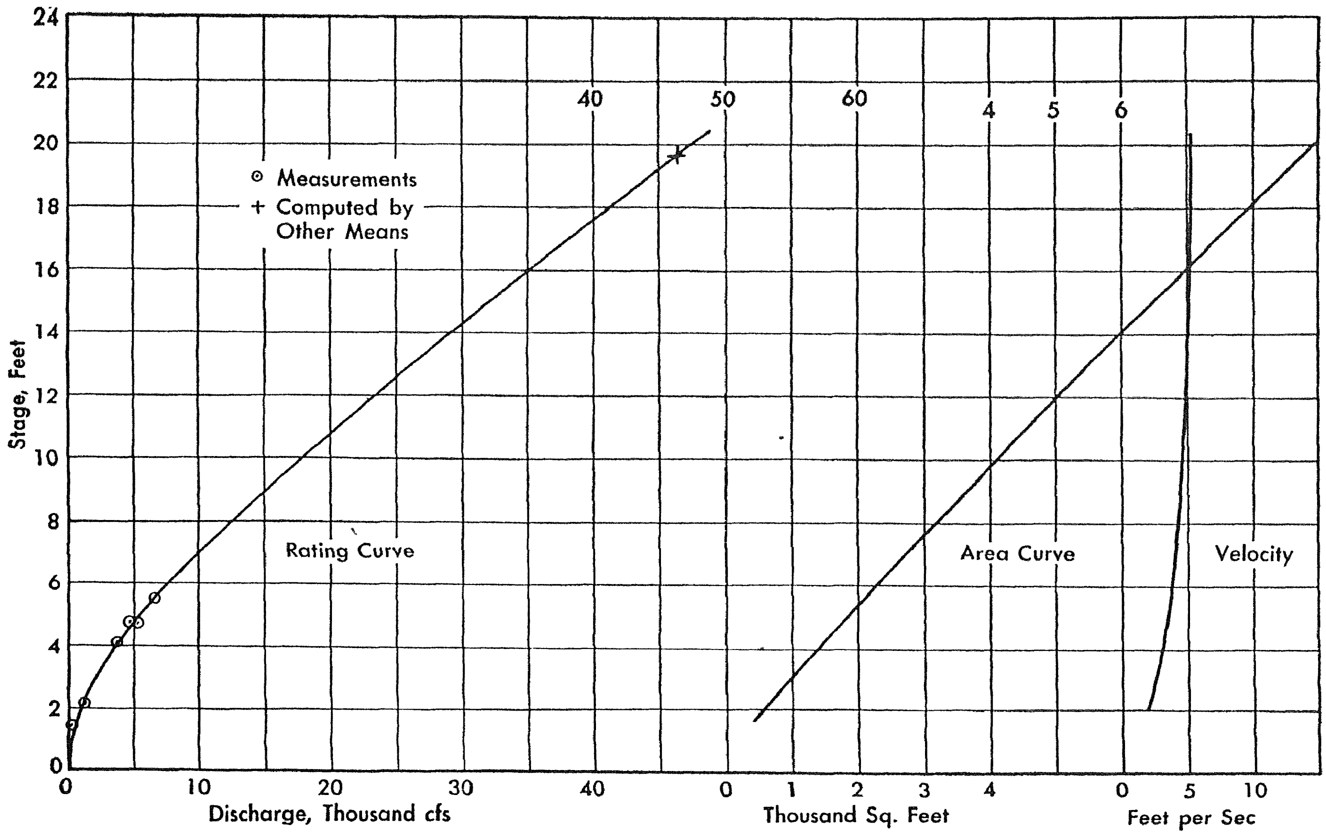



FIGURE 109. Rating, Area, and Velocity Curves, Tallapoosa River, Cragford, Ala.

climates is the shifting of sandy or erodible beds. This difficulty is very irregular since it is brought about by high stages resulting from floods, freshets, or rises caused by storm runoff. Aquatic growths in channels and on fixed controls are troublesome in summer operation, and trees and other long-lived vegetation sometimes form an impediment to flow in the smaller streams. Still other disturbing factors are improvements by man, such as bridges, channel improvements, dikes, encroachment on the channel by buildings, dams, ditches, and levees. All these obstructions must be taken into account in evaluating river discharges.

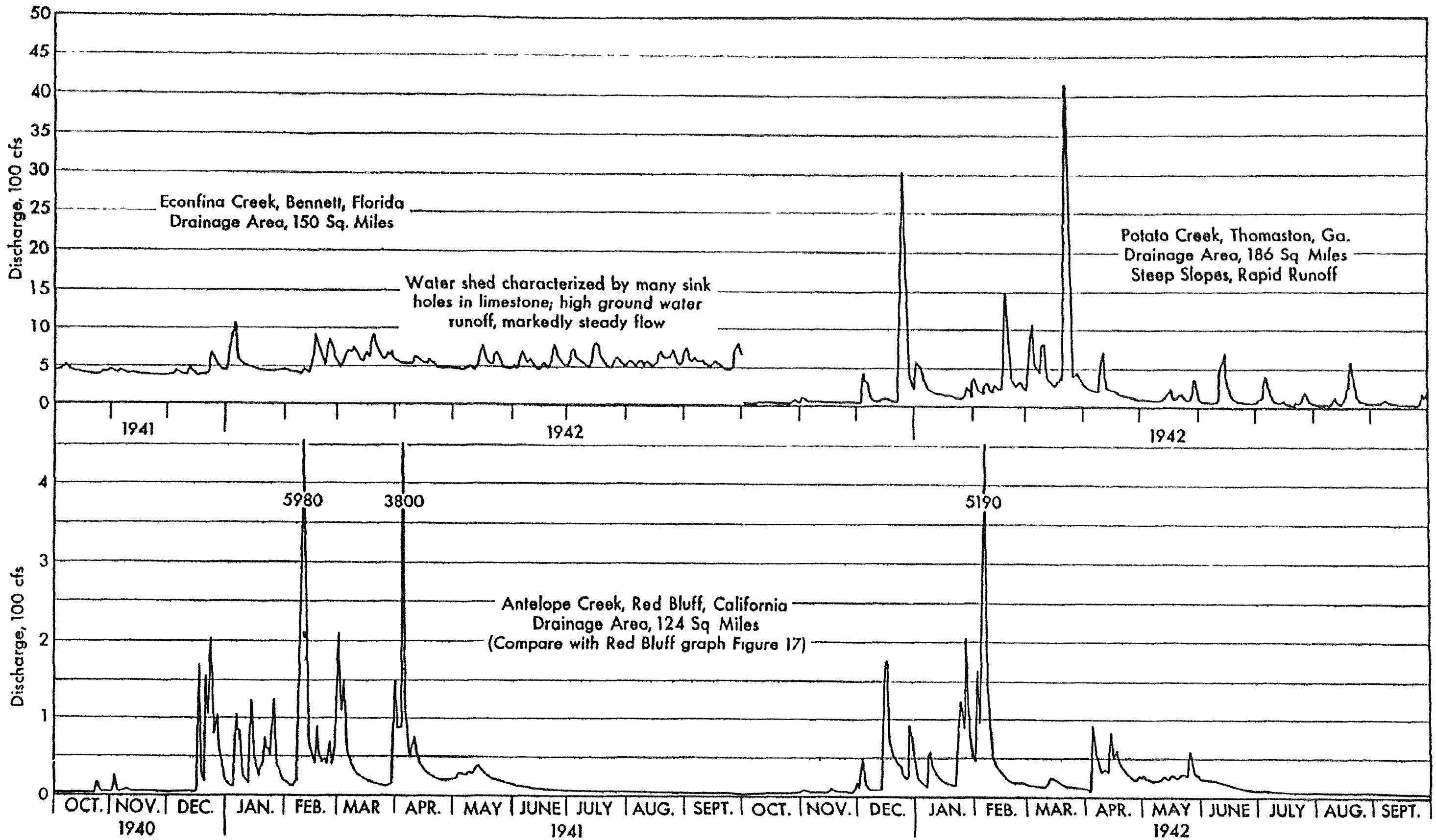
*Total Runoff.* Total runoff is the volume of water obtained in any given period by adding the increments of discharge, usually mean daily discharge, and applying the proper coefficients to convert rates of discharge into volume. It is the total volume from all sources of stream flow and as such is a variable quantity, changing from year to year, month to month, day to day, and often hour to hour. In small flashy streams it changes constantly, depending upon the sources of water and is sensitive almost immediately to changes in supply conditions. In Figure 111 there is shown a hydrograph of daily discharge for the Chattahoochee River to illustrate the changing flow during the water year from October 1, 1931, to September 30, 1932. A series of daily hydrographs of different streams is shown on the Plates.

*Sources of Runoff.* The immediate sources of runoff may be divided into three types:

First, the residual water from rainfall, that is, the water not lost by evaporation and infiltration into the soil after falling. This portion of the runoff is intermittent, its occurrence being dependent entirely upon sufficient rainfall.

Second, snowmelt, or the residual portion from snowfall that is not lost by evaporation and infiltration. Runoff from this source may be a very small proportion and may continue for only a month or two in the spring. However, in colder climates or in mountainous areas that are favorably situated with respect to winter moisture, it may be a source of flow well into summer and produce as much as 50 per cent or more of the annual runoff (see Figure 101). Water from glaciers may be included in this category. Water from glaciers flows as long as temperatures are warm enough to melt ice.

Third, ground water runoff which comes from the subterranean storage. This source is the steadiest of all as it continues throughout the year unless freezing in winter is deep enough to prevent it from flowing. All sources of runoff, however, are derived ultimately from precipitation.


Econfina Creek, Bennett, Florida
Drainage Area, 150 Sq. Miles
Water shed characterized by many sink holes in limestone; high ground water runoff, markedly steady flow
Potato Creek, Thomaston, Ga.
Drainage Area, 186 Sq Miles
Steep Slopes, Rapid Runoff
Antelope Creek, Red Bluff, California
Drainage Area, 124 Sq Miles
(Compare with Red Bluff graph Figure 17)
5980
3800
5190
Discharge, 100 cfs
Discharge, 100 cfs
50
45
40
35
30
25
20
15
10
5
0
4
3
2
1
0
1941
1942
1942
OCT.
NOV.
DEC.
JAN.
FEB.
MAR
APR.
MAY
JUNE
JULY
AUG.
SEPT.
OCT.
NOV.
DEC.
JAN.
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MAR.
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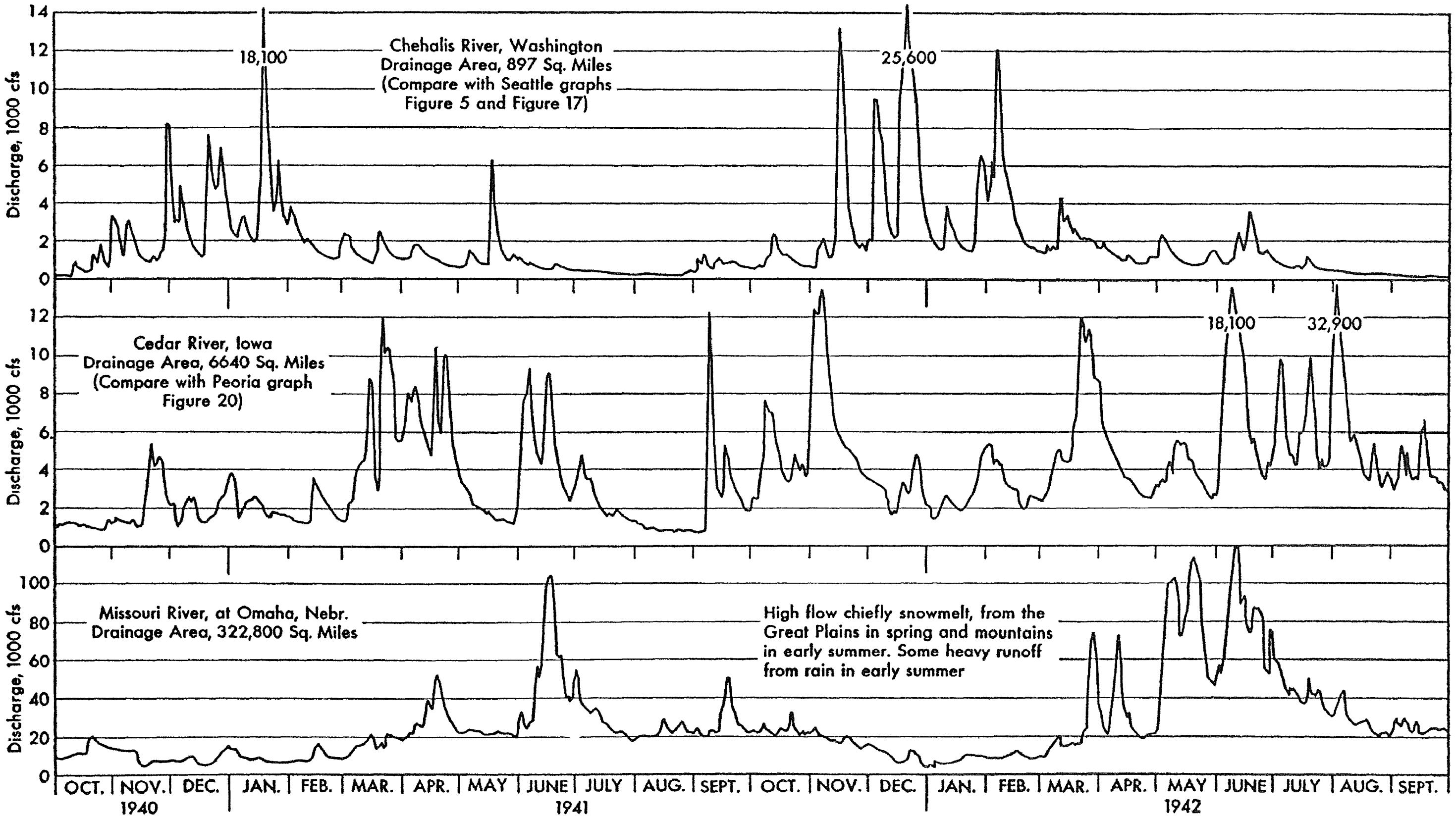
MAY
JUNE
JULY
AUG.
SEPT.
1940
1941
1942

PLATE


Chehalis River, Washington
Drainage Area, 897 Sq. Miles
(Compare with Seattle graphs
Figure 5 and Figure 17)
18,100
25,600
Discharge, 1000 cfs
0
2
4
6
8
10
12
14
Cedar River, Iowa
Drainage Area, 6640 Sq. Miles
(Compare with Peoria graph
Figure 20)
18,100
32,900
Missouri River, at Omaha, Nebr.
Drainage Area, 322,800 Sq. Miles
High flow chiefly snowmelt, from the
Great Plains in spring and mountains
in early summer. Some heavy runoff
from rain in early summer
20
40
60
80
100
OCT.
NOV.
DEC.
JAN.
FEB.
MAR.
APR.
MAY
JUNE
JULY
AUG.
SEPT.
1940
1941
1942

PLATE

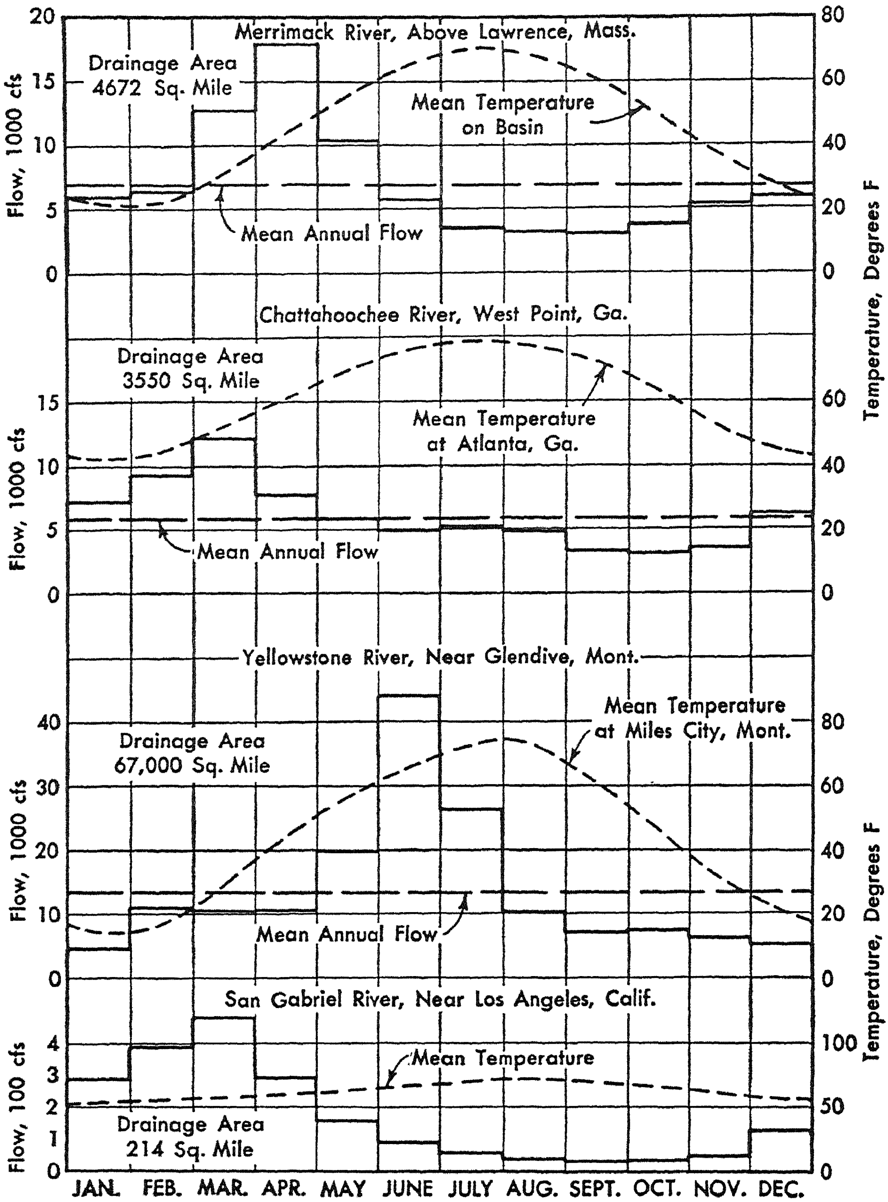



FIGURE 110. Hydrographs of Mean Monthly Flows

Figures 110 to 115 show the various aspects of runoff mentioned above. All discharge shown in these graphs was derived from rainfall.

Figure 111 shows the total runoff which is divided into the two constituent parts, surface runoff, and base flow, which is mostly ground water runoff. The dashed line marking the latter is drawn through the stable lower portions of the hydrograph and in this case is considered

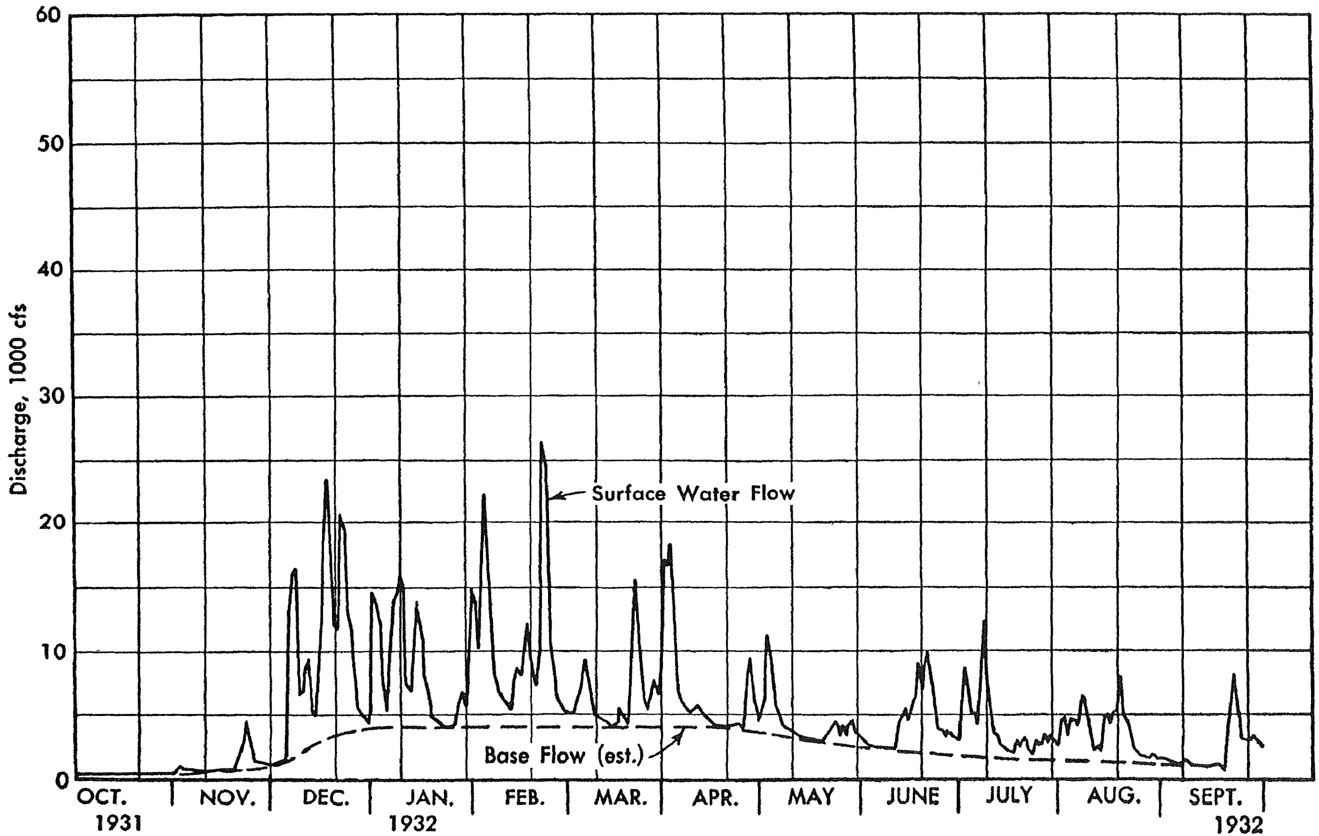



FIGURE 111. Daily Hydrograph, Chattahoochee River, West Point, Ga.

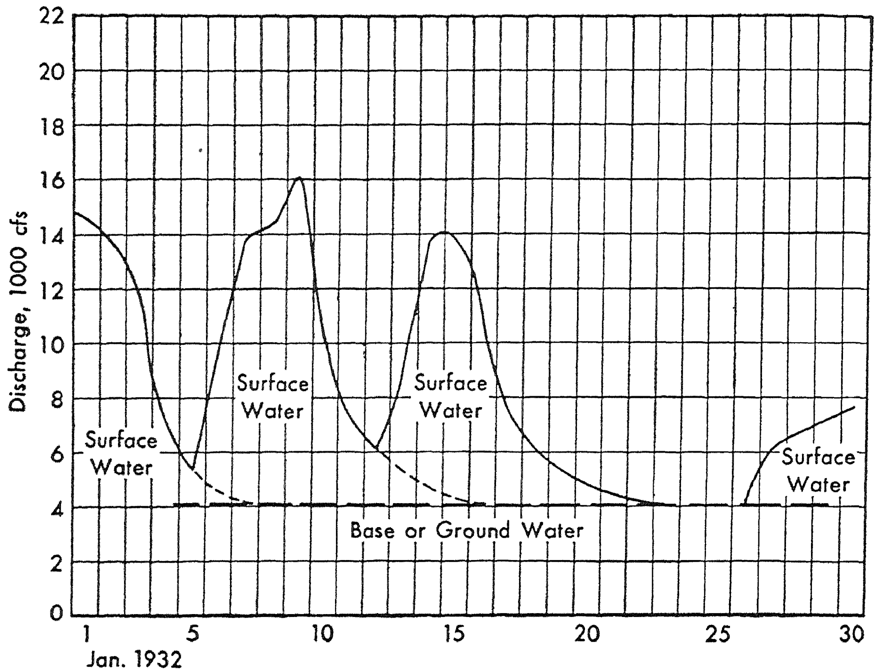



FIGURE 112. High Water Hydrographs, Chattahoochee River, West Point, Ga.

to be rather definitely fixed. In Figure 112, three hydrographs of high water of separate storms are shown to bring out the relation between the surface runoff and the base flow, chiefly from the ground water. In Figure 113, a single storm hydrograph of two different streams are shown to illustrate the relationship between base flow, surface runoff, and the rainfall causing the high water; these hydrographs show the lag between the time of the runoff and the rains, particularly in peak values. Hydrographs of runoff and pluviographs of rainfall, together with graphs of the accumulated rain and discharge plotted against time are shown in Figures 114 and 115 for a small drainage area of 1.03 acres. Because of the small area and high elevation of the surface above the water table, there is no base flow. The accumulative graphs show the difference between the total rain and runoff, which difference constitutes the losses due to evaporation (principally of precipitation caught on vegetation and referred to as "interception") and infiltration. It will be noted that in each case there is a lag of time between the inception of the rain and the beginning of the runoff. This is caused by the necessity of saturating the surface of the soil, filling surface depressions, and temporary surface and channel storage, with some loss (usually small) by evaporation, before runoff can begin.

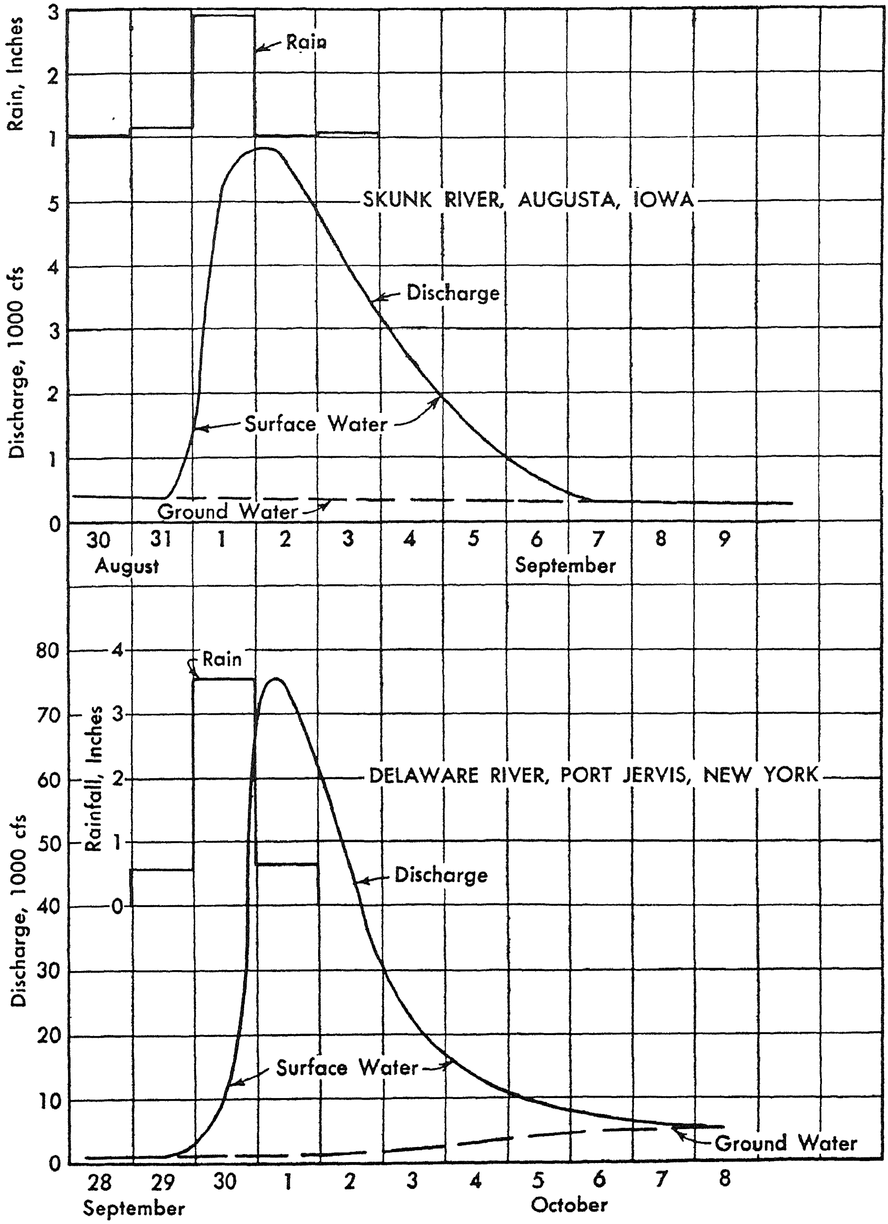



FIGURE 113. High Water Hydrographs, Delaware River, Port Jervis, N. Y.

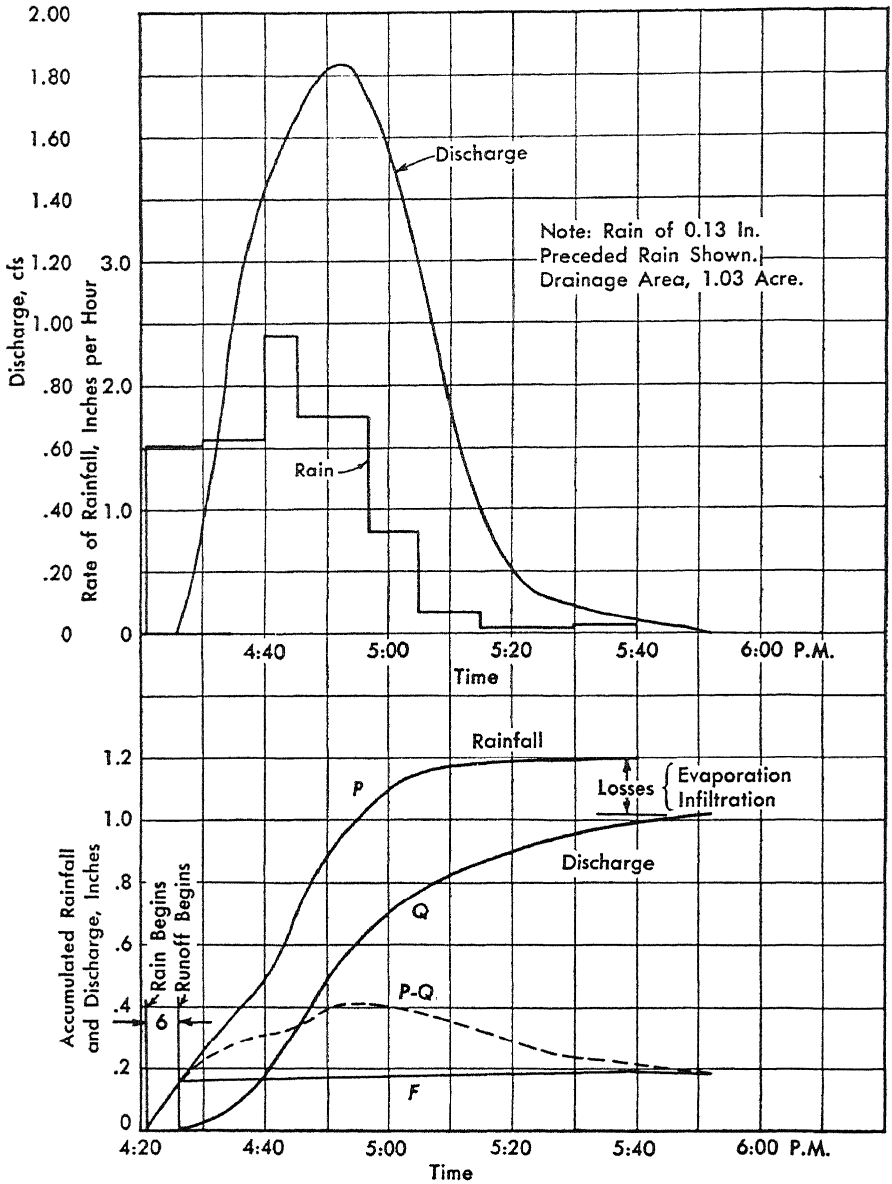



FIGURE 114. Hydrograph, Storm of June 26, 1932, Hays, Kan.

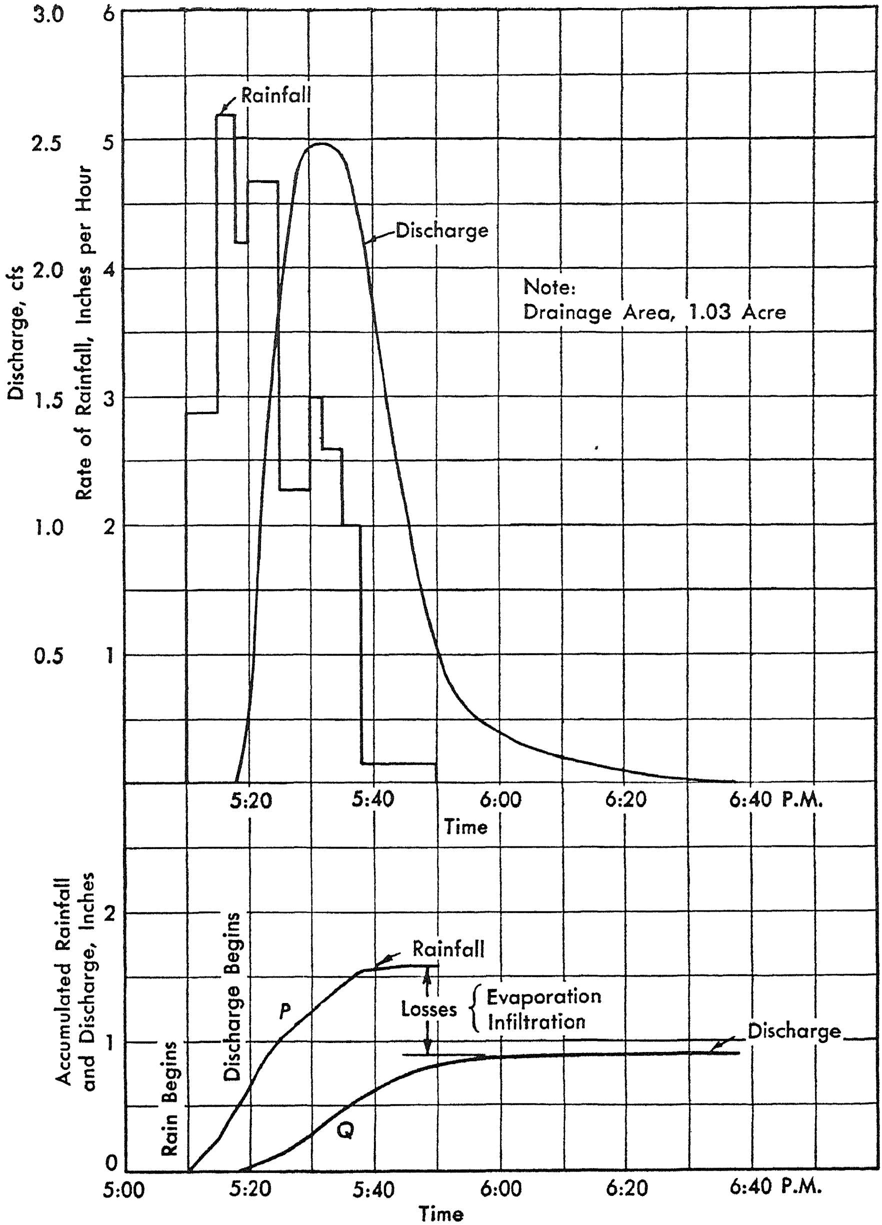



FIGURE 115. Hydrograph, Storm of August 1, 1932, Hays, Kan.

*Surface Detention.* Reference has been made previously to the initial action of rainfall in saturating the surface. Following this action, the depressions of the surface are filled with water in excess of the current infiltration rate. When the depressions are filled the subsequent rainfall builds a layer of water on the surface which, as soon as it is deep enough, begins to flow off towards the nearest channel. This surface layer of water has been designated "surface detention" by Horton (90) and the name has been retained by other investigators of runoff phenomena (106). The term "detention" is better than "storage" because it emphasizes the transitory nature of the phenomenon, and for that reason is used hereinafter. The flow over the surface prior to reaching a well-defined channel is called "overland flow."

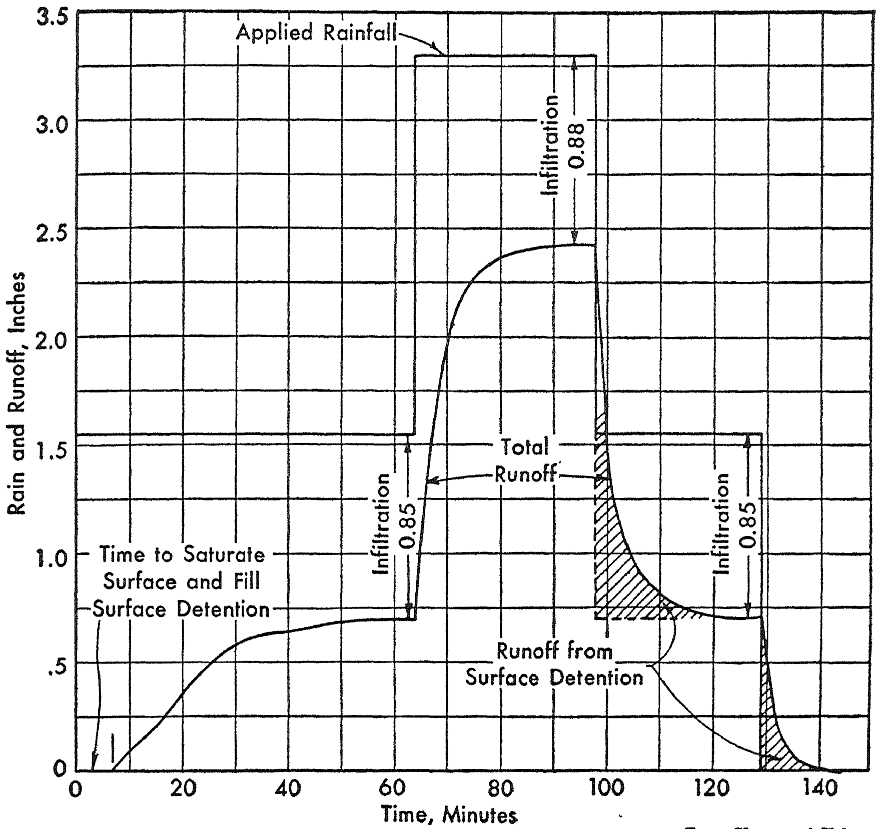
Horton (90) found that the relationship between overland flow over a small plot could be expressed by the equation

$$Q_s = K_s \delta_a^M$$

where $Q_s$ represents the rate of overland flow; $\delta_a$, the average depth of the surface detention; $M$, a coefficient depending upon the nature of the flow (i.e., laminar, turbulent, or mixed); and $K_s$, a coefficient depending upon the characteristics of the watershed. From Manning's equation for discharge, $M$ may be determined to be 5/3 for flow that is fully turbulent. For laminar flow, the velocity of which varies as the square of the depth, and rate of flow as the cube, $M$ equals 3. Other degrees of turbulence will have $M$ varying between those values. An important and probably common condition of turbulence is that having a degree of 75 per cent, for which $M$ would be 2.0.

Other investigators have confirmed the above equation for overland flow. The conditions of turbulence and values of $M$, however, are due largely to Horton but the values may in general be substantiated upon consideration of the accepted formulas of flow.

The accumulation of water as surface detention, and its relation to runoff and rainfall are most clearly demonstrated by simulated rainfall on small experimental plats. Figure 116 shows the graphical results obtained by Sharp and Holton (163) on sprinkled plats. The simulated rainfall was moderately heavy, 1.55 and 3.30 inches per hour. Upon application of 1.55 inches per hour some seven minutes were required to saturate the surface and accumulate the surface detention water before runoff started. The runoff increased gradually for 50 minutes, at which time a steady rate of 0.70 inches per hour was reached. This rate continued until the rate of rainfall was increased to 3.30 inches per hour, whereupon the runoff rose to 2.42 inches per hour. Upon cessation of the rainfall the runoff decreased rapidly, but did not stop imme-




*From Sharp and Holton*

FIGURE 116. Rainfall and Runoff from a Plat Experiment

diately since the water accumulated by surface detention required time to drain.

The depths of surface detention are not great in ordinary storms, but range from approximately 0.01 to 0.1 inch. With these depths it is probable that only sufficient water lies on the surface to flow around the grains of the soil and small irregularities rather than over them. However, during intense storms which cause disastrous floods the depths would be much greater and sufficient to flow over the smaller surface irregularities.

These details are generally obscured in hydrographs from natural streams which usually have much greater drainage basins. Nevertheless the same factors operate in natural basins. These small artificial plats with simulated rainfall may appropriately be considered a differential of a natural watershed; the observed runoff of a natural stream as represented by the recorded hydrograph can well be looked upon as the integration of discharge from all the differential plats of a

drainage basin together with all variations in rainfall, losses, and all conditions producing the runoff.

*Infiltration*. In the preceding sections infiltration was mentioned as a factor that reduced the amount of water available to surface runoff. By the term "infiltration" is meant the absorption of water into the soil from either rainfall or snowmelt. This infiltration or absorption is an important element in the disposal of precipitation: it is by this process that water becomes available for all forms of land vegetation and for the replenishment of ground water. Infiltration is of great importance to engineers and hydrologists because of its reduction of flood flows and the surface runoff which would otherwise be available for water supply and other utilization.

Infiltration capacity has been defined by Horton (90) as the "maximum rate at which the soil, in a given condition, can absorb falling rain." Since the absorptive capacity of the soil varies greatly and since there may not always be sufficient rain or snowmelt to supply the maximum rate, it seems desirable in most cases to speak of the "infiltration rate." This practise will be followed hereinafter unless the full absorption capacity is meant.

*Determination of Infiltration Rate.* Rates of infiltration have been measured in a number of ways. They have been measured directly by means of lysimeters. These instruments are essentially containers holding a quantity of soil to which a known quantity of water is applied and the infiltration is measured. In recent years, however, two other methods have been developed in an effort to obtain the rates of infiltration for the soil in its natural condition. One of these methods makes use of an instrument or devise called the "infiltrometer," and the other approaches the problem by comparing rates of rainfall with the runoff hydrograph.

The use of infiltrometers permits the determination of the actual rates of infiltration of the soil in place. These instruments are essentially tight enclosures placed around a small plot of land to which water is applied at a known rate by sprinklers, and from which the runoff is caught and measured. Several types have been developed which have, according to Wilm (194), varying degrees of accuracy. Wilm concluded that infiltration rates are characteristically variable and that the instruments can be expected to give only relative estimates of the true infiltration. Nevertheless, in spite of that none too optimistic conclusion, the infiltrometer is the only instrument that to date can give even an approximate idea of the detailed operation of infiltration.

Data obtained by experimenters from infiltrometers are consistent in showing high rates of absorption at the beginning of a rain, then

a rapidly diminishing rate until a fairly constant ultimate value is obtained. The initial and ultimate rates, as well as time to reach the latter, vary greatly, being dependent upon the many factors that affect such rates. Some curves found by Beutner, Gaebe, and Horton (20) in their experiments on soils in Arizona are shown in Figure 117. In these experiments sprinklers were used to simulate rainfall on small plots of different types of soil in both wet and dry conditions. The resulting data show typical infiltration curves and varying rates of ultimate steady infiltration.

Horton has derived a type of equation to fit the infiltration curve, namely,

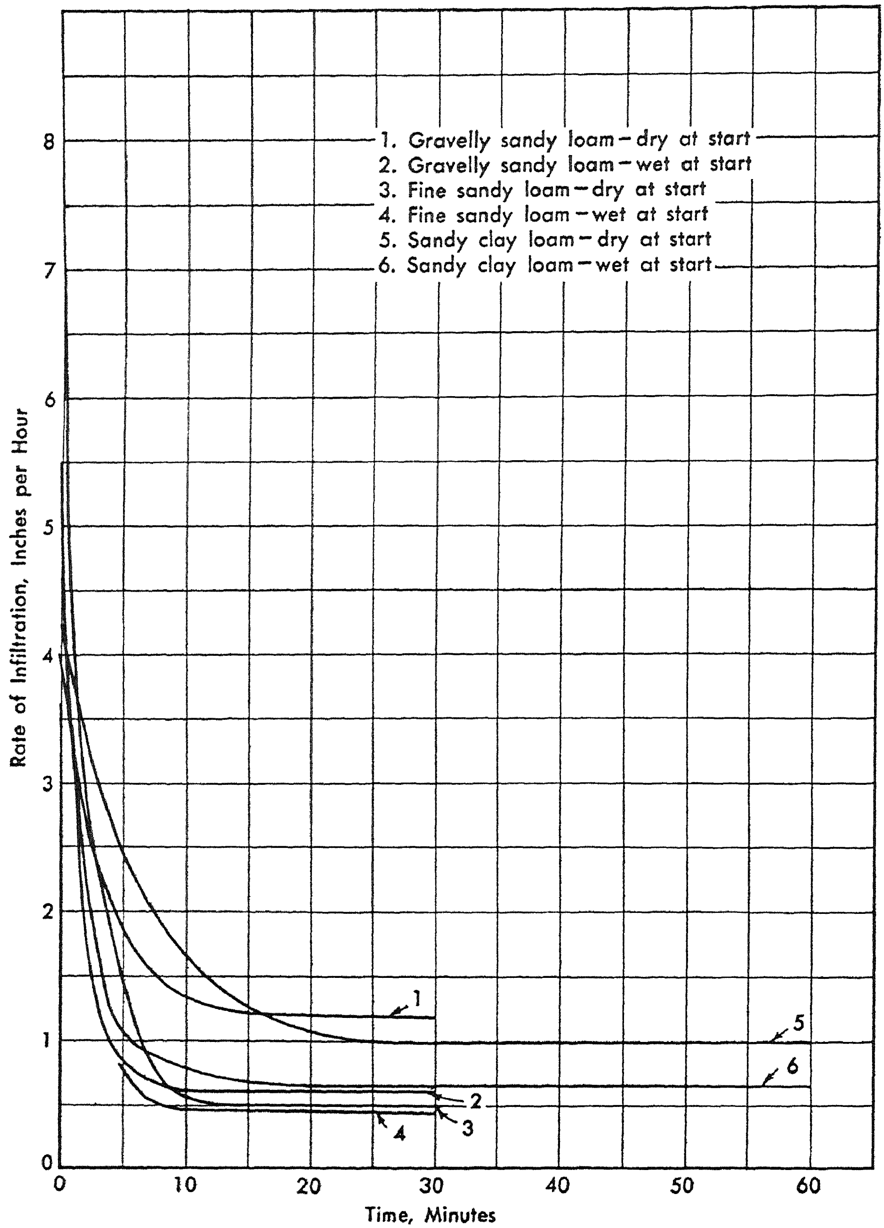
$$f = f_c + (f_o - f_c)e^{-Kt}$$

in which $f$ is the infiltration rate; $f_c$, the ultimate or steady rate; $f_o$, the high initial rate; $K$, a constant, and $t$, the time elapsed since the beginning of the rain.

It will be noted from the curves in Figure 117 that the high initial rate of infiltration has in most cases diminished to its relatively low ultimate steady rate within 10 minutes. Although this reduction is more rapid than usual, all similar experiments show a like short period necessary to reach the ultimate rate, the time usually ranging from 10 minutes to one hour. This situation indicates that in a common rainstorm of several hours or days duration the high initial rate is of minor importance in the total amount of water deducted from the flood flows, and that the really significant feature in accomplishing such reduction is the ultimate steady rate which is attained shortly after the beginning of the rainfall.

A number of investigators has analyzed stream flow hydrographs and rainfall pluviographs for the purpose of determining with reasonable accuracy the rate of infiltration and the shape of the infiltration curve. This approach is admittedly more or less approximate and depends much upon a knowledge of the shape of the infiltration curve as determined from the experiments on sprinkled plots. The details of the procedure has varied somewhat among different investigators.

Sherman and Mayer (166) start from the average loss rate, designated $f_{av}$, which is computed by dividing the difference between the total rainfall and the total surface runoff by the time between the beginning of the runoff and the end of the rainfall. In the storm the graphs of which are depicted on Figure 114, for example, this average loss would be the total losses (evaporation and infiltration) divided by the time between 4:26 and 5:40 P.M.; the total losses are 0.18 inch and the time 1 hour and 14 minutes, making the average loss rate $f_{av}$ equal




*From Beutner, Gaebe, Horton*

FIGURE 117. Infiltration Curves, Arizona Soils

to 0.146 inch per hour. For many purposes of hydrograph analysis this computation is sufficient.

Sherman and Mayer continue from the above result, and by a series of approximations trace the infiltration curve. This determination depends upon a set of empirically derived curves which cannot readily be reproduced and for which the reader is referred to the original paper (166).

Sharp and Holtan (162) approach the determination somewhat differently, their method being founded on their experiments with sprinkled plots (163). Essentially their procedure was to plot summation curves of the rainfall $P$ and the runoff $Q$, as in Figures 114 and 115, and then compute a third curve from the differences $P$-$Q$ between the two former curves. A straight line was then drawn from a point on the $P$-$Q$ curve on the ordinate at the beginning of the runoff to a point on the $P$-$Q$ near the end of the runoff, the straight line being tacitly assumed on basis of the experiments on sprinkled plots. This straight line $F$ was assumed to give the summation of the infiltrated water at the ultimate rate. Differences in the ordinates between the curves $F$ and $P$-$Q$ were caused by the surface detention, the values of which were shown to be consistent when plotted against runoff as described previously; this consistency substantiates the method used for determining the shape of the infiltration curves.

*Variation in Infiltration Rates.* As is the case in all hydrological data there are many factors operating to produce variations in the infiltration rate. The first in importance is the porosity of the soil itself. For example, Musgrave and Norton (140) state that infiltration in the Marshall silt loam was 7 to 10 times that of the Shelby silt loam. The state of cultivation has an important bearing, but the effect is transitory; infiltration is high at the beginning of a rainstorm but diminishes rapidly, apparently because the pores on the surface quickly become clogged. For a similar reason turbid water reduces infiltration. Organic matter on the surface and growing vegetation promote infiltration greatly. Duley and Kelly (49) found in their experiments, using Marshall silt loam, that native sod with good grass cover and soil with a straw cover had an infiltration rate several times that of the bare cultivated soil. The presence of moisture in the soil appears to reduce infiltration somewhat in proportion to the amount of soil moisture. On theoretical ground, at least, temperature must have an effect on the rates of infiltration because of a variation of viscosity of water with temperature.

It is evident that with these various facts operating on the rates of infiltration their determination is a matter for individual investigation

for each basin and for each storm or flood. Even a relatively small area provides many types of soil, vegetal cover, condition of cultivation, and antecedent condition of soil moisture. Some of these items vary greatly with the season and corresponding temperature, vegetation, and state of cultivation, so that in the study of infiltration rates consideration should be given to the seasons. This is obvious in colder regions where the almost impervious frozen ground in winter contrasts strongly with the permeable ground surface in the summer season of flourishing vegetation.

A few investigators, such as Wilm (194), have utilized the theory of statistics for analyzing data of infiltration. In general there has not been great enough accumulation of data of infiltration made available to the public on which to make the most efficient use of statistical analysis, since such methods are predominantly useful for large masses of data. Because of the various uncertainties and approximations in the determination of infiltration capacities, there will be undoubtedly a serious need for use of such methods when sufficient data are available.

*Effect of Ground Water on Surface Streams.* The hydrology of ground water constitutes a formidable subject in itself, and while it cannot be covered adequately here, it will be discussed in some detail in Chapter 11 with only a few comments here which may be necessary to understand its effect on the hydrographs of surface flow.

Wherever a stream channel intersects the upper surface of the ground water (the usual condition in humid regions) it becomes the steady and equalizing contributor of flow. It contributes constantly during storm runoff and between storms. The rate of contribution varies, however, according to the height of ground water above the outlet channels, being great in wet seasons and diminishing slowly during seasonal dry period. The contribution of ground water to stream flow is shown on hydrographs of Figures 111 to 113. As shown by the hydrographs of the small areas, Figures 114 and 115, streams of small watersheds may have intermittent flow because the channels are entirely above the zone of ground water.

*Ground Water Recession Curve.* Horton (89, 90) has shown that the ground water flow for a given basin declines at a steady rate in the absence of further replenishment and that this declining discharge may be represented well by an empirical equation of the type

$$q = q_o e^{-ct}$$

where $q$ is the rate of flow at time $t$; $q_o$, the rate of flow at a selected initial time when the ground water discharge is high; $c$, a coefficient; and $t$, the time elapsed since the initial flow $q_o$ obtained. Horton adds

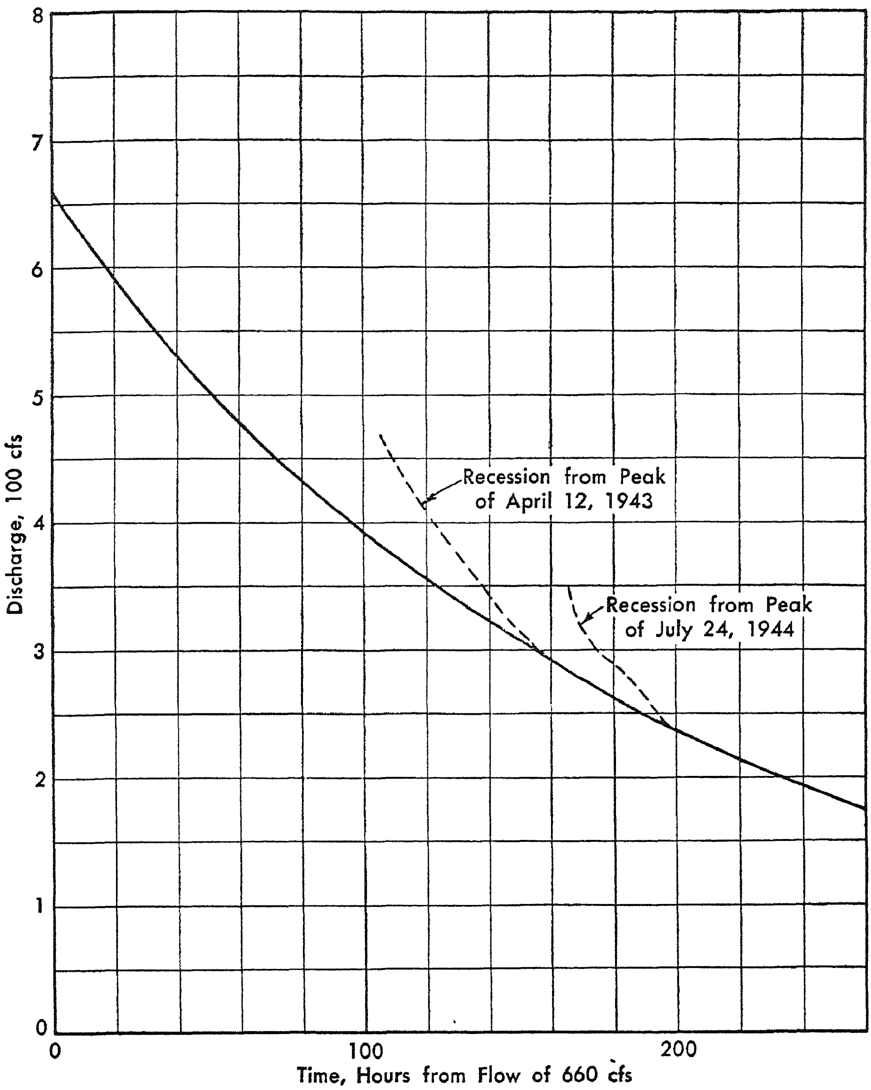



FIGURE 118. Recession Curve, Little Blue River, Endicott, Nebr.

(90) that where a drainage basin consists of many ground water sub-basins and declining flow may be represented better by the equation

$$q = q_o e^{-ct^n}$$

where the notation is the same as given above with $n$ as a constant exponent. Both of these equations can be derived from curves representing the ground water outflow with respect to time, or the ground water hydrograph.

The curve represented by the first of the above equations is a straight

line on arithmetic-logarithmic coordinate paper. The curve is readily derived from the portion of the plotted hydrographs continuing after the surface runoff has ceased. A number of such hydrographs should be available and should have a range of ground water outflow from the usual highest to the lowest. Portions of the ground water recession curves of the various hydrographs can be fitted together by sliding a master graph horizontally until the various portions fit together in a continuous line. This is a simple task if recorder charts are available. From this master graph the points of time and discharge can be picked off to plot for the recession curve on semilogarithmic coordinates, which is a desirable step in order to align and check the points that have been plotted. The final curve on rectangular coordinates with a suitable scale can then be taken from the semilogarithmic curve. A recession curve for the Little Blue River, Endicott, Nebr., is shown in Figure 118.

*Summary of Action of Rainfall to Runoff.* The transition of rainfall into runoff may now be summarized by the following sequence:

1. Upon falling, the rain first saturates the soil surface and vegetal covering, the losses of which may include interception by a forest or vegetal canopy.
2. Infiltration begins and continues through the storm period.
3. Initial detention fills the surface depressions.
4. Storage by surface detention is accumulated.
5. Runoff from land surface begins and subsequently the channel storage is filled.
6. Flow begins at the outlet or gaging station. By this time considerable water has accumulated by surface detention and channel storage. Surface detention increases as the rainfall increases and decreases as the latter diminishes. Channel storage and discharge increase until a crest is reached, which in the case of the larger drainage basins, may occur at a considerable interval after the cessation of rainfall.
7. After the crest, flow continues from surface detention and channel storage at a diminishing rate until the supply is exhausted and the flow is supplied from the ground water only.
8. Ground water flow continues throughout the period of rainfall and runoff. Prior to the rain it is dependent upon the recession from the preceding rain period, but as the water infiltrating during a rain reaches and raises the water table, the ground water flow increases until the supply from infiltration stops.

All the factors affecting runoff are variable. As has been shown heretofore, the occurrence of rainfall is extremely variable and from

that fact comes the variation in depth of surface detention. While storage in surface detention tends to make surface runoff more uniform, it eliminates only the minor changes which would otherwise result from the variable rates of rainfall. On a given plot of ground the ultimate rate of infiltration, upon reaching equilibrium with the conditions of the given storm, appears to be largely fixed by the type of soil, vegetal covering, and geological conditions which remain essentially constant for the duration of the storm. However, the initial rates of infiltration, being dependent to a large degree upon prior conditions of soil moisture, are variable and adjust themselves until equilibrium is established in the ultimate steady rate. The initial interception of precipitation, likewise being dependent upon vegetation and other ephemeral moisture conditions, is highly variable. In contrast with these variable factors the characteristics of the drainage basin, such as shape, tributaries, amount of channel storage, and lag of time between analogous points of a given rainfall and the resulting runoff, are all dependent mainly upon the fixed features of the watershed and are substantially constant under uniform rainfall. These definitely fixed physical features of a watershed which control rates of flow are the basis for defining storm runoff characteristics by what is called the "unit graph" or "distribution graph."

*The Unit Hydrograph.* The unit hydrograph is defined by Sherman (123) as "the hydrograph of surface runoff (not including ground water runoff) on a given basin due to effective rain falling for a unit of time." Effective rainfall is the portion that goes into surface runoff and is the total rainfall reaching the ground, less infiltration and such evaporation of the ground moisture as may occur. The unit of time to be used depends upon the area of the drainage basin, and in any case it should be less than the time elapsing from the occurrence of the rainfall to the time the crest of the flow is reached. It may be a day or less for moderate areas.

The concept of the unit hydrograph was introduced by Sherman (165) in 1932, and has since been developed into an efficient and valuable tool for analyzing runoff and computing the peak discharge and volume of floods where the only other data available are depths and distribution of rainfall.

The basic concept of the unit hydrograph is that, for a given watershed, the runoff from the rainfall of single storms occurring within the selected unit of time produces hydrographs of equal length in time, and of such a shape that the ordinates are proportional to the rainfall, and that homologous time increments are always the same proportion of the total runoff, regardless of the depths of rainfall and magnitude of

the resulting runoff. If rainfall continues through two or more unit periods of time, the hydrographs representing the runoff from each of the individual units may simply be added in the proper order of time corresponding to the unit periods to obtain the hydrograph of the total runoff. To correspond to the basic assumptions and in order to reproduce strictly proportional results, the rainfall should be evenly distributed over the basin and occur at a uniform rate throughout the unit period with uniform losses.

The unit hydrograph is based on the premise that the fixed physical characteristics of a given basin effectively control the rates of runoff. These characteristics, such as slope, distribution of tributaries, and topography, are for all practical purposes fixed and do control the runoff rates to a point within practical limits. Aside from the assumption of an ideal storm, there are some variable factors operating on runoff that tend to upset the balance, but they are not sufficient to preclude effective use of the unit hydrograph concept.

*Development of the Distribution Graph and Factors.* Bernard, (19) taking advantage of the proportionality of the unit hydrograph, introduced a feature that simplified its use. He computed the proportion of the total runoff that occurred in each unit period of surface runoff for floods that were produced by rainfall of unit time duration. When these proportions were found for a number of such storms they were averaged. These average values were designated "distribution factors" and from them the so-called "distribution graph" was constructed to show the proportionate daily distribution in per cent of the total runoff from a storm of unit time over the basin.

As originally developed by Sherman and Bernard, the unit hydrograph and distribution factors were based on the rainfall and subsequent runoff of a storm of a duration of one unit of time. The basic conditions for selecting the storms from which the factors should be derived are the same as for the unit hydrograph, namely: a rainfall uniform over the watershed and throughout the duration of the unit time, there should be a period of no rain immediately preceding and following the storm to be used, the storm runoff should not be interrupted by low temperatures or artificial storage, or mixed with water from melting snow or ice. Skill and good judgment are necessary for the selection of storms for deriving these factors. Later developments in the technique of distribution factors have provided methods of securing distribution factors from multiple-period storms which have some substantial advantages in that such factors are more likely to be based on storm conditions similar to those of serious floods. The discussion given hereinafter is based first upon the single

unit storms for simplicity in presenting the principles, and followed by a description of methods for deriving factors from multiple-unit storms.

The distribution graph or factors have been widely used in computing runoff from rainfall. In order to show how they have been developed, distribution graphs are prepared from hydrographs obtained at gaging stations on a number of streams. However, only a discussion of the elementary principles is given. The development of the distribution graph and unit hydrograph is still in progress and new and better techniques are being discovered from time to time, so it would be futile to attempt at this time to give a complete exposition of all methods of derivation and detailed applications of the distribution graph and factors.

*Procedure for Deriving Distribution Factors.* The first step in obtaining one-day distribution factors is the careful selection of the storms producing adequate runoff and high peak flows and those concentrated in essentially a one-unit period. In addition there should be negligible precipitation for several days before and after the period of the storm so that the base stream flow will have receded to approximately its normal discharge for the period. As many storms as can be obtained should be used; it is not likely with the usual length of record available that many suitable storms and high water periods will be found after the undesirable ones are eliminated. The individual sets of distribution factors desired from each of as many storms as are obtainable may then be averaged to give a set more generally applicable to other similar situations. The calculation of the mean value should be applied only for the purpose of averaging observational variations which occur in the determination of the peak flow of the runoff, the depths and particularly the distribution of intense rainfall, and all other factors entering into the determination of the surface runoff. Variations due to different types of storms are another matter and should not be averaged by statistical methods.

The second step is the plotting of the hydrograph of total runoff. In order to gain a better conception of the relation between rainfall and runoff, it is desirable to plot also on the same graph the rainfall, showing the period of the storm. The scales of both should be large enough to reduce drafting errors below limits allowable from the data.

The third step is to determine the base flow which is mainly from ground water. Frequently this can be done by simple inspection of the hydrograph, particularly where the floods are of sufficient size to minimize errors in the assumptions regarding base flow. For more accurate work or in more intricate storm situations, the base flow can

TABLE 64. DISTRIBUTION FACTORS, JAMES RIVER, BUCHANAN, VA.

| DATE | TOTAL(a) FLOW RECORDED | BASE FLOW dsf | DEDUCT FOR OTHER RAIN-FALL, dsf | SURFACE RUNOFF dsf | DISTRIBUTION FACTOR Per Cent | Σ OF FACTORS Per Cent |
|---|---|---|---|---|---|---|
| 1/16/24 | 2,080 | 2,080 | none | 0 | .... | .... |
| 1/17/24 | 31,000 | 2,200 | | 28,800 | 55.3 | 55.3 |
| 1/18/24 | 15,200 | 2,300 | | 12,900 | 24.8 | 80.1 |
| 1/19/24 | 8,740 | 2,400 | | 6,340 | 12.2 | 92.3 |
| 1/20/24 | 5,370 | 2,550 | | 2,820 | 5.4 | 97.7 |
| 1/21/24 | 3,900 | 2,700 | | 1,200 | 2.3 | 100.0 |
| 1/22/24 | 2,830 | 2,830 | | 0 | 0.0 | .... |
| Totals | 69,120 | 17,150 | | 52,060 | 100.0 | .... |
| 11/15/26 | 1,090 | 1,090 | .... | 0 | | |
| 11/16/26* | 7,000 | 1,200 | .... | 5,800 | 9.9 | 9.9 |
| 11/17/26 | 31,400 | 1,400 | .... | 30,000 | 51.2 | 61.1 |
| 11/18/26 | 12,300 | 1,600 | .... | 10,700 | 18.3 | 79.4 |
| 11/19/26 | 7,810 | 1,750 | 0 | 6,060 | 10.3 | 89.7 |
| 11/20/26 | 7,570 | 1,900 | 2,300 | 3,370 | 5.7 | 95.4 |
| 11/21/26 | 6,650 | 2,050 | 2,900 | 1,700 | 2.9 | 98.3 |
| 11/22/26 | 5,580 | 2,200 | 2,400 | 980 | 1.7 | 100.0 |
| 11/23/26 | 4,380 | 2,300 | 2,080 | 0 | .... | .... |
| 11/24/26 | 2,900 | 2,300 | 600 | 0 | .... | .... |
| Totals | 86,680 | 17,790 | 10,280 | 58,610 | 100.0 | |
| 10/21/29 | 641 | 641 | none | 0 | .... | .... |
| 10/22/29* | 9,240 | 740 | | 8,500 | 20.7 | 20.7 |
| 10/23/29 | 19,200 | 850 | | 18,350 | 44.8 | 65.5 |
| 10/24/29 | 8,190 | 1,000 | | 7,190 | 17.5 | 83.0 |
| 10/25/29 | 4,580 | 1,100 | | 3,480 | 8.5 | 91.5 |
| 10/26/29 | 3,170 | 1,200 | | 1,970 | 4.8 | 96.3 |
| 10/27/29 | 2,400 | 1,350 | | 1,050 | 2.6 | 98.9 |
| 10/28/29 | 1,950 | 1,500 | | 450 | 1.1 | 100.0 |
| 10/29/29 | 1,680 | 1,680 | | 0 | 0 | .... |
| Totals | 51,051 | 10,061 | | 40,990 | 100.0 | .... |
| 4/30/32 | 1,260 | 1,260 | none | 0 | .... | .... |
| 5/1/32* | 6,900 | 1,400 | | 5,500 | 14.0 | 14.0 |
| 5/2/32 | 20,000 | 1,500 | | 18,500 | 47.2 | 61.2 |
| 5/3/32 | 8,790 | 1,600 | | 7,190 | 18.4 | 79.6 |
| 5/4/32 | 5,480 | 1,700 | | 3,780 | 9.6 | 89.2 |
| 5/5/32 | 4,040 | 1,800 | | 2,240 | 5.7 | 94.9 |
| 5/6/32 | 3,280 | 1,950 | | 1,330 | 3.4 | 98.3 |
| 5/7/32 | 2,780 | 2,100 | | 680 | 1.7 | 100.00 |
| 5/8/32 | 2,320 | 2,320 | | 0 | .... | .... |
| Totals | 54,850 | 15,630 | | 39,220 | 100.0 | .... |
| 11/18/32 | 1,650 | 1,650 | none | 0 | | |
| 11/19/32* | 3,170 | 1,700 | | 1,470 | 6.8 | 6.8 |
| 11/20/32 | 10,900 | 1,800 | | 9,100 | 42.2 | 49.0 |
| 11/21/32 | 7,770 | 1,900 | | 5,870 | 27.2 | 76.2 |
| 11/22/32 | 4,880 | 2,000 | | 2,880 | 13.4 | 89.6 |
| 11/23/32 | 3,620 | 2,100 | | 1,520 | 7.1 | 96.7 |
| 11/24/32 | 2,840 | 2,200 | | 640 | 3.0 | 99.7 |
| 11/25/32 | 2,370 | 2,300 | | 70 | 0.3 | 100.0 |
| Totals | 37,200 | 15,650 | | 21,550 | 100.0 | .... |

* Rain on this date.

(a) Units of volume of runoff are taken as "day-second-feet," (abbreviated dsf), that is, the volume accumulated from flow at the rate of one cubic foot per second flowing for one day. This unit is used for convenience in the computation; it equals 1.9835 acre-feet, but is commonly taken as 2 acre-feet.

be delineated by means of a recession curve plotted to the same scale as the hydrograph.

The fourth step is determination of the hydrograph of the surface runoff used in the actual computation of the distribution values. It is of interest to note that such a hydrograph produced by a storm of unit period is the unit hydrograph expressed in units of discharge, and is directly convertible to the distribution graph expressed in percentage. The subsequent steps are matters of computation and are best shown by illustration.

*Distribution Factors for James River, Buchanan, Virginia.* The derivation of a set of distribution factors for the James River at Buchanan, Va., with a drainage area of 2,080 square miles, is shown in Table 64 and Figures 119, 120, and 121.

Although the computations in Table 64 are relatively simple, a brief description of them is not out of place. The first two columns, "Date" and "Total Flow Recorded" are obtained directly from the published data of the *Water Supply Papers,* U. S. Geological Survey. The values of the third column, "Base Flow, dsf" are the estimated discharges of the ground water flow, and are taken from the hydrographs in Figures 119, 120, and 121. The ground water discharge, together with the deductions for other rainfall, are subtracted from the total flow to give the surface flow in the fifth column. The total surface runoff is then divided into the daily surface flow to give the distribution factors. The last column, "$\Sigma$ of Factors" is obtained by summing the daily distribution factors from the beginning of the storm to each successive day.

Diagrammatic hydrographs of the total flow given in the second column of Table 64 are shown as histograms in the Figures 119, 120, and 121. These hydrographs were plotted primarily for the purpose of determining the ground water and the recession of the runoff from a subsequent rain as shown for one hydrograph, Figure 119. Since the distribution factors are computed from the data in the table, actual shapes of the hydrographs are not required. The selected storms were all of one-day rains, so that no difficulty was met in determining the base flow except for the storm of November 1926; for the latter type of storm, which is complicated by the runoff from other storms, it is essential that the recession curve be drawn.

The sets of distribution factors for all floods shown in Figures 119, 120, and 121 were averaged by means of the summation curves of daily values shown in Figure 121. This procedure was adopted to obtain the averages of the observational fluctuations, and in deriving the means the greatest weight was given to the mid-portions between approximately the points of 20 and 80 per cent on the curves. Where circum-

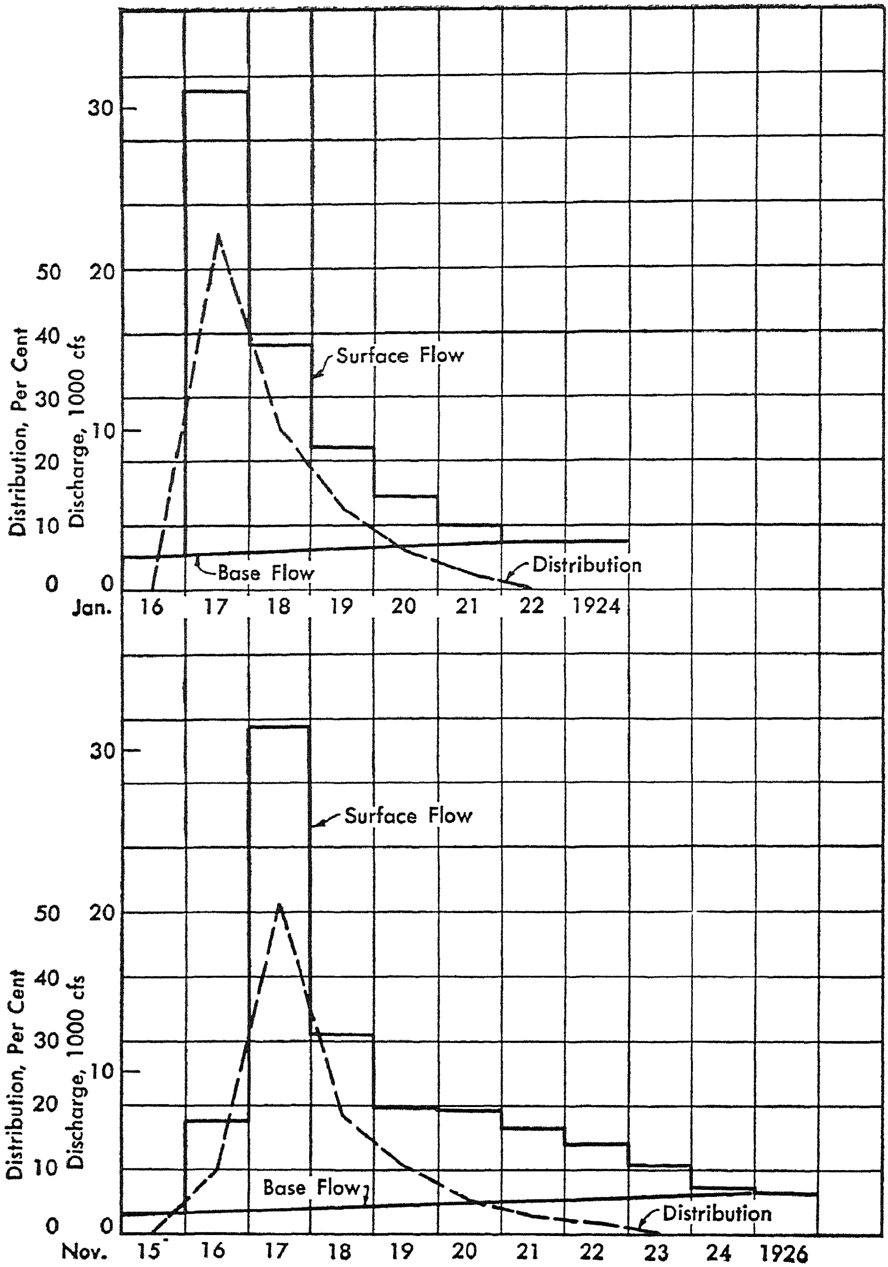



FIGURE 119. Hydrographs, James River, Buchanan, Va.

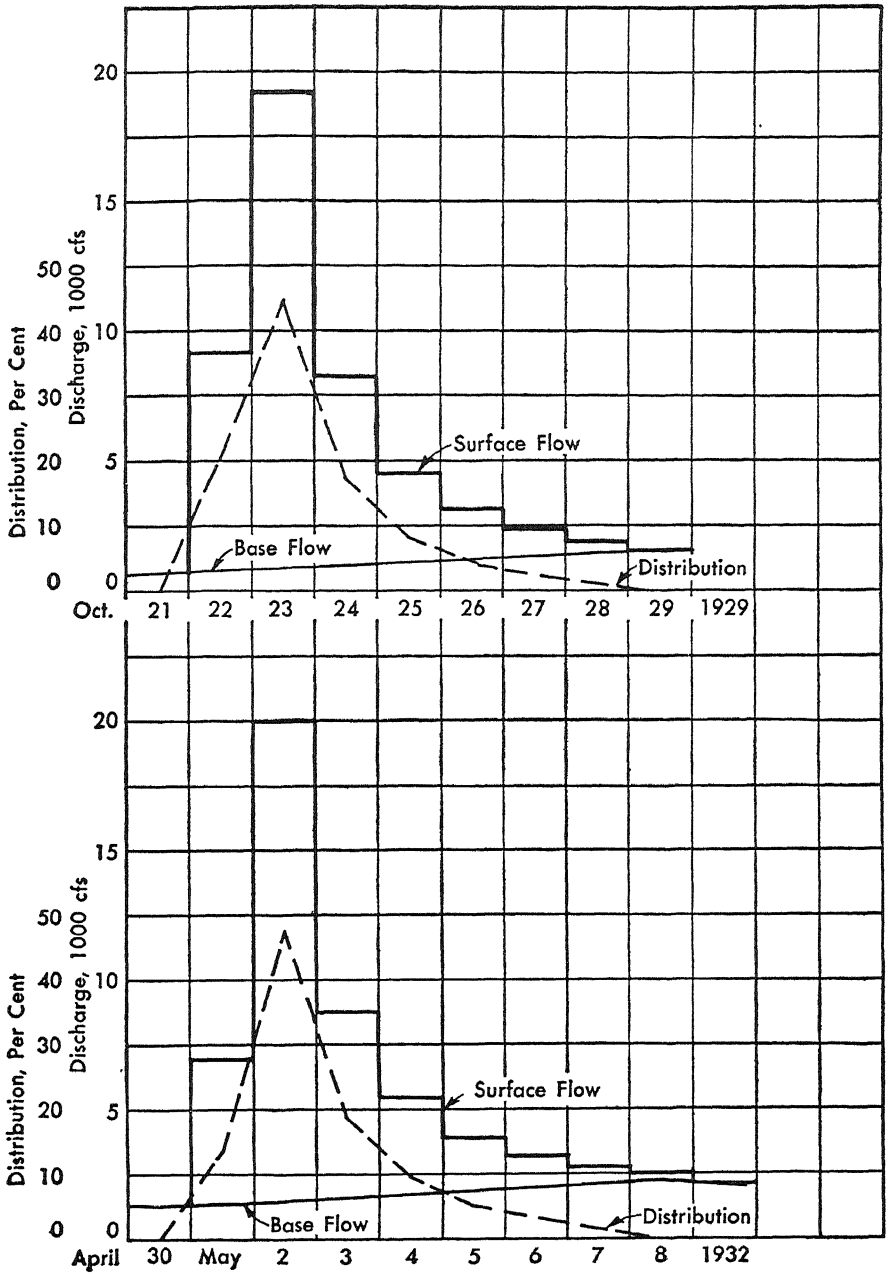



FIGURE 120. Hydrographs, James River, Buchanan, Va.

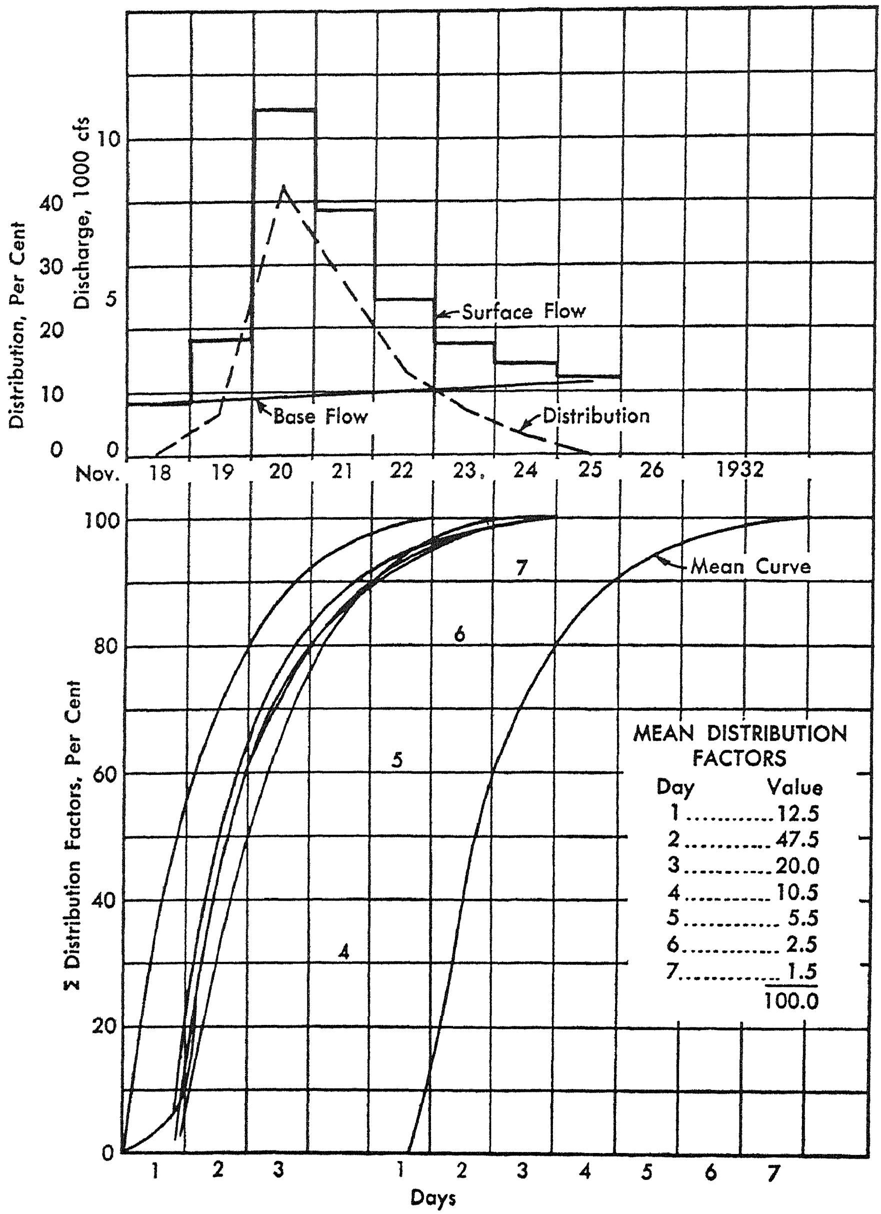



FIGURE 121. Distribution Factors, James River, Buchanan, Va.

stances of timing of a hydrograph with respect to the occurrence of rainfall indicates that the arbitrary one-day periods of observation have upset the most probable sequence of time, it can be moved to the right or left before being averaged in the group. A suitable group of summation distribution graphs should present reasonably good conformity to a common shape since there should be only observational errors to be eliminated by averaging. In the lower and central portions of the curves the average time from an arbitrary origin should be found. The upper portion, say 10 or 15 per cent, can be averaged on the ordinates since this part represents the final portion of the actual hydrograph, and equal ordinate values may be subject to relatively large errors in time.

The foregoing procedure has several advantages that justify its use. The problem of making the individual peaks of the hydrographs coincide in time is eliminated entirely, and the inaccuracy inherent in the use of one-day values of flow is reduced. It is easier to compare visually the summation curves that the distribution graphs when a number are superimposed on the same coordinates. The values of time for corresponding percentages are more easily averaged without distorting peak values, and if necessary, it is easier to adjust the summation curves for time of occurrence of the distribution graphs.

Figure 121 shows the five individual summation curves of the distribution graphs and the mean curve derived from them. From the latter curve the mean daily values of the distribution factors were obtained as shown in Table 65.

TABLE 65. MEAN DAILY DISTRIBUTION FACTORS, JAMES RIVER, BUCHANAN, VA.

| DAY | Σ OF DAILY VALUES *Per Cent* | DAILY FACTORS *Per Cent* |
|---|---|---|
| 1 | 12.5 | 12.5 |
| 2 | 60.0 | 47.5 |
| 3 | 80.0 | 20.0 |
| 4 | 90.5 | 10.5 |
| 5 | 96.0 | 5.5 |
| 6 | 98.5 | 2.5 |
| 7 | 100.0 | 1.5 |
| | Total | 100.00 |

In accordance with the convention used heretofore, the daily distributions are plotted at the mid-point of the day. The summation values, on the other hand, are plotted at the end of the day, since they represent the totals up to that time.

It may appear that distribution factors for periods shorter than one day could be derived from the cumulative mean curve in Figure 121.

Such values obtained for fractional-day periods would not be correct because it would not be possible to reproduce from them the original hydrograph, which reproduction is the criterion of correctness of distribution factors. Since the mean cumulative curve is derived from flow of one-day storms, it can be used only for one-day factors. In accordance with the theory of the unit hydrograph, the distribution factors must correspond to the same period of time as the rain causing the runoff. The reason for this will be made clear later in this chapter.

The daily distribution factors for the James River at Buchanan, Va., were derived entirely from data of daily discharge as published in the *Water Supply Papers* of the U. S. Geological Survey, with little reference to the rain other than the length and date of the storm. The relating of the time of the runoff to the rainfall causing it is another problem that must be solved before the distribution factors can be used in the synthesis of a flood hydrograph.

Since runoff bears a definite relationship to the causative rainfall, the daily distribution factors must be correlated with the rain when they are used to compute a flood hydrograph based on this rain. For that reason the relationship between the runoff factors and the corresponding rainfall causing the runoff must be determined. Where sufficient accurate data from recording stream and rain gages are available, it is not difficult to determine that relationship. Where only daily records of stream flow and rain are available, this basic relationship is not so easily determined. In nature, rain practically never falls for exactly one calendar or recording day, or through a day that coincides with the proper period to produce runoff through a day for the flow record. Adjustments are necessary so that there can be obtained a good average relationship between observed rainfall and runoff.

*Distribution Factors, Delaware River.* To illustrate a procedure to use where daily records of rainfall and runoff only are available, and to demonstrate the general principle of correlating rainfall and runoff, two sets of distribution factors are computed for the Delaware River, Port Jervis, N. Y., from data given in *Water Supply Paper No. 772*, (94). The rainfall reported for each of the two storms was observed at nine stations three of which measured the 24-hour catch in the morning, five, in the afternoon, and one regular Weather Bureau station for which the rainfall is reported for the period from midnight to midnight. The stations, with amounts of precipitation and their overlapping periods of reading, are shown in Figures 122 and 123; the times of readings are taken to be 7:00 A.M., 7:00 P.M., and midnight. On the basis of the distributions shown in Figure 122, the precipitation of the storm of September 28 to October 1, 1924, was taken mainly from the

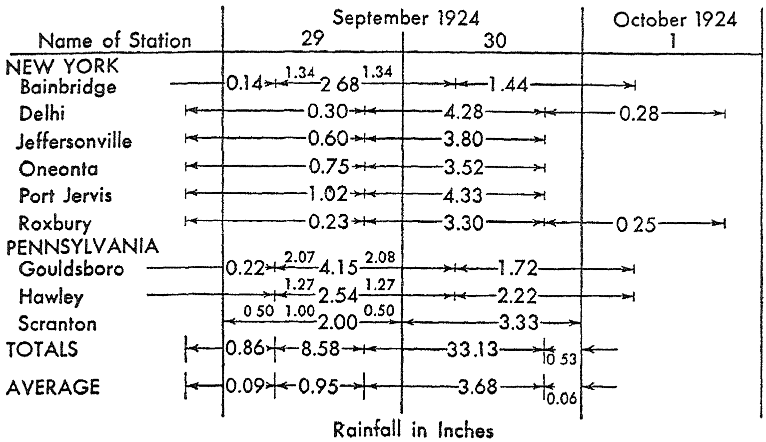
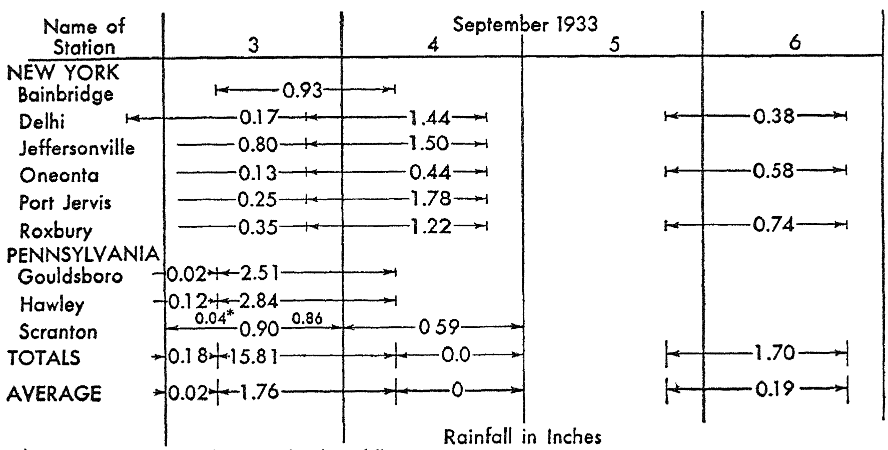



FIGURE 122. Rainfall on the Delaware River Basin, above Port Jervis, N. Y.




*0.04 inch out of 0.90 inch estimated to have fallen between midnight and 7:00 A. M.

FIGURE 123. Rainfall on the Delaware River Basin, above Port Jervis, N. Y.

catches of Jeffersonville, Oneonta, and Port Jervis, as lasting from 7:00 P.M. September 28 to midnight September 30, a total of 53 hours instead of the four days on which precipitation was recorded. Apparently the bulk of the rain fell in the night, from 7:00 P.M. September 29 to 7:00 P.M. September 30. In similar manner, from the distribution shown in Figure 123, rainfall was taken to last from midnight September 2, to 7 A.M. September 4, a total of 31 hours, since stations reading at 7:00 A.M. reported no rain after 7:00 A.M. September 4. The distribution is reasonably definite. The small showers on September 6 are not effective.

The hydrographs and distribution graphs which were computed

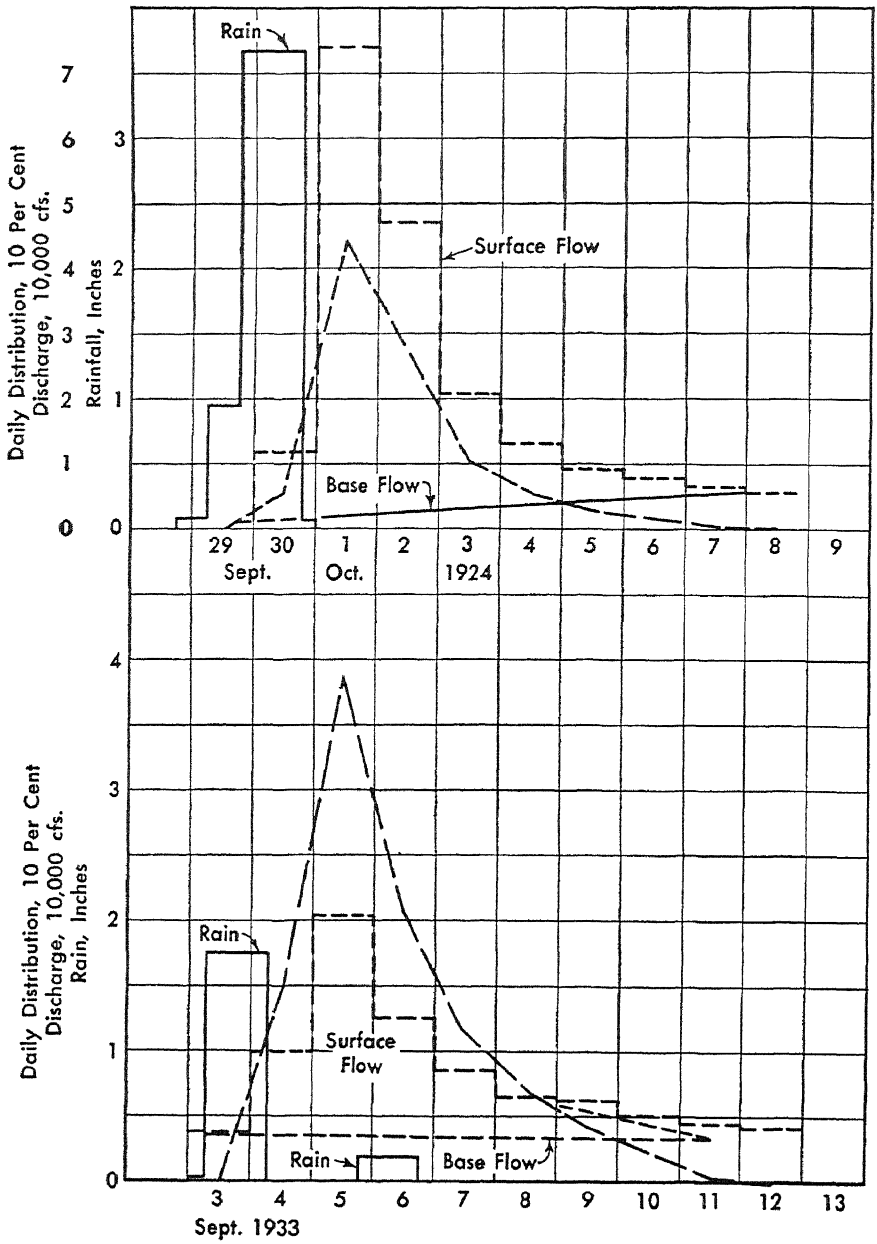



FIGURE 124. Distribution Graphs, Delaware River, Port Jervis, N. Y.

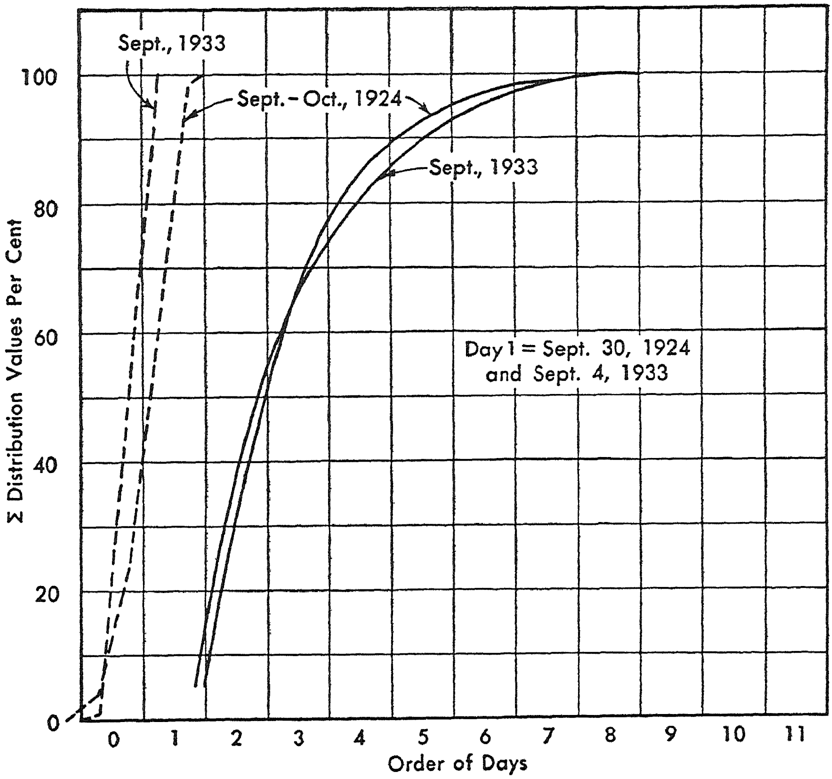



FIGURE 125. Accumulative Distribution Valves, Delaware River, Port Jervis, N. Y.

exactly as were those for the James River above, are shown in Figure 124. The accumulative graphs of rainfall and runoff are shown in Figure 125. The rainfall data derived on Figures 122 and 123 were then used to plot the cumulative rainfall curves. The rainfall was accumulated and plotted in accordance with the periods given for average rainfall. These graphs show an average lapse of time, or lag, of 47.4 hours between the rainfall and runoff at the points in time when 50 per cent of the rainfall and 50 per cent of the runoff occurred.

When greater accuracy is desired, or when units of time of a few hours to one-half day are required, it is necessary to work from the charts of recording gages both for rainfall and runoff, or from readings taken at sufficiently frequent intervals to define accurately the pluviograph and hydrograph. The recorder charts for runoff hydrographs eliminate all uncertainties from the stream flow except minor instrumental errors. When stream flow data of this sort are available, the principal source of uncertainty that remains is in the rainfall and its areal distribution.

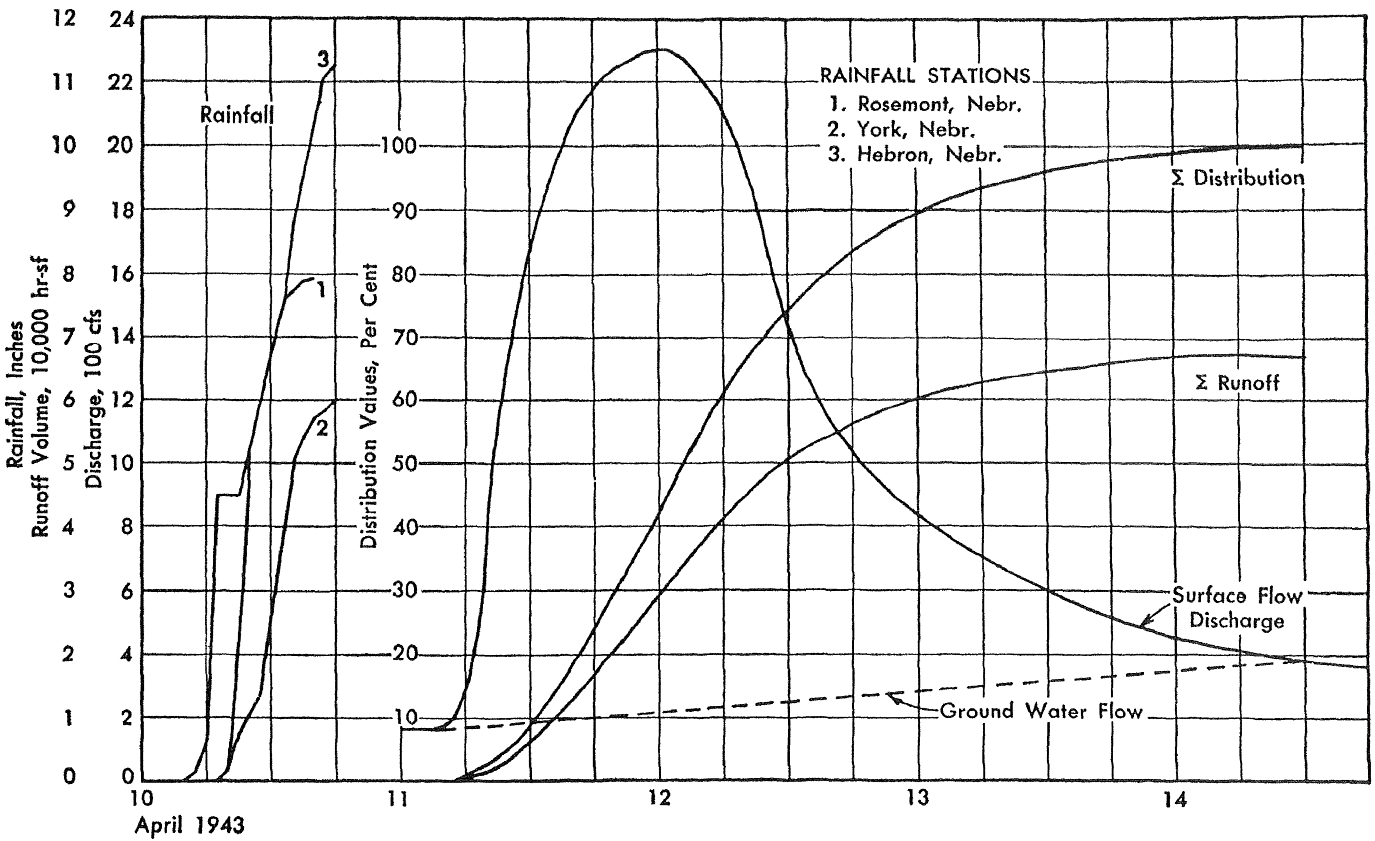



FIGURE 126. Hydrograph and Distribution Graph, Little Blue River, Endicott, Nebr.

*Distribution Factors, Blue River.* To illustrate a method of deriving distribution factors directly from recorder charts, such factors were computed from two rises on the Little Blue River at the recording station near Endicott, Nebr. The rainfall, hydrographs, and accumulative factors are shown on Figures 126 and 127, all plotted in accordance with time of occurrence. Although the rainfall over the basin was rather spotty during the two storms, as is frequently the case in the subhumid climate of the Blue River basin, the hydrographs are satisfactory.

The distribution factors are derived by the following steps:

1. From the recorder charts, take the stage at suitable intervals to define the hydrograph, determine the corresponding discharge from a rating table, and plot the hydrograph.
2. Delineate the ground water flow, preferably by using a ground water recession curve, and delineate the recession of runoff from other storms if they affect the hydrograph under consideration.
3. Compute the accumulative surface runoff from the discharge hydrograph and plot the results in proper sequence of time. The accumulative runoff shown on Figures 126 and 127 was computed for hourly values in units of hour-second-feet; the hour-second-foot is the unit volume of water produced by a flow of one cubic foot per second for one hour.
4. Determine the period of rain which caused the surface runoff. The period of time for which the distribution factors are computed is determined by the length of effective rainfall.
5. Using the length of the effective rainfall as the period, compute the accumulative distribution factors, similar to those shown in Figure 121.
6. Compute the individual distribution factors for each successive period of time by subtracting the value of the first, then the second, third, and each successive accumulative value from the next following. The forms of computations and distribution factors computed for two storms on the Little Blue River are given in Table 66.

The effective rainfall of April 1943 was taken to be of 12 hours duration as shown in Figure 126. Therefore the accumulative factors were computed for unit time of 12-hour intervals beginning at 4:00 P.M., April 11, 1943, dividing the value on the ordinate of the "$\sum$ Runoff" curve by the total runoff, 67,000 hour-second-feet. The ratios so obtained were used to plot the curve for "$\sum$ Distribution." By subtracting each 12-hour value of the latter curve from the next following point, the individual distribution factors were obtained as given in Part B, Table 66. The rain of this storm was fairly well centered over

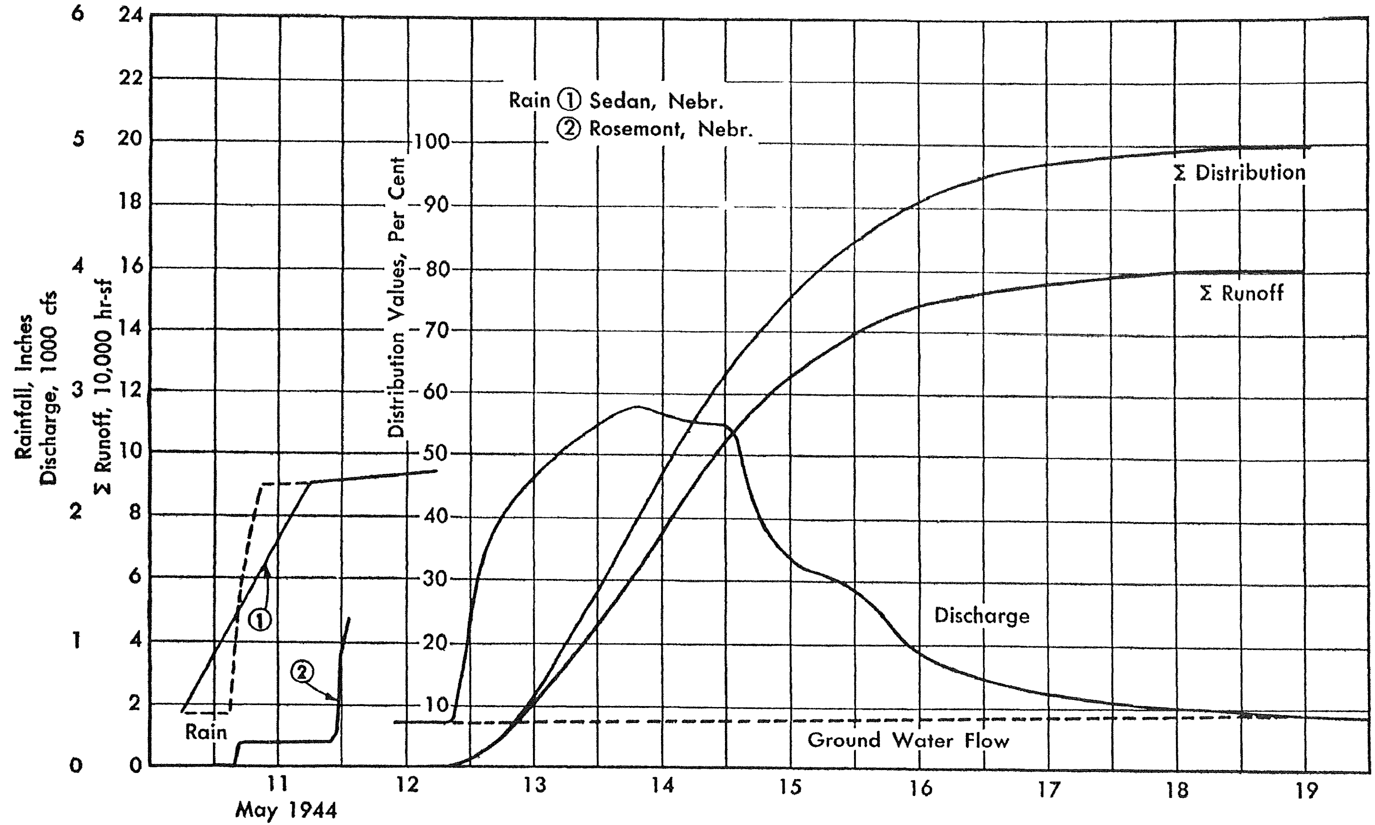



FIGURE 127. Hydrograph and Distribution Graph, Little Blue River, Endicott, Nebr.

TABLE 66. DISTRIBUTION FACTORS, LITTLE BLUE RIVER

| DATE APRIL Day | Hour | SURFACE DISCHARGE 100 csf Hour | Mean | Σ SURFACE RUNOFF *100 Hr-sf* | Σ DISTRIBUTION FACTORS *Per Cent* | 12-HOUR DISTRIBUTION FACTORS *Per Cent* |
|---|---|---|---|---|---|---|
| PART A: COMPUTATION FORM | | | | | | |
| 11 | 4 P | 0 | 0 | 0 | .... | .... |
| | 5 | .4 | 0.2 | 0.2 | .... | .... |
| | 6 | 1.2 | 0.8 | 1.0 | .... | .... |
| | | And so forth | | | | |
| 12 | 4 A | 18.6 | 18.25 | 119.25 | 17.5 | 17.5 |
| | | And so forth | | | | |
| 12 | 4 P | 20.0 | 20.2 | 362.45 | 53.3 | 35.8 |
| | | And so forth | | | | |

PART B: DISTRIBUTION FACTORS OF TWO STORMS

| STORM DATE | PERIOD OF FACTOR | DISTRIBUTION FACTORS, PER CENT, FOR UNIT PERIODS *1st* | *2nd* | *3rd* | *4th* | *5th* | *6th* | *7th* |
|---|---|---|---|---|---|---|---|---|
| April 1943 | 12 hours | 17.5 | 35.8 | 25.9 | 11.0 | 5.8 | 3.5 | 0.5 |
| May 1944 | 24 hours | 23.1 | 38.4 | 23.2 | 9.6 | 3.7 | 1.5 | 0.5 |

the basin as shown by the rain-summation curves (1) and (3), although there was a lapse of a few hours between times of the fall of equal proportions of the total rain. This rain was as uniformly distributed over the basin as is likely to be found for short storms. This hydrograph may be accepted as satisfactory for the derivation of distribution factors.

The distribution factors for the storm of May 1944 were computed in the same manner as those of the preceding storm, except that 24-hour periods were used beginning at 8:00 P.M. on May 12, since the rain fell at Rosemont from 3:00 A.M., May 11, to 1:00 A.M., May 12, with a likelihood of small amounts following the latter hour at Sedan. Although fairly well centered on the basin, this rain was plainly somewhat irregular as to time of occurrence as shown by the cumulative record of the rain at Rosemont. The station at Sedan is non-recording so that the dotted portion of the line had to be drawn in on the basis of the Rosemont record. The effect of the irregularity of occurrence of the rainfall is plainly discernible on the hydrograph. This storm is not so satisfactory as the preceding one because the peak value was delayed, that is, shown at a later period than it should have been, but the set of distribution factors can be accepted with some adjustment on the basis of those of the storm of April, 1943.

*Distribution Factors for a Small Plat.* The runoff of small plats may be computed by means of distribution factors as well as that of the larger drainage basins. Data were obtained from the records of the Soil

Conservation Service to derive distribution factors for an area of 1.03 acres, located at the experiment station at Hays, Kansas. The plat was approximately 65 feet wide and 700 long and had a slope of 5 to 6 per cent. The two hydrographs and summation distribution graphs, which were computed for 10-minute periods on the basis of rainfall of like time, are shown in Figure 128. The individual distribution factors and the averages are listed below:

| PERIOD NUMBER | June 18, 1932 | June 25, 1932 | AVERAGE |
|---|---|---|---|
| 1 | 64.2 | 52.2 | 58.2 |
| 2 | 25.5 | 39.0 | 32.2 |
| 3 | 8.5 | 7.8 | 8.2 |
| 4 | 1.8 | 1.0 | 1.4 |
| Totals | 100.0 | 100.0 | 100.0 |

*Distribution Factors from Multiple-Period Storms.* Although in some of the foregoing examples distribution factors have been computed on the basis of unit time of one day, it frequently happens that such factors are wanted on the basis of a half or smaller portion of a day. Since rain does not fall to fit desired units of time, it is usually difficult to find storms of the desired length, and many storms may have to be discarded because they cover too long a period of time for single unit distribution factors. Furthermore, it is probable that the rainfall of multiple-period storms would be more uniformly distributed over the watershed, and thus provide more accurate distribution factors. Therefore it is desirable to have some means of readily dividing or breaking down distribution factors of a longer period into factors for a smaller unit of time, or deriving distribution factors from the more frequent multiple-period storms.

Before proceeding with the analysis, a restatement of the theory of the distribution factors and unit hydrograph is given to form the basis of the method of deriving distribution factors from rainfall of more than one unit of time. The unit hydrograph is formed by the runoff from one unit of effective rain falling on the whole basin; each unit of rain produces the same total volume of runoff directly proportional to its magnitude and in the same proportions through corresponding units of time. From the last condition are derived the distribution factors. Then if one unit of effective rain falls on the basin in the first unit of time, it alone will produce the first unit portion of the runoff. If another unit rain occurs, the second time-unit of runoff will consist of the second portion of runoff from the first unit rain and the first portion of runoff of the second unit rain. This process will be repeated until the surface runoff is gone, each day's runoff consisting of portions

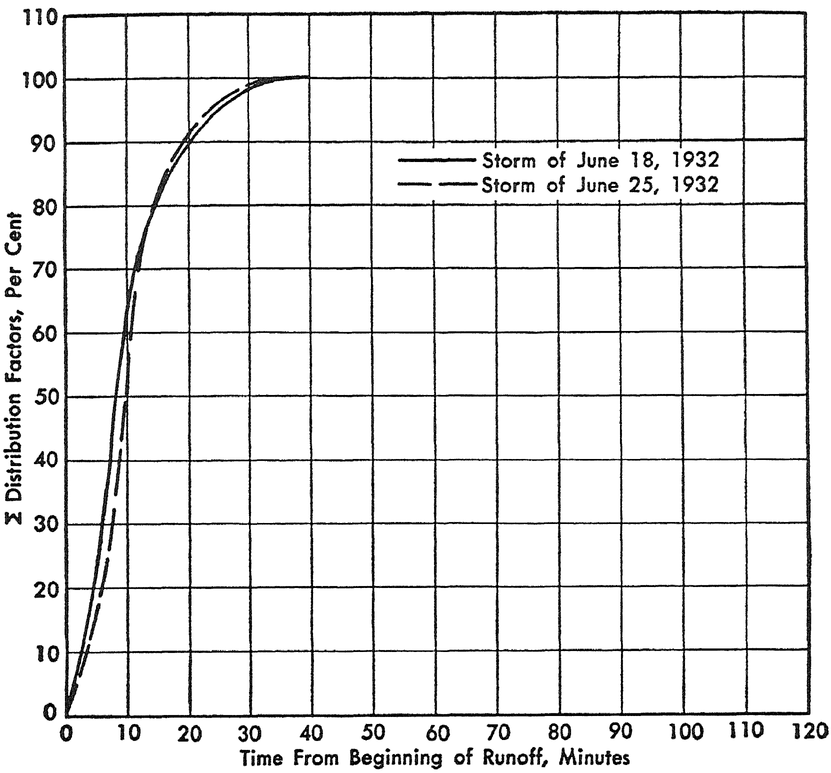



FIGURE 128. Distribution Factors from a Small Area, 1.03 Acres

of runoff from the two units of rain. If more rain occurs there will be more portions of runoff, each from a separate unit of rain.

By taking advantage of the foregoing situation it is possible to derive distribution factors for a unit of time shorter than the period of rainfall, from a mean cumulative curve such as shown on Figure 121. The accuracy of such factors, like all others, will depend upon the availability and accuracy of the data of rainfall and runoff. Particular attention should be given to obtain the best data for deriving such factors, and it is important to fix the time of the beginning of runoff as accurately as possible. Such distribution factors can be computed from the formulas given below, which are set up for deriving unit distribution factors from a 3-unit rainfall such as shown on Figure 129.

Let $r_1$ = the effective rain on first small unit of time for which the new distribution factors are wanted.

$r_2$ = the effective rain on the second period.

$r_3$ = the effective rain on the third period.

Let $R$ = the total effective rain, which is that portion that goes to make surface runoff. The hydrograph should be plotted, the base flow should be deducted and the losses of rainfall by infiltration and evaporation accounted for so that the volume of total effective rainfall equals the volume of runoff.

Then let $v_1$ = the total flood runoff on the 1st time unit

$v_2$ = the total runoff on the 2nd time unit

$v_n$ = the total runoff from the $n$th time unit

and $V$ = the total runoff; this equals the total effective rainfall by definition. The $r$'s and $v$'s are known values, since they are obtained from records.

Let $D_1$ = the proportion of the total flood runoff occurring in the 1st time unit

$D_2$ = the proportion of the total flood runoff occurring in the 2nd time unit

$D_n$ = the proportion of the total flood runoff occurring in the $n$th time unit.

The $D$'s can be computed from the hydrograph of the storm runoff. Let $d_1, d_2 \cdots, d_n$ be the proportions of runoff occurring on the 1st, 2nd, $\cdots$ $n$th units of time of runoff respectively, from one time-unit of rain. These $d$'s are the distribution factors as defined heretofore, and are the unknown values to be found.

Then $$D_1 = \frac{v_1}{V} = \frac{v_1}{R}, \text{ and hence } v_1 = D_1R.$$

But in this instance all the runoff comes from the first unit of rain, therefore

$$v_1 = d_1r_1.$$

Substituting the value $D_1R_1$ for $v_1$, and transposing

$$d_1 = \frac{D_1R}{r_1} = \text{first distribution factor;}$$

then $$D_2 = \frac{v_2}{V} = \frac{v_2}{R} = \frac{d_1r_2 + d_2r_1}{R}$$

from which, $$d_2 = \frac{RD_2 - d_1r_2}{r_1} = \text{second unit distribution factor;}$$

$$D_3 = \frac{v_3}{V} = \frac{v_3}{R} = \frac{d_1r_3 + d_2r_2 + d_3r_1}{R}$$

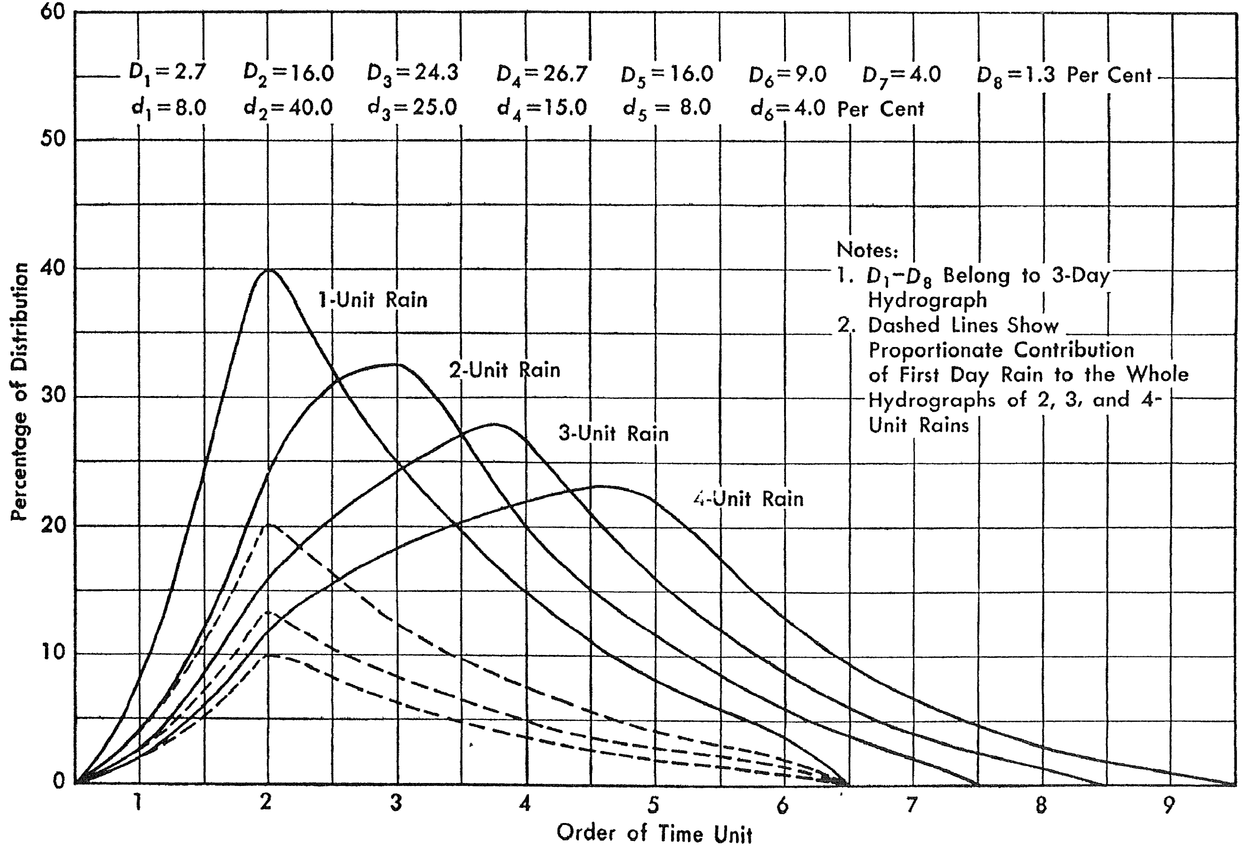



FIGURE 129. Hypothetical Distribution Graph

from which, $$d_3 = \frac{RD_3 - d_1r_3 - d_2r_2}{r_1} = \text{third unit distribution factor;}$$

$$D_4 = \frac{v_4}{V} = \frac{v_4}{R} = \frac{d_2r_3 + d_3r_2 + d_4r_1}{R}$$

from which, $$d_4 = \frac{RD_4 - d_2r_3 - d_3r_2}{r_1} = \text{fourth unit distribution factor;}$$

$$D_5 = \frac{v_5}{V} = \frac{v_5}{R} = \frac{d_3r_3 + d_4r_2 + d_5r_1}{R}$$

from which, $$d_5 = \frac{RD_5 - d_3r_3 - d_4r_2}{r_1} = \text{fifth unit distribution factor;}$$

$$D_6 = \frac{v_6}{V} = \frac{v_6}{R} = \frac{d_4r_3 + d_5r_2 + d_6r_1}{R}$$

from which, $$d_6 = \frac{RD_6 - d_4r_3 - d_5r_2}{} = \text{sixth unit distribution factor.}$$

It is assumed here for purposes of illustration that there are only six periods of runoff from a one period rain as shown in Figure 129; in this case if one more distribution factor, $d_7$, is computed, it should be zero. However, when working with actual field data, some adjustment will probably be needed to smooth out the factors, which may be done readily by constructing a new cumulative curve of the distribution factors of the reduced periods of time.

*Distribution Factors by the Method of Least Squares.* Distribution graphs may be derived from storms of two or more time units by using the method of least squares. The same assumptions and procedures are used to develop the basic data of effective rainfall and flood hydrograph as previously described. From the hydrograph of the flood runoff the total number of time units of surface runoff is then obtained. Furthermore, the number of unit distribution factors is known from the theory of the unit hydrograph to be $m - (n - 1)$ where $m$ is the number of time units of surface runoff from a given storm, and $n$ the number of time units of rain.

For illustrating this use of the method of least squares, assume a two-day storm with effective rain of $r_1$ and $r_2$ inches on the first and second days respectively, and that the resulting surface runoff lasts six days. The number of daily distribution factors is $6 - (2 - 1) = 5$.

Let $d_1$, $d_2$, $d_3$, $d_4$, and $d_5$ be the five successive distribution factors. The observation equations can be set up as in Table 67.

TABLE 67. OBSERVATION EQUATIONS FOR DISTRIBUTION FACTORS BY LEAST SQUARES

| ORDER OF DAYS | RAIN | OBSERVATION EQUATIONS | DAILY RUNOFF |
|---|---|---|---|
| 1 | $r_1$ | $r_1d_1$ | $= q_1$ |
| 2 | $r_2$ | $r_2d_1 + r_1d_2$ | $= q_2$ |
| 3 | | $r_2d_2 + r_1d_3$ | $= q_3$ |
| 4 | | $r_2d_3 + r_1d_4$ | $= q_4$ |
| 5 | | $r_2d_4 + r_1d_5$ | $= q_5$ |
| 6 | | $r_2d_5$ | $= q_6$ |

From the above observation equations, the normal equations can be derived by any standard method. Merriman (132) sets up the method of weighting the individual observation equations and totaling the coefficients. Lipka (116) and Arkin and Colton (9) illustrate a tabular method of achieving the same results.

The normal equation for each unknown, that is, each distribution factor, in the above observation equations can be obtained readily by multiplying each observation equation by the coefficient of the unknown factor (the $d$'s) and adding all coefficients to give the coefficient of the normal equation. The process would be repeated for each of the five normal equations.

In the above example the same results probably could be derived more readily by using the method previously given, since $d_1$ could be computed from the first observation equation from known quantities and substituted in the second to obtain $d_2$. However, if rain occurred on several days it would probably be advisable to use the method of least squares to derive the distribution factors in order to obtain the best values. One disadvantage of deriving distribution factors by the method of least squares is the large amount of work involved in solving the normal equations.

The distribution factors of the last portion of the hydrograph computed by the method of least squares are likely to be irregular because of inaccuracies of the original data, particularly the rainfall, and the deduction of the base flow and infiltration and other losses. Some adjustment usually is needed to produce smooth, consistent hydrographs from distribution factors derived by this method.*

*Variations in Distribution Factors.* Distribution values for any given river basin or gaging station are subject to considerable variation not attributable to the fixed basin characteristics. There are first the observational errors of the data of stream flow and precipitation, such as are inherent in all measurements. This type of error is accidental in any carefully operated gaging station; the discharge hydrograph at a

* The originator of this method is not known.

first-class recording station is indeed the most accurate of all data involved, so that, except for uncertainties of the magnitudes of the peaks at stations which are read only once or twice a day, the variation from observational errors at good gaging stations may be considered negligible in comparison with other sources of error. On the other hand, the equally important data of rainfall on the basin are subject to greater observational errors because rainfall observations are made at points and must serve as samples of the areal catch of the basin.

However, bigger deviations of the values of the distribution factors come from violation of the assumptions made in the basic concept of the unit graph. Rain is seldom evenly distributed over the watershed but is more frequently concentrated in one portion or another. Heavy rain concentrated on the lower portion near the gaging station results in quick flashy rises at that point. The total time of runoff is less, and the earlier periods contain proportionately larger amounts of the total runoff. If, on the other hand, the rain is concentrated on the upper portions of the basin the runoff is slow and the later periods contain larger proportions of the total flow. Similarly, where the actual distribution of the rainfall in time departs from the assumed uniform rate over the unit period a different set of distribution values is obtained, the nature of which is dependent upon the concentration of rain during the period. Concentration of rain in the beginning of the period would produce one set of values, and concentration near the end would produce another. The use of such distribution factors should be confined to conditions similar to those from which they were derived.

Although the same proportion of the flow should run off in the same unit periods, it is not likely that this is always true even with uniformly distributed rainfall. There is some reason to believe that runoff from the larger storms flows off more quickly than that from small storms. The reason for this is to be found in consideration of the formulas for velocity of stream flow. Take Manning's formula, for example,

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}.$$

It may be seen that the velocity varies as $R^{2/3}$. While the slope will remain substantially constant, the hydraulic radius for the main channel increases as the flow increases and directly as the stage rises; therefore the velocity must increase. The hydraulic radius of overland flow also increases with depth of water. This tendency to increase in velocity may be offset to some extent by increased roughness of channel and cross currents due to flow among trees, shrubs, and other

obstructions on the edge of the channel, which roughness would result in higher values of $n$. The net result of these contradictory tendencies is difficult to evaluate, but consideration of them indicates that the distribution factors to be used for estimating large floods should be derived from data of as large freshets as can be found and used.

Another source of error arises from the difficulty of eliminating ground water. A well-developed ground water recession curve helps materially in determining the end of the surface runoff, but does not indicate where the ground water begins to rise as a result of accretions to the water table from the current storm. However, if larger storms are selected for investigation, the ground water flow is a small part of the total flow and no serious error is likely from this source.

Likewise, separation of the surface water of preceding and subsequent storms leaves some uncertainty as to the true volume of runoff from the storm under investigation. This uncertainty can be reduced to small proportions by selection of well isolated storms.

Although the data derived from storms for computation of distribution factors may be analyzed by the method of statistics, they are usually too few to justify more than the computation of simple averages. The derived factors should be checked invariably by computing the runoff from a known storm, which should be one not used for the derivation, and then comparing the computed hydrograph with the observed one.

*Formula for Runoff from Small Areas.* From an analysis of runoff from experiments on sprinkling plats, Horton (91) has derived a formula that is applicable to small watersheds having lengths of a few hundred feet. This analysis starts with the equation for overland flow in inches per hour at the edge of a well-defined channel:

$$q_s = K_s \delta_c^M$$

where $\delta_c$ represents the depth of flow at the edge of the channel. This depth equals 3/2 of the average depth of surface detention under the assumptions that a parabolic curve represents the profile of the water surface and that a condition of mixed flow prevails. The remaining notation is as defined previously. The so-called "storage equation," which states that inflow equals outflow plus or minus change in storage, is then utilized in the differential form which upon substitution of the above equation in it becomes

$$\sigma \, dt = K_s \delta^M \, dt + \gamma \, d\delta_c$$

in which $\sigma$ represents the supply which is the effective rainfall; $\gamma$ is the ratio between the average depth of surface detention and the depth at

the channel; $t$, time in minutes; the other notation as defined previously.

In continuing the development, Horton assumes an overland flow with a degree of turbulence of 75 per cent, for which $M$ equals 2.0. Upon substituting 2.0 for $M$, integrating the storage equation, making other notational substitutions in the process, there is obtained the equation of flow

$$q_s = \sigma \tanh^2 \frac{3}{2} (\sigma K_s)^{1/2} t$$

in which the notation is as previously defined.

For flow involving other degrees of turbulence, the equation may be expressed as

$$q_s = \sigma \tanh^M \frac{M+1}{M} (\sigma K_s)^{1/M} t.$$

The constant $K_s$ is dependent upon the characteristics of the watershed, including slope, roughness of the surface, length, and degree of turbulence; it may be expressed as

$$K_s = \frac{1020\sqrt{S}}{InL}$$

where $S$ represents the slope; $n$, the roughness factor; $L$, the length of an elemental strip; and $I$, the turbulence factor which is expressed by the equation

$$I = \frac{3}{4} (3.0 - M).$$

For turbulence of 75 per cent, and upon putting $q_s$ in cubic feet per second, the equation for $q_s$ reduces to

$$q_s = \sigma \tanh^2 0.922t \left(\frac{\sigma}{nL}\right)^{1/2} (S)^{1/4}.$$

Hathaway (82) gives the following values for $n$ to use in the above equation:

| SURFACE | VALUE OF $n$ |
|---|---|
| Smooth pavements | 0.02 |
| Bare packed soil, no stones | 0.10 |
| Poor grass cover or rough bare surface | 0.20 |
| Average grass cover | 0.40 |
| Dense grass cover | 0.80 |

The foregoing equations for overland flow are relatively new and have not yet been widely adopted, although Hathaway (82) states that they are being used for estimating flow in airport drainage. That use is limited to areas not over 600 feet long. It seems that these formulas should be useful also in determining flow from small areas such as those considered for storm sewers.

*The Rational Formula.* Prior to the development of the unit hydrograph and distribution factors, runoff from rainfall was computed by means of the so-called "rational formula," which is as follows:

$$Q = CIA$$

in which $Q$ is the discharge in inches per acre or cubic feet per second; $A$, the area of the watershed; $I$, the rate of rainfall in inches per hour or cfs; $C$, a coefficient representing the portion of rainfall that forms the surface runoff.

The critical factor in this formula is the coefficient $C$. It is usually estimated on the basis of previous experience with similar areas and watersheds, since it must represent many elements in runoff. It has to serve for the following modifications:

1. Infiltration losses
2. Equalization of flow caused by surface detention
3. Equalization of flow caused by valley and channel storage
4. The effect of the various physical factors of the watershed on the flow, including slope, shape, distribution of tributary channels, vegetal cover, etc. It is evident that successful use of this rational formula depends entirely upon the skill and judgment of the engineer in estimating suitable coefficients. It has been largely superseded in estimation of flows for airport drainage by the more recent methods.

*Effect of Snow on Runoff.* Since flood hydrographs have been analyzed in detail it is possible to gain some idea of what effect melting snow has on flood runoff. Snow retains water from rainfall and its melting as its density increases until its capacity for retention is filled, whereupon water is released for runoff. Furthermore, early in the spring, snow melting is confined largely to the daytime hours when air temperature is above freezing, and ceases or markedly decreases at night. Such snow melting may continue for a number of days during warm weather and then stop entirely upon the advent of cold weather. These conditions are reflected in the formation of the hydrograph of the runoff.

To illustrate more clearly the nature of runoff from rainfall and melting snow, the hydrograph in Figure 130 was prepared from meteorological data and runoff of the Pemigewasset River at Plymouth, N. H., for March, 1936, when the area was visited by two intense storms. (See

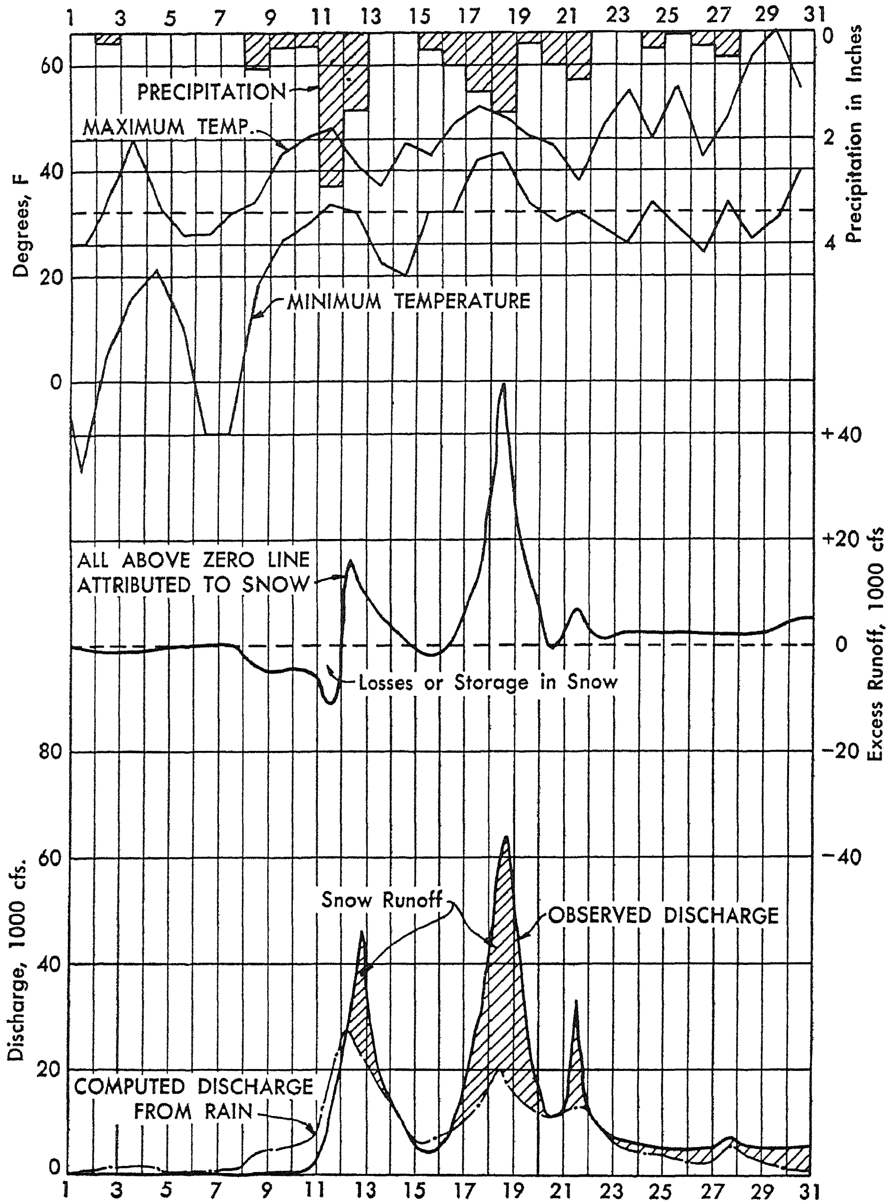



FIGURE 130. Snow Runoff, Pemigewasset River, Plymouth, N. H., Month of March, 1936

also the isohyetal maps in Figures 46 and 47.) The daily values of precipitation and maximum and minimum temperatures are plotted at the top of Figure 130. Although not indicated on the figure, there was a moderately heavy snow cover over the entire drainage area at the beginning of the rain period.

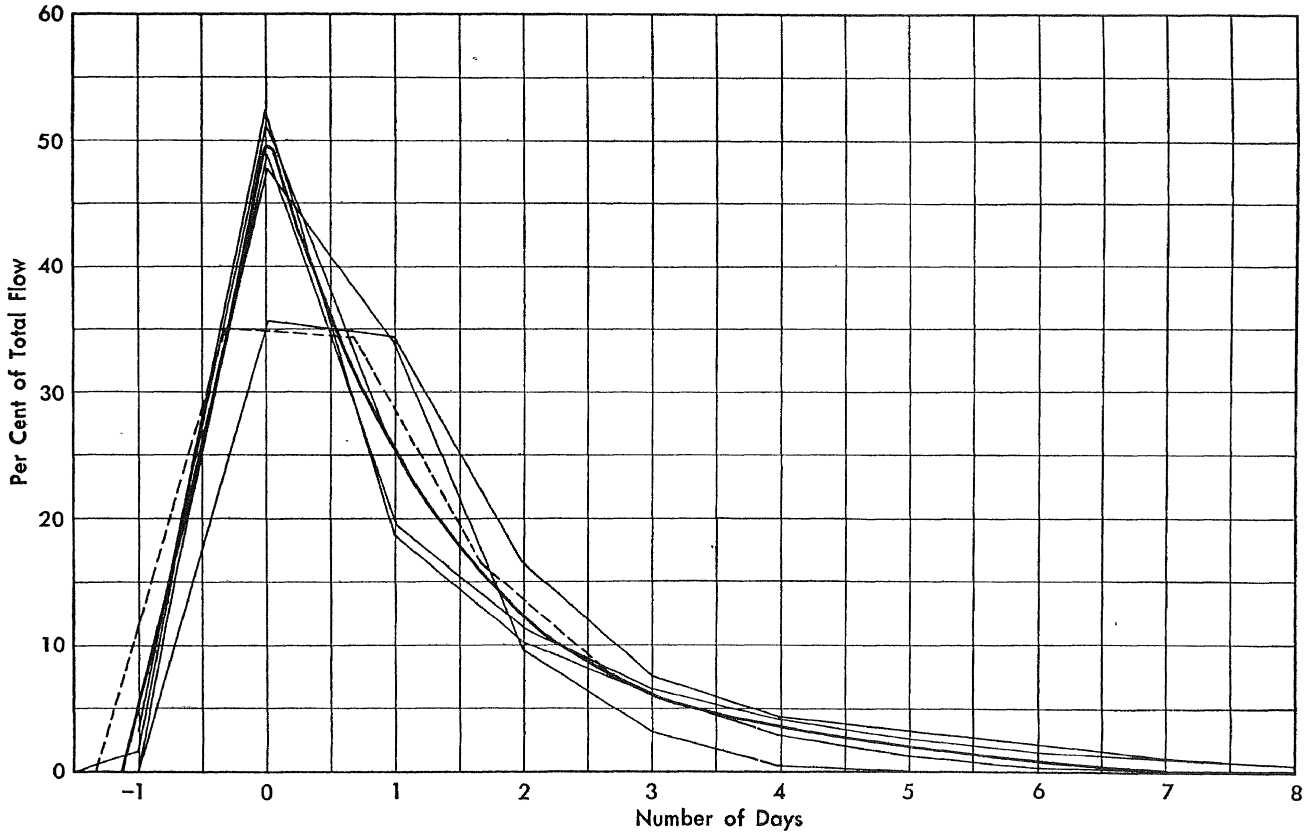



FIGURE 131. Distribution Graphs, Pemigewasset River, Plymouth, N. H., for Summer Season with no Snow on the Ground

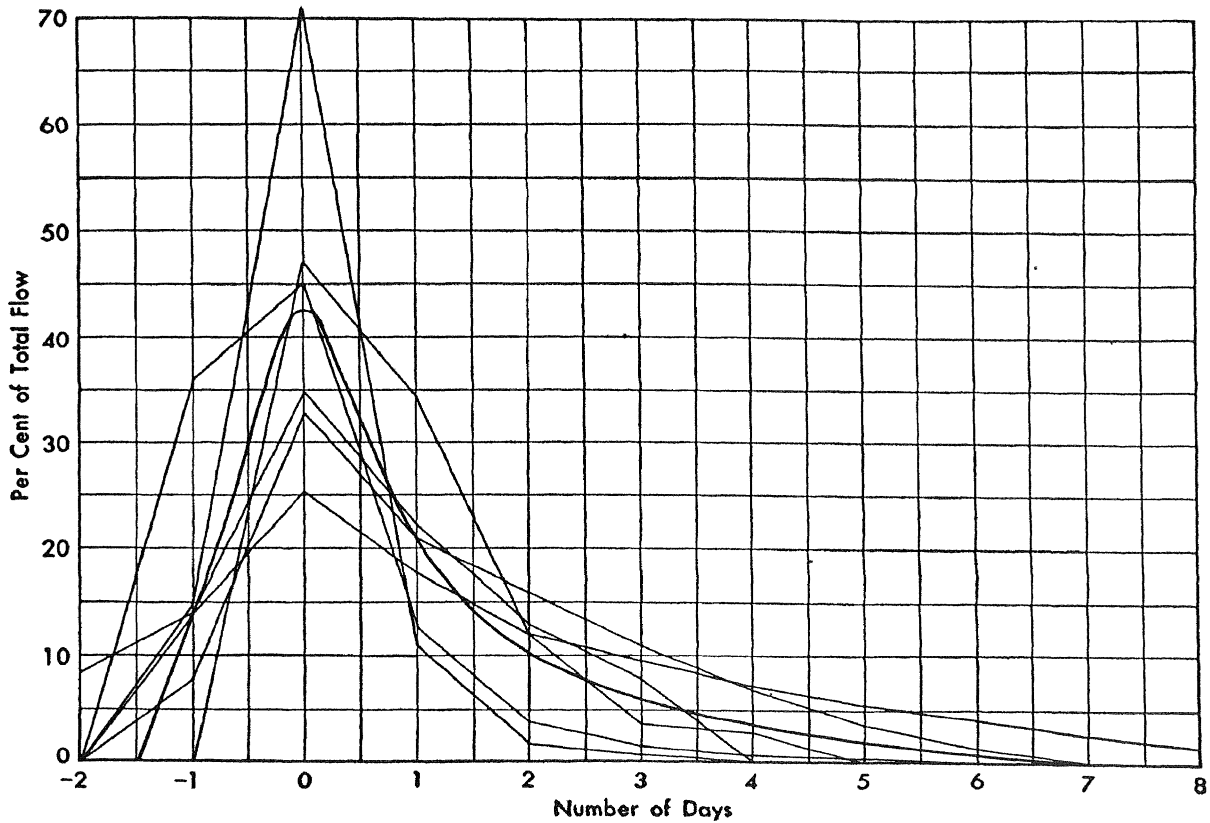



FIGURE 132. Distribution Graphs, Pemigewasset River, Plymouth, N. H., for March and April with Snow on the Ground

To make an approximate estimate of the snow contribution to the floods, the runoff from rainfall was computed by means of a set of distribution factors, such as those shown in Figure 131, derived from hydrographs of freshets with runoff from rain only. It was assumed that all rainfall went into runoff, that is, no losses were assumed since the current information indicated that the ground was frozen. The hydrograph of the computed runoff is shown at the bottom of Figure 130. This computed runoff was then subtracted from the observed stream flow, and any positive remainder was designated "Excess Runoff" and attributed to snowmelt. This division of the runoff is shown by the curve plotted about the axis in the middle of the figure; the upward portion of the curve is the runoff in excess of that attributed to a runoff of 100 per cent of the rainfall, and the downward portion consists of losses or storage of rain in the snow. The snow runoff is shown again by the crosshatched areas on the bottom hydrograph.

On the above basis it was estimated that snow contributed 30 per cent of the runoff of the first peak, March 2–15, and 70 per cent of the second peak, March 7–20. This estimated snow runoff should be considered a minimum, since the computed runoff was based on 100 per cent of the rainfall. If there had been losses of rainfall due to evaporation or infiltration the snowmelt would have been increased in like amount.

Subsequently to the development of the above method of estimating runoff from snowmelt, a method basically similar with some added improvements was used in the calculation of the basin snowmelt coefficients given in Table 56.

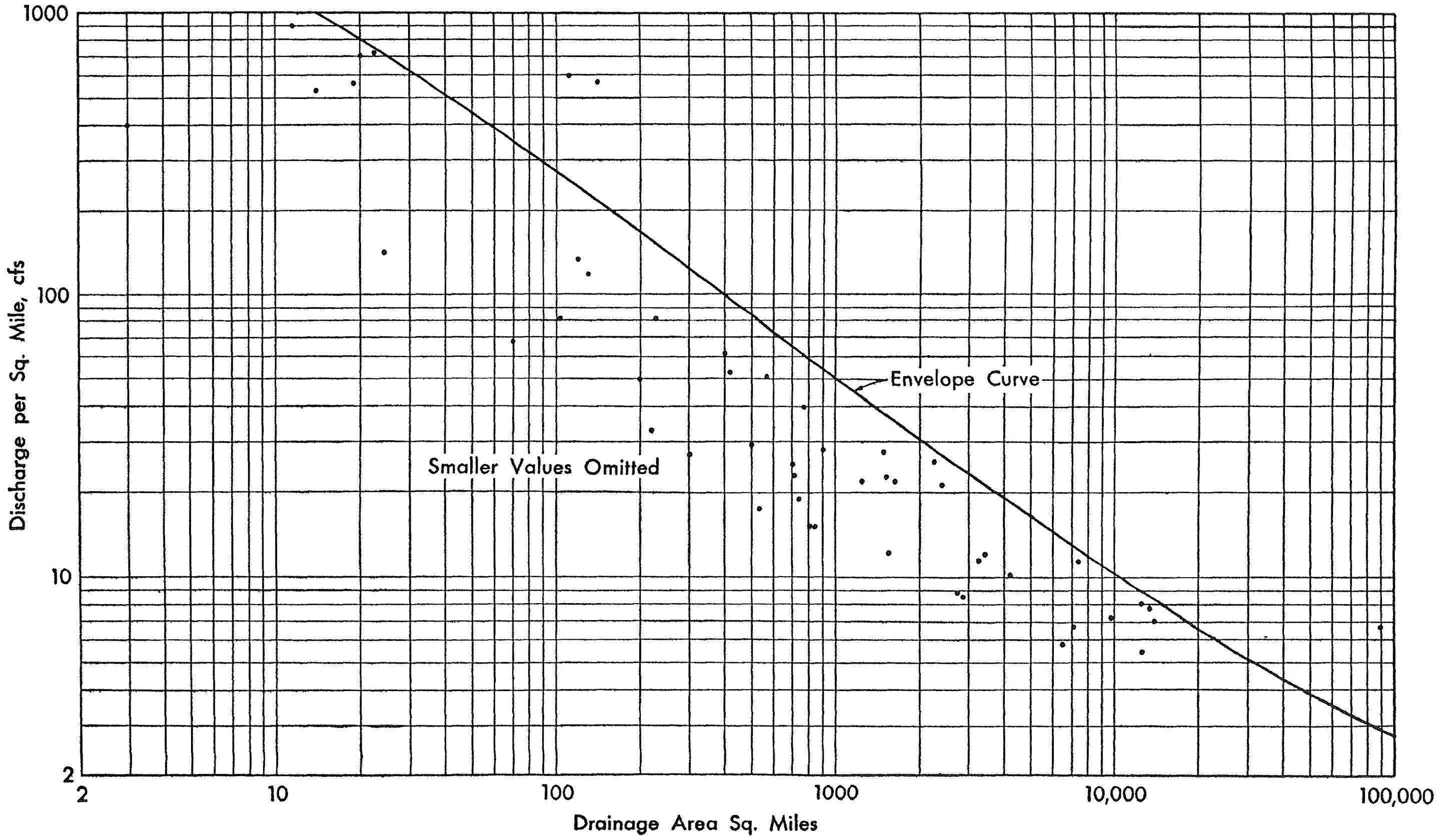
In view of the effect that melting snow has on the subsequent runoff, it is evident that freshets containing snowmelt should not be used to compute the distribution factors of a stream. They can, of course, be computed from such freshets but they are more widely spread and obviously would be unreliable. A comparison of two sets of one-day distribution factors of the Pemigewasset River, New Hampshire, is shown graphically in Figures 131 and 132. The former figure has the distribution graphs derived entirely from runoff produced by rain. Figure 132 is the graph of spring freshets containing snowmelt, and the impossibility of segregating the effect of snow is shown by the widely spread graphs. The importance of snow as a producer of runoff on the Pemigewasset River is shown in Figure 101.

# 10 FLOODS

*Definition of Flood.* The term "flood" designates the excessive stream flow or stage which from various causes exceeds the capacity of the normal channel. Floods are invariably caused by surface flow of the types heretofore discussed. The line of distinction between ordinary surface runoff which causes moderate high water and a flood is indefinite, and is determined largely by the average height of the banks and the economic development of the adjacent land. Commonly, discharge that overtops the banks is considered a flood, but the height of the banks and consequently the capacity of the channel vary greatly within comparatively short reaches of the stream. The definition of floods is therefore arbitrary and in accordance with a convenient rule that a flood is flow greater than that which is usual. For many localities on larger streams, the Weather Bureau has established so-called "flood stages" which are stages at which the stream is deemed to be in flood. However, these stages are purely arbitrary elevations. The only differences in the stages and the discharges below and above the fixed points are those of magnitude.

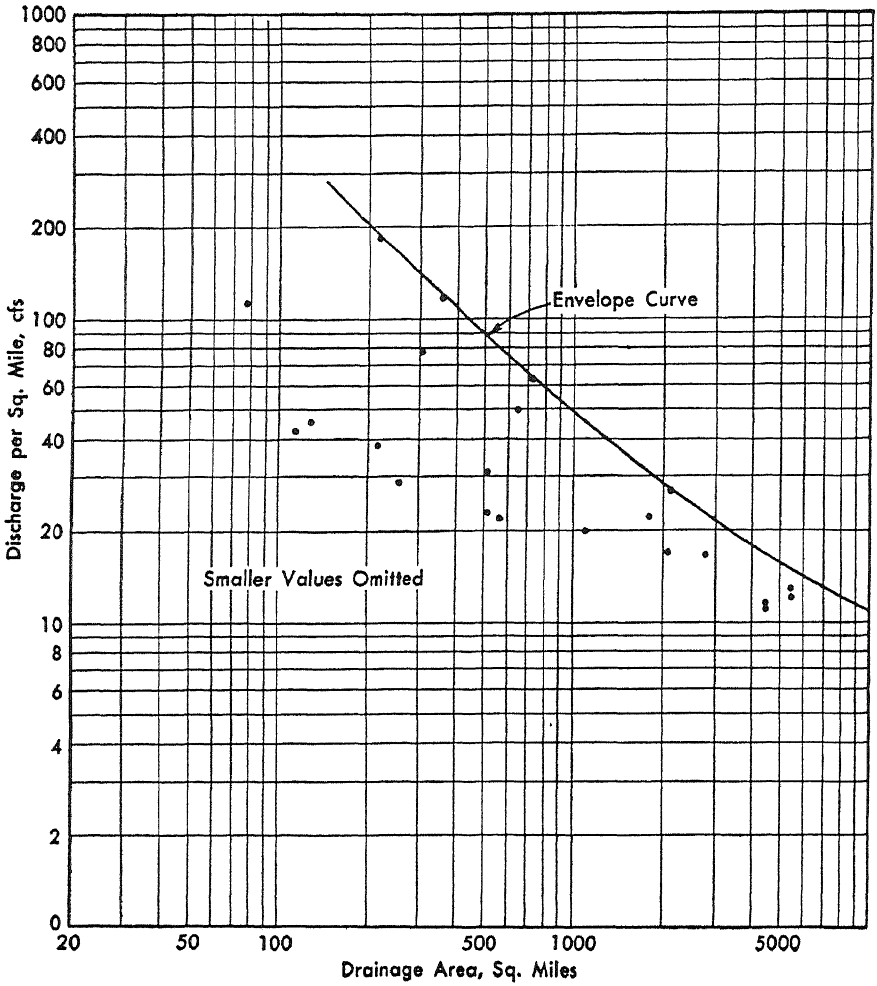
As may be expected, the magnitudes of flood peaks depend very much upon the size of the drainage area. There is not, however, a straight-line relationship between peak discharge and unit area, but on the contrary the rate of discharge per unit of area diminishes rapidly as the total area increases.

*Maximum Observed Floods.* Frequently it is desirable to compare peak flood discharges of different streams, since the relatively short records are not likely to contain an observation of the maximum flood on any given basin. Comparison of peak discharge of floods on different streams is made most effectively on a basis of rate of discharge per unit area, since the comparison is then based on the intensity of rainfall and the physical characteristics of the basin other than magnitude. A comparison of this sort is instructive, but it should be limited to a region of homogeneous climate, subject to the same or closely similar




*After Crawford and Whitaker*

FIGURE 133. Maximum Discharges for Iowa Streams to January, 1942




*After Lenz*

FIGURE 134. Maximum Flood Flows in Wisconsin

types of storms, and to regions of similar topography. One should expect to find, for example, greater floods from the same area in Texas, which is subject to tropical hurricanes, than in Montana which is beyond the range of those storms.

The comparison can be made most effectively by plotting the observed peak discharges, reduced to rate per unit of area, against total drainage area. One such plotting of observed storms is shown in Figure 133, which was taken from Crawford and Whitaker (42). Another comparison is shown in Figure 134 and was taken from Lenz (114).

Although these graphs present an informative picture of flood flows

as they have been observed, they do not necessarily indicate the magnitude of future floods on a given basin. It is possible that the uppermost points may indicate or approximate the probable maximum floods on the few basins from which the data were obtained, but other basins on which lower flood peaks have been observed may not have received anything like the maximum flood within the short period of record. Moreover, since topography has a great influence on peak flows, a discharge of a given flood on one stream is not necessarily indicative of possibilities on another. These facts should be considered when such graphs are used to find possible peak floods on another or even the same basin.

It should be pointed out, however, that the two graphs, shown in Figures 133 and 134, are each within a region of reasonably homogeneous climate, so that comparisons need not take into account diversity of climatic factors.

*Causes of Floods.* The causes of natural floods may be grouped in three general classes:

1. Excessive rainfall; this class is the principal cause of floods throughout the world in all climates except possibly in the polar regions. With sufficient rainfall a flood will occur on any stream. The amount of precipitation necessary to cause a flood varies greatly with many factors, such as the topography, size and shape of the basin, distribution of tributaries, season of the year, condition of the ground surface, temperature, and the intensity and distribution of the precipitation.

2. Rapid melting of deep snow cover; snow melting can cause floods only in the colder temperate and frigid zones where precipitation occurs as snow which accumulates in considerable depth during the winter season. It is found as a contributing cause wherever there is an appreciable snowfall. However, as a primary cause of serious floods it is found only in regions where heavy snow cover may be preserved until late in spring. The volume of water released from snow of a given depth is dependent entirely upon the amount of heat received, so that this source of danger is intensified in regions so located that they may be invaded by warm moist tropical air masses when the ground is covered with snow.

3. Ground conditions; ground conditions do not, of course, supply water to make a flood, but the quantity of water remaining from precipitation or snowmelt and available for runoff depends largely upon the condition of ground surface. When the ground is frozen nearly all surface water drains off. At other times the surface may be dry, the soil absorbent, and the rate of infiltration high. Then the runoff may be

reduced to such a small volume that no flood occurs. Ground conditions are so important that they can cause or prevent a flood.

*Other Minor Causes.* Other circumstances may modify flood flow. In northern regions ice jams may contribute to higher floods; that is, they may cause higher stages or discharge, although not greater total runoff. This condition is to be expected more frequently in regions which are subject to tropical air masses in spring at or before the usual spring breakup; the flood resulting from rain and melted snow in such a case carries out the ice before it has time to melt, and it may pile up behind any obstruction. The danger of ice jams is intensified on a few rivers which have headwaters towards the south and drain to the north, so that the snow and ice melt first in the headwaters areas and the resulting runoff is forced to flow in an ice-covered channel as it moves downstream. The unmelted ice cover causes higher stages by backwater, even without any noticeable jam. An example of this condition is found in the Red River of the North, which flows northerly along the eastern border of North Dakota into Lake Winnipeg, Canada.

*Artificial Causes.* Conspicuous among artificial causes of floods are failures of dams. When dams fail during a flood, by overtopping or undermining, the water in the reservoir is added in a concentrated mass to the natural storm runoff. In exceptional cases, the breaking of a dam may be the principal or even sole cause of a flood, but generally speaking, dam failures are only minor contributing causes to flood crests. In other places artificial obstructions to channels, such as bridges, buildings, and railway and highway grades, raise flood stages by backwater to greater heights than would prevail under natural conditions. Improper manipulation of reservoirs may also affect flood peaks by either diminishing or increasing them.

*Debris in Floods.* Whereas floods are primarily an excess of water, they not infrequently are greatly augmented in stage by the addition of debris or detritus that has been eroded from the surface of the watershed or the channels of the stream. This situation is most marked in regions with easily erodible soil and steep slopes, and subject to intense rainfall. Southern California and the southern Appalachian Mountain regions are examples. In such regions the detritus may constitute so large a portion of the flow that it is liquid mud; such flows are called "debris flows" and behave differently from water overflows. But not all debris flows occur in mountainous regions. In the central plains, flows from the loessial soil of western Iowa have been observed to carry 10 to 20 per cent of sediment by weight, which is a material increase in volume of flow.

*Importance of Floods.* Floods are of great economic importance because of their destructive effects on human life and property. The design of many structures depends much upon the magnitudes of expected floods; among these structures are bridges which must be provided with ample openings to pass flood discharge, and spillways of dams which must have sufficient capacity to pass the largest possible flood. The loss of life and damage caused by floods is immense and protection against those ravages of nature is work of nationwide importance.

The important attributes in the study of floods are the magnitudes of peaks and volumes and frequencies of their occurrence.

*Distribution of Flood Data.* The histograms of a number of distributions of flood data consisting of all peaks above a basic discharge are shown in Figures 135, 136, and 137. The streams from which the data were collected are located along the Atlantic seaboard and the western Great Plains along the eastern slope of the Rocky Mountains. These examples represent the usual distribution of flood data. The first class of magnitude is much the greatest and the number per class diminishes rapidly as the magnitude increases. In each case it can be seen that to the left of the mean there is a large number of the smaller floods, while to the right there is the diminishing number of larger floods with the extreme flood peaks considerably in excess of the mean.

*Other Types of Distributions.* Many methods of computing flood frequencies use either a series of annual floods or the so-called "partial duration" series (98). The series of annual floods consists of the selection of the highest flood of each year to represent all floods of the like period. This procedure is selective and produces a series averaging somewhat higher than an array consisting of all flood peaks. The distribution is therefore distorted so that the true probability of occurrence or frequency cannot be obtained. The so-called "partial duration series" is an array consisting of a number of the highest floods observed in a period such that the number of floods equals the number of years in the record. This method of selection includes all high floods and omits the low ones; such a distribution is even more distorted than that obtained by using the highest annual flood, and hence less accurate results can be obtained. For most accurate results all flood data should be used.

*Frequency of Floods.* The problem of flood frequencies has been attacked by many investigators. Many of the methods have been entirely empirical and involved only a gradation of data and simple calculations to determine the observed number of floods of various

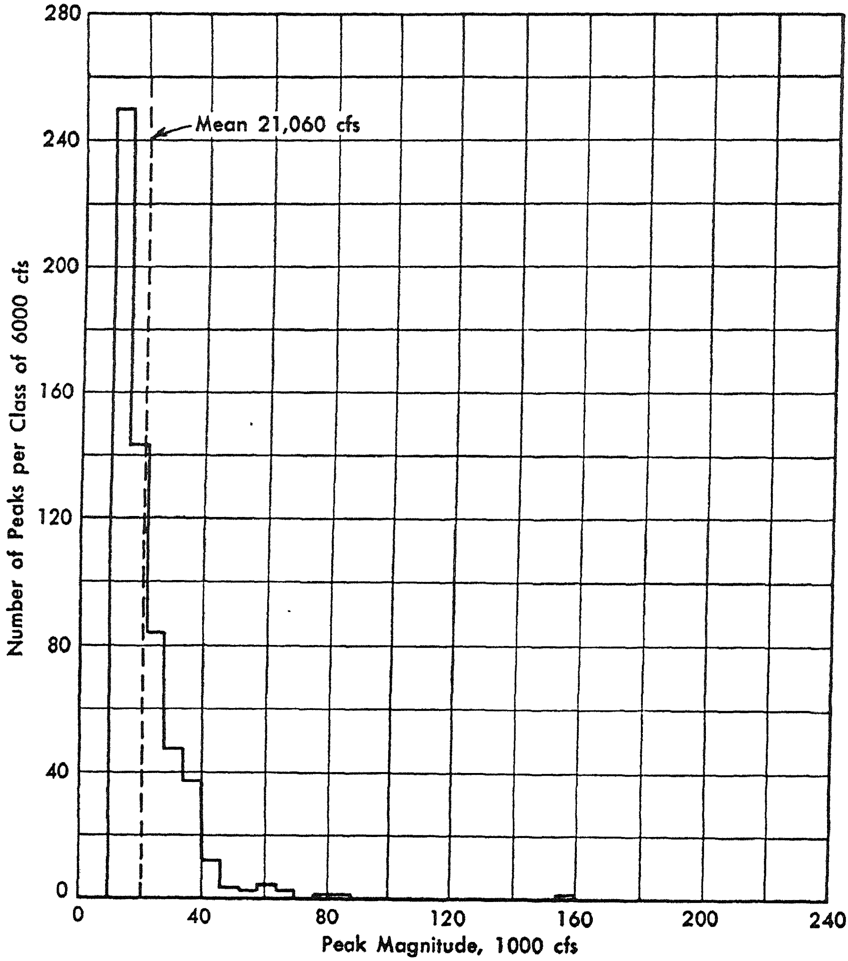



FIGURE 135. Histogram of Flood Data, Merrimack River, Lawrence, Mass., 1879–1938

sizes each year. An extensive summary of the various methods of calculation of frequency is given in *Water Supply Paper No. 771, Floods in the United States* (98). Except that some discussion is given to the approximate arithmetic computation of frequency because of its rather wide acceptance, these methods are not considered further.

*The Approximate Method.* This method is the simplest, the most readily applied, and the least reliable. It appears in various forms. Commonly, the data are selected on one or the other of two bases; first, the highest flood in each unit of time, usually one year; second, a number of the highest floods may be selected to equal the number of years of record. These data are arranged in order of magnitude, the

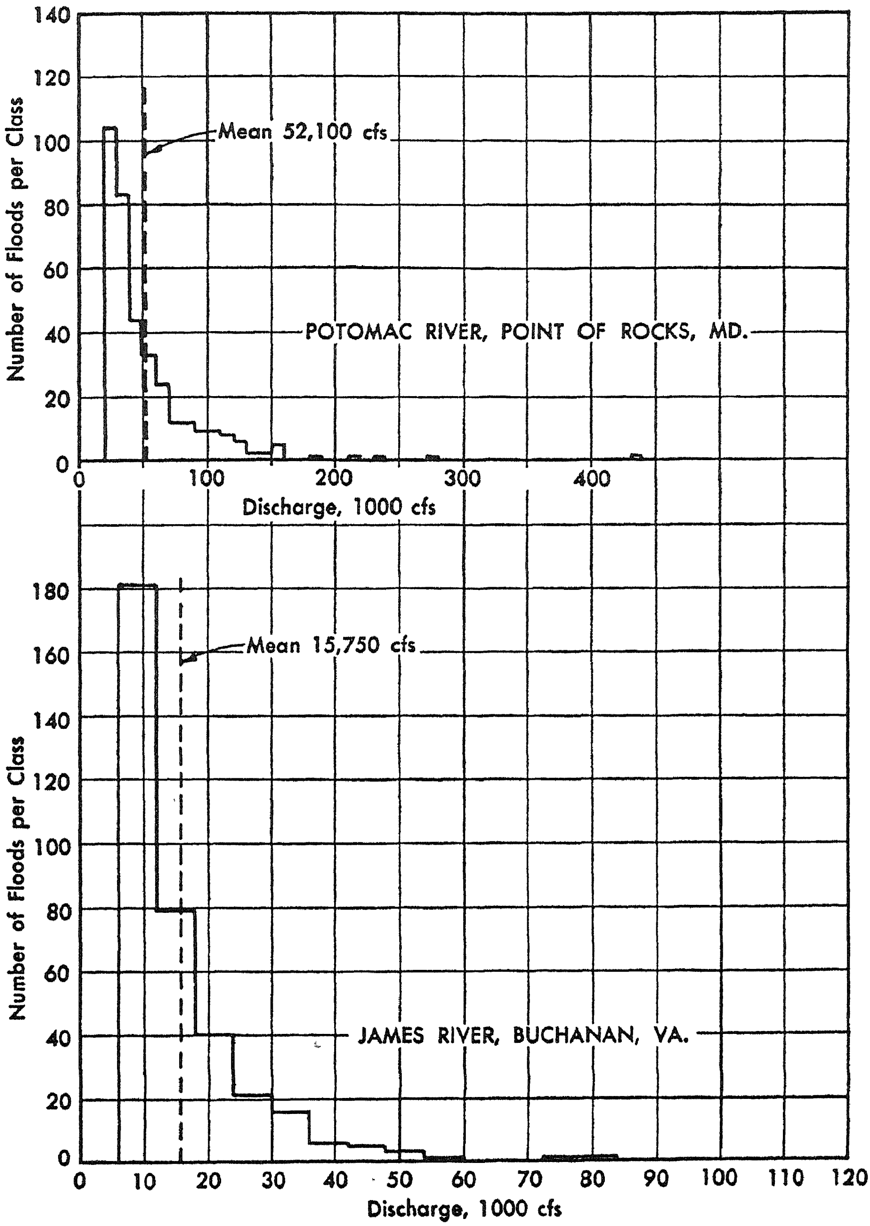



FIGURE 136. Histograms of Flood Data, Middle Atlantic States

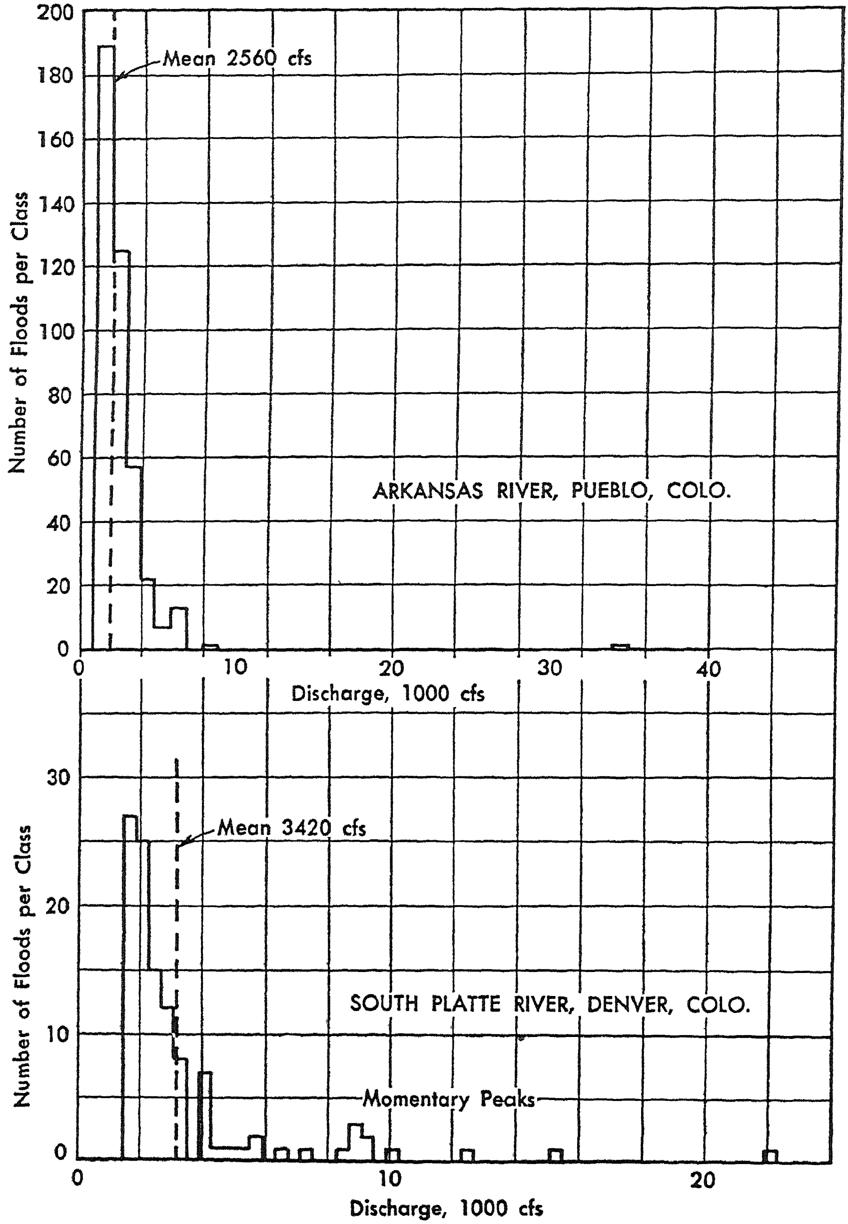



FIGURE 137. Histograms of Flood Data, Rivers in Colorado

highest being placed first. Then the frequency is computed by the equation

$$F = \frac{m - 0.5}{n}$$

in which $F$ is the frequency, $n$ the number of data, and $m$ the order of magnitude.

In *Water Supply Paper No. 771*, five variations of the approximate method are described:

1. Annual Flood. The maximum daily flows for each year of the period of record are arranged in order of magnitude and plotted as a cumulative frequency curve, that is, to give the frequency equal to or exceeding a selected value.

2. Monthly Flood. The maximum daily flows in each month are treated in the same manner as the annual peaks. The time then equals the total number of months in the period of record.

3. Daily Flows. The daily flows above a selected base are used in the same manner as (1) and (2). The time or number of observations equals the total number of days in the period. This cannot be considered a true frequency curve in the sense in which the term is commonly used, because the consecutive daily flows are not independent events.

4. Flood Event. All floods above an arbitrary base are arranged in order of magnitude and computed for plotting in the same manner as the annual floods. In this case, the denominator $n$ is the number of floods and not the years of record. Since time is not entered in any manner in this method, the formula used with such an array of data does not give frequency but a probability. To obtain frequency, the element of time must be entered; it may be done by multiplying the results by the average number of floods per year.

5. Average Number of Floods per Century. This procedure is practically identical with the procedure outlined in (4) above, with the element of time included by taking the average number of floods per 100 years.

*Criticism of the Approximate Methods.* The methods described are open to a number of objections. First, the arbitrary term 0.5 (or 1.0) is inserted with little theoretical or observational basis. It tends to decrease the computed frequency since it reduces the numerator. For the maximum observation it makes the frequency once in a number of years twice the period of record, which is not in accord with theory or experience. An objection arising from the nature of the formula itself is the fact that the factor $n$ is a cardinal number representing the number of years of record or number of occurrences, while the factor $m$

in the numerator is an ordinal denoting the order of magnitude, and hence cannot logically operate in arithmetical processes. However, this objection may be obviated by assuming $m$ to be of a different nature, and to this there is no valid objection, since the formula is flexible enough to cover almost any assumed situation. A third objection is serious. The two independent factors of the formula are $n$ and $m$, the first representing the total period of record and the other representing the order of magnitude from largest to smallest. There is no connection expressed between the variation of magnitude and length of time. With a given period of years (or other unit of time) the maximum observation will have the same frequency regardless of its size relative to the other observations. In other words, a given record may by chance have included a maximum flood peak only moderately greater than the others, but no difference would be shown in its frequency and that of a peak much greater. This is manifestly contrary to observation. In view of these considerations, the use of the approximate method is plainly out of place where good results are desired.

A distinction from the above statements must be made for a similar formula which may be used to calculate average number of events per year directly from the observation. This is

$$P = \frac{m}{n}$$

where $P$ is the average number of data per year equaling or exceeding a given base value, $m$ is the total number of observations of a certain attribute exceeding the base value, and $n$ the total number of years over which observations were made. This formula is useful for plotting observed points for comparison with computed frequency curves.

*Steps Toward Theoretical Probability.* Mr. W. E. Fuller is given credit (1914) for the first formula for computing the magnitude of floods and their frequency of occurrence. His formula is

$$Q = Q_{\text{ave}}\,(1 + 0.8 \log_{10} T)$$

where $Q$ = maximum 24-hour peak

$Q_{\text{ave}}$ = average annual floods, 24-hour peak

$T$ = years, period in which the flood is to be expected.

This formula is strictly empirical; that is, it is based entirely upon observed data without including any concept from the theory of probability.

In a discussion of Mr. Fuller's paper, Brigadier General G. B.

Pillsbury suggested the use of the normal probability integral,

$$P = \frac{1}{\sqrt{2\pi}} e^{-z^2} dz$$

without, however, developing the equation to fit the extreme skew of distributions of flood magnitudes.

In his paper on *Storage to be Provided in Impounding Reservoirs for municipal Water Supply* (98), Dr. Allen Hazen utilized certain elements of the theory of probability in the solution of hydrologic problems. For this purpose he devised a special form of coordinate paper which had for the ordinates an arithmetic scale, and for the abscissas a scale spaced proportionately to the integral values of the normal probability equation. The ordinates represented the magnitudes of the observations as runoff in gallons per day per square mile, and the abscissas represented percentages of time in which the given observations prevailed. The percentage of time $P$ was found by the equation

$$P = \frac{2m - 1}{2n}$$

in which $m$ is the order of magnitude of the observation (beginning with the largest), and $n$ is the total number of observations.

Later Dr. Hazen developed this method of frequencies to apply to floods. In this development a logarithmic scale was used for the ordinates in place of the arithmetical scale. In order to plot straight lines from the data of floods, he developed a series of skew factors which were based on the values of the probability integral. The percentage of time of each observation was computed as before. It may be seen that although this method utilized some features of theoretical probability, it is primarily empirical. Nevertheless, the coordinate paper that he developed is a useful tool in studying flood data.

Mr. L. S. Hall utilized a probability coordinate paper in computing the yearly variation of runoff of California streams. The ordinates were measured by an arithmetical scale and the abscissas were measured by a scale proportional to the values of the integral of the normal probability equation. The percentage of time of each observation plotted on the abscissa axis was found by the equation

$$P = \frac{2m - 1}{2n}.$$

This method is also primarily empirical, and furthermore was found not to be of universal applicability.

*Method based on Pearson's Probability Functions.* The first notable paper presentĕd on the full use of theoretical probability or distribution curves for calculating flood frequencies was made by H. Alden Foster (67), who used two of the family of probability functions developed by Dr. Karl Pearson of London, England. Although designated here as theoretical curves, they differ from the theory based on the Laplace function and are not to be confused with the theoretical methods hereinafter developed. Pearson's curves, which have been discussed in Chapter 1, have been used extensively in statistical work in widely different fields of knowledge. Types I and III, which were the ones used by H. Alden Foster, are as follows:

$$\text{Type I}: y = y_o(1 + x/a_1)^{m_1}(1 - x/a_2)^{m_2}.$$

$$\text{Type III}: y = y_o e^{-px}(1 + x/a)^{pa}.$$

*Basis of Theoretical Methods.* Consider any one river in the north temperate zone. The causes of floods on such a stream may be divided into two classes: excessive rainfall, and rapid melting of snow. These causes may operate singly or in combination. In addition, either or both can be combined with ground conditions that may be favorable or unfavorable to rapid runoff. Furthermore, behind these primary causes there are many contributing factors which may unite to cause the particular observed flood.

Since excessive rainfall is undoubtedly the primary cause of great floods, a summary of the events leading up to a flood is in order. The study of air masses and their interactions shows that excessive precipitation such as that which causes floods is the result of the overrunning of a cold air mass by one of warm moist air. Both cold and warm air masses move from their respective source regions along their respective general paths, or rather zones, of travel. There is much variation within each of the zones, so that the meeting of the different masses in a given locality is largely a matter of chance. The mere meeting of two diverse air masses does not necessarily mean excessive precipitation; there may, in fact, be none. The occurrence of precipitation and its quantity depend upon many diverse features making up the storm, such as moisture content of the air mass, thermal stability of the air or lack thereof, velocity and duration of the winds from the regions of moist air, (which in turn depend upon difference in barometric pressure in the areas of the storm), and the rate of lifting of the warm air (which itself depends upon the slope of the cold front, and more remotely upon the difference in temperature of the two air masses). Observations of rainfall show that the quantity varies from practically

nothing to amounts received only once in periods of several hundred years. Severe storms may occur in any season of the year in the temperate zones, and in enough seasons in other zones to encounter diverse conditions of ground surface. Surface conditions vary to the extent that 10 per cent or less of the rainfall becomes runoff, or on the other hand, 75 per cent or more may be runoff. Likewise, snow cover may be nothing or it may be much; it may contribute the principal portion of the flood runoff or it can reduce the runoff by absorbing and storing a considerable portion of the rainfall for subsequent release. The areas covered by storms vary greatly; heavy precipitation may be concentrated on a few square miles or it may be spread over an extensive area. Furthermore, the center of the storm may coincide with the center of one watershed or it may be divided among several. Considering the many variable factors entering its makeup, any flood is seen to be the net result of a chance combination or chance variations of the multifarious contributing causes.

The preceding statement does not in any way abrogate the law of causality which must necessarily operate in the genesis of floods as in other physical phenomena. It does mean that the occurrence of floods will vary, and both the number and magnitude will fluctuate about some norm which depends upon the occurrence of the various contributing causes prevailing in any given locality. In view of these considerations, the applicability of the methods of theoretical probability to problems of flood frequency is readily recognized, for all these methods utilize the mean as a base for computing the distribution of variations. This will be seen in the example to be given later.

Let $p_1, p_2, p_3, \cdots p_n$ be the probabilities of the various factors contributing to any flood and $q_1, q_2, q_3, \cdots q_n$ be the probabilities of the factors failing to contribute. Out of a total of $m$ hydrological and meteorological factors operating, a sufficient number must occur to produce the flood; that is, there must be $n$ factors with favorable probabilities, $p_1, p_2, p_3, \cdots p_n$. The favorable factors constitute a chance combination of events, the probability of which may be expressed thus,

$$P(C_n^m) = (p_1 \times p_2 \times p_3 \times \cdots \times p_n)(q_1 \times q_2 \times q_3 \times \cdots \times q_{(m-n)}).$$

$C_n^m$ is the combination of $m$ things taken $n$ at a time and $P\ (C_n^m)$ is the probability of the combination. An average value $p$ may be assumed for the various probabilities contributing to the flood, and an average value $q$ for the probabilities of those not contributing. Then the above equation may be expressed as

$$P = C_n^m\ p^m q^{(m-n)}.$$

*Statistical Methods.* The methods of computing flood frequencies designated here as "statistical methods" all utilize the processes of mathematical theory of probability and mathematical statistics. The term "statistical methods" is applied to any method which utilizes any one of several frequency or distribution functions which have been developed for skew distribution, either as a series or with logarithmic transformation of the variate. The method of H. Alden Foster should properly be included since the functions of Dr. Karl Pearson are well known in statistical fields. However, since that method has been discussed previously, only the methods developed from or similar to the Laplace probability function are considered here.

One function derived by the logarithmic transformation of the normal law is given by Mr. Arne Fisher (59). Another logarithmic function was developed by Professor J. J. Slade, Jr. (169). This is the same function that was used in Chapter 6 for calculating the frequency of rainfall. The method of its use with flood data is similar to that with rainfall.

*Applicability of Functions with the Logarithmic Transformation of the Variate.* In view of the availability of different types of probability functions, it is pertinent to inquire into the suitability of functions based on logarithmic transformation of the variable. In this aspect the problem of finding a function to fit distributions of flood data is similar to the same task pertaining to data of daily rainfall (Chapter 6), except that the skew of flood data is not so extreme as that of daily precipitation. The use of the normal probability function may be ruled out at once because it is limited to distributions with approximately equal positive and negative variations, a feature which is distinctly not found in the case of flood data.

A series such as Gram-Charlier likewise does not provide sufficient skew for distributions. The Poisson function deals with homograde integral data only and is not, strictly speaking, applicable to data of flood magnitudes. Furthermore, although it provides considerable skew it is not enough for flood data. There remain therefore, the functions with the logarithmic transformation of the variate, and Pearson's functions.

*Distributions of Data of Flood Peaks.* Before proceeding with a discussion of type of function to be used, it is desirable to examine the distributions by plotting the data on probability graph paper, as was done with data of daily precipitation. On Figures 138 to 150 are plotted the data of floods on a number of rivers in various parts of the United States. The same data for the various streams are plotted on arithmetic probability and on logarithmic probability coordinate paper. On the first type of paper the data would plot as a straight line

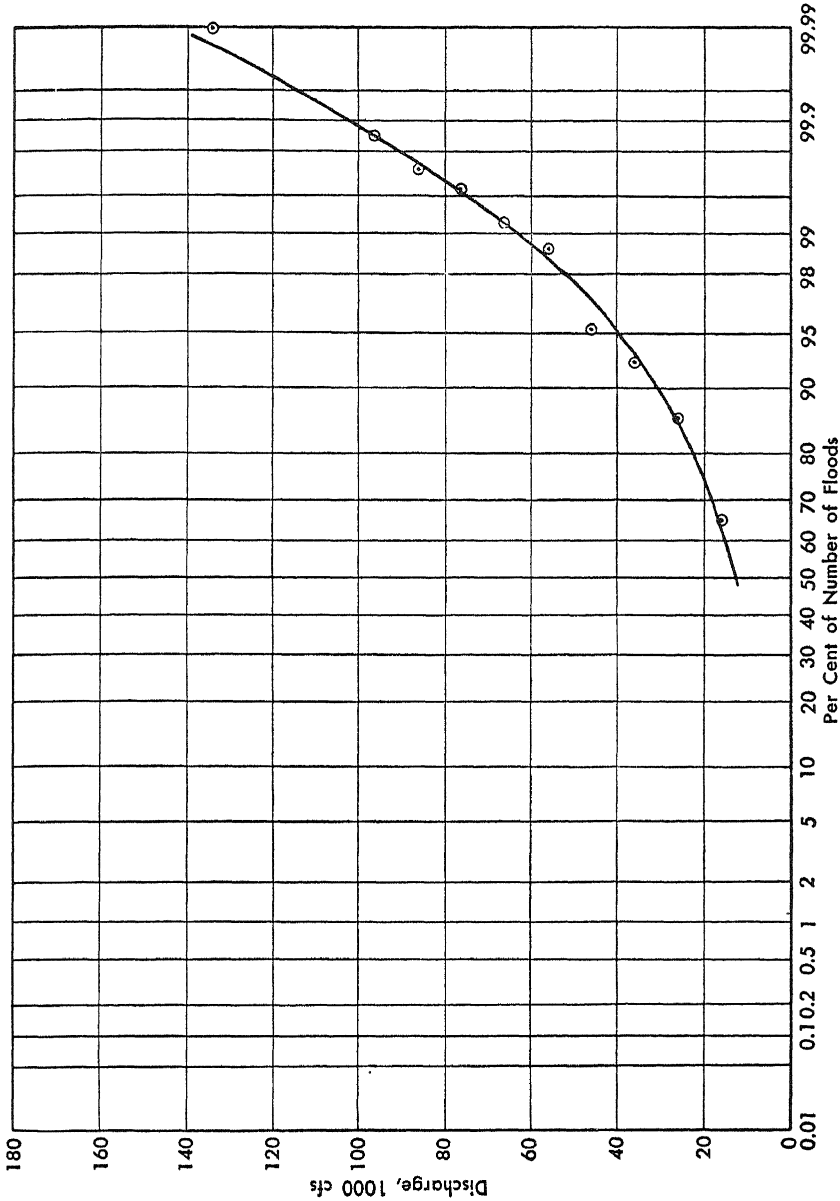



FIGURE 138. Distribution of Flood Data, Chattahoochee River, West Point, Ga., One-Day Peaks

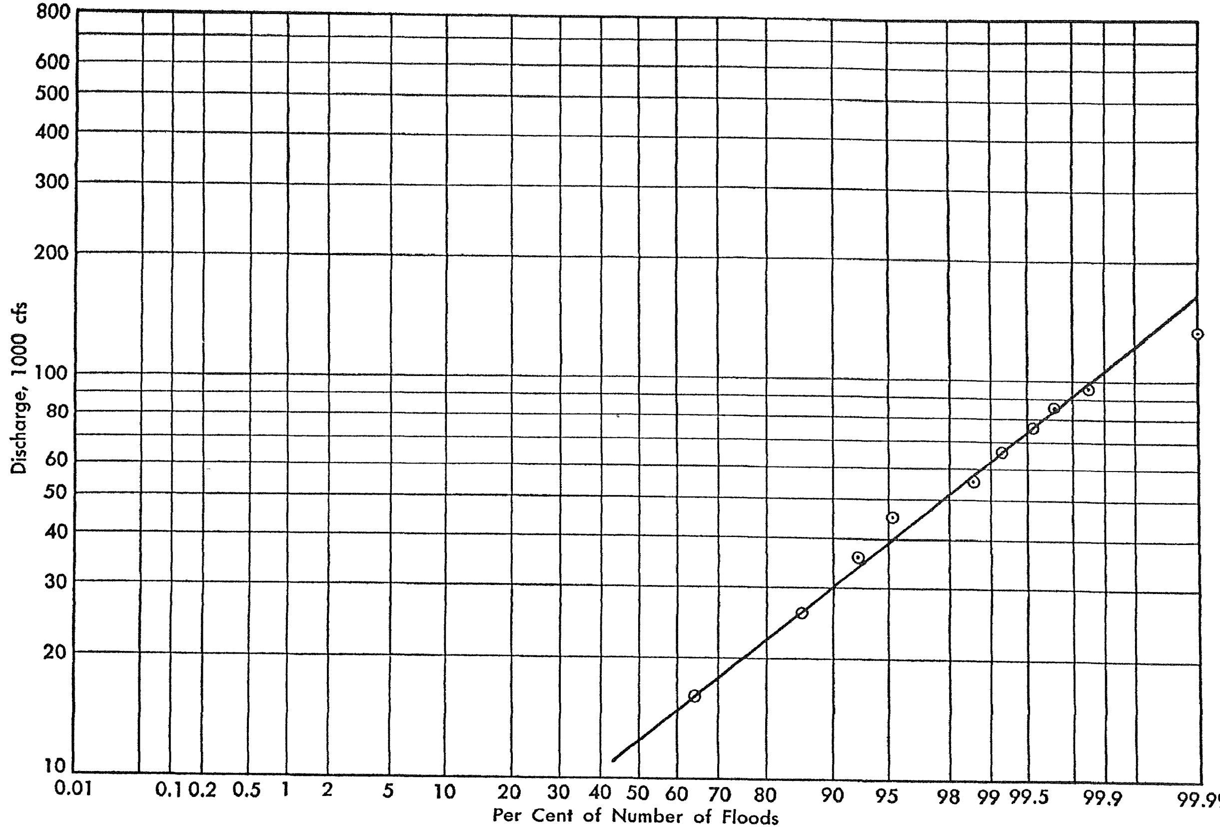



FIGURE 139. Distribution of Flood Data, Chattahoochee River, West Point, Ga., One-Day Peaks

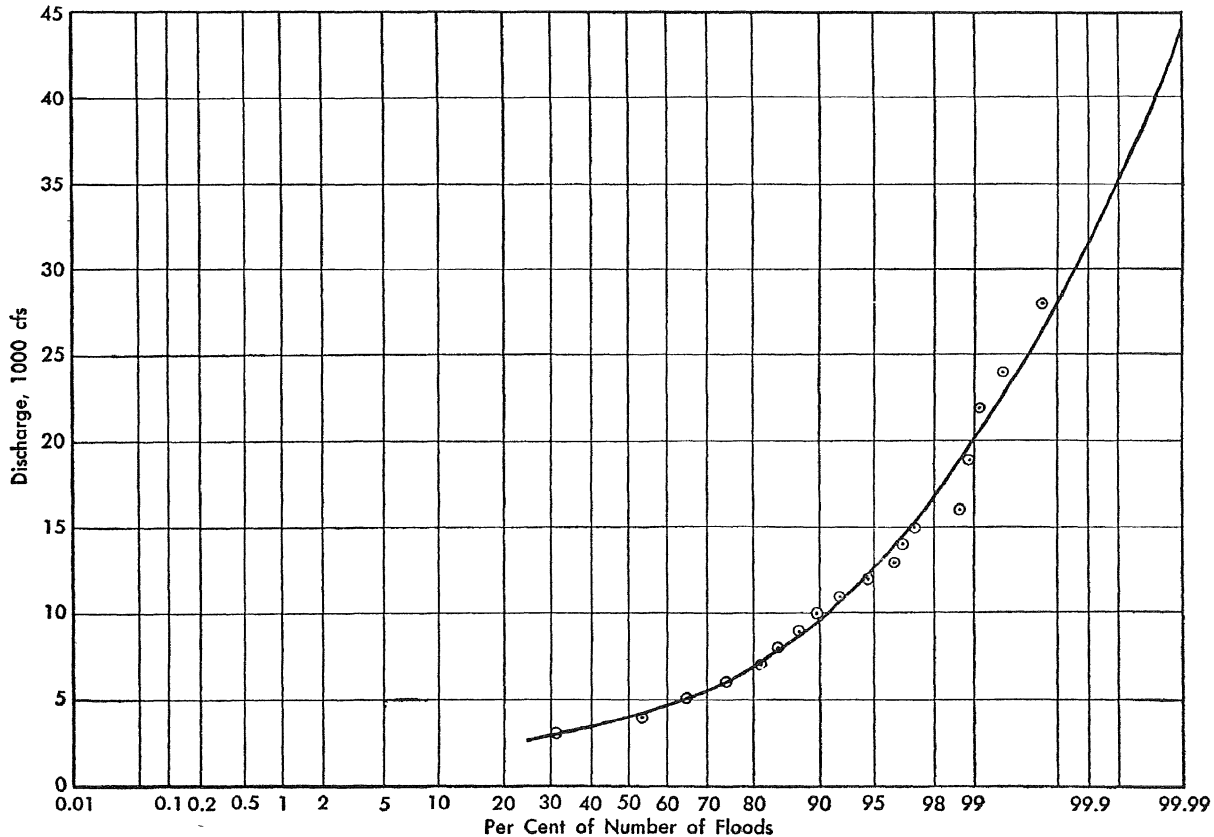



FIGURE 140. Distribution of Flood Data, Potomac River, Point of Rocks, Md., One-Day Peaks

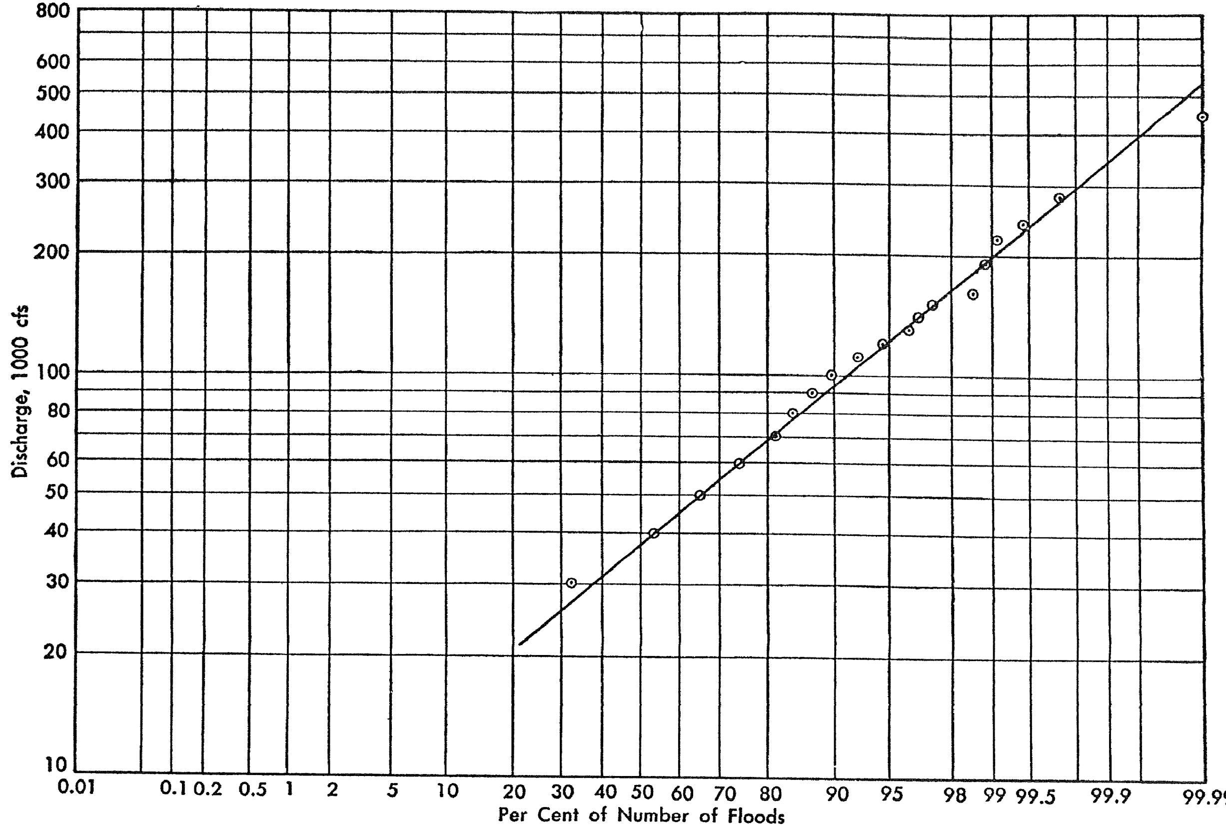



FIGURE 141. Distribution of Flood Data, Potomac River, Point of Rocks, Md., One-Day Peaks

if the distribution approximated the normal probability function. On the other hand, if the distribution is definitely curved on that paper, the distribution cannot be fitted to that function without some modification. Moreover, if the data plot a fairly straight line on the logarithmic probability paper, it may reasonably be concluded that a logarithmic transformation of the variate should fit the observed distributions.

In Figures 138 to 141 are shown the distributions of data of the Chattahoochee and Potomac Rivers plotted on both types of coordinates; the data consist of magnitudes of one-day peaks. In Figures 142 and 143 graphs are shown for one-day peaks and volumes of five-day runoff on the James River. The floods on these three streams are caused chiefly by runoff from intense rainfall, except that those in the winter and spring may have some runoff from snowmelt. All three streams produce substantially straight lines on logarithmic probability coordinates and all are curved on arithmetic probability coordinates. In drawing the curves little weight has been given to the uppermost point because it was the maximum flood of record, and the proper value of time could not be determined, hence it was plotted arbitrarily on the last line of the sheet.

In Figure 144 there are shown the distributions of four streams of the Great Plains. Except the South Platte, these are substantially straight lines on logarithmic probability paper, suggesting a single dominant causative factor, although both rainstorms and snowmelt are causes in all streams. Apparently the two causative factors produce similar distributions of floods, with a possible exception of the South Platte River.

The distribution of the South Platte River is an illustration of the freakish arrays that one may find occasionally and that one must watch for. There is not at the time of this writing sufficient information available to account for its peculiar shape. The drainage area, 3855 square miles, is partly in the mountains and partly on the Great Plains, with one sizable tributary entirely on the Plains. Snowmelt can make a substantial portion of the runoff from the mountains but probably only a small part of the runoff from the Plains. The watershed is subject to small but intense thunderstorms, called "cloudbursts," as well as milder but more prolonged rains. The distribution shown may be the result of these diverse elements or it may simply be an accidental grouping of those flood occurrences, and will become a more normal distribution with additional observations. The curve on Figure 144 should be compared with Figure 150 showing the distribution of momentary peaks.

Two streams in New England are shown in Figures 145 to 148.

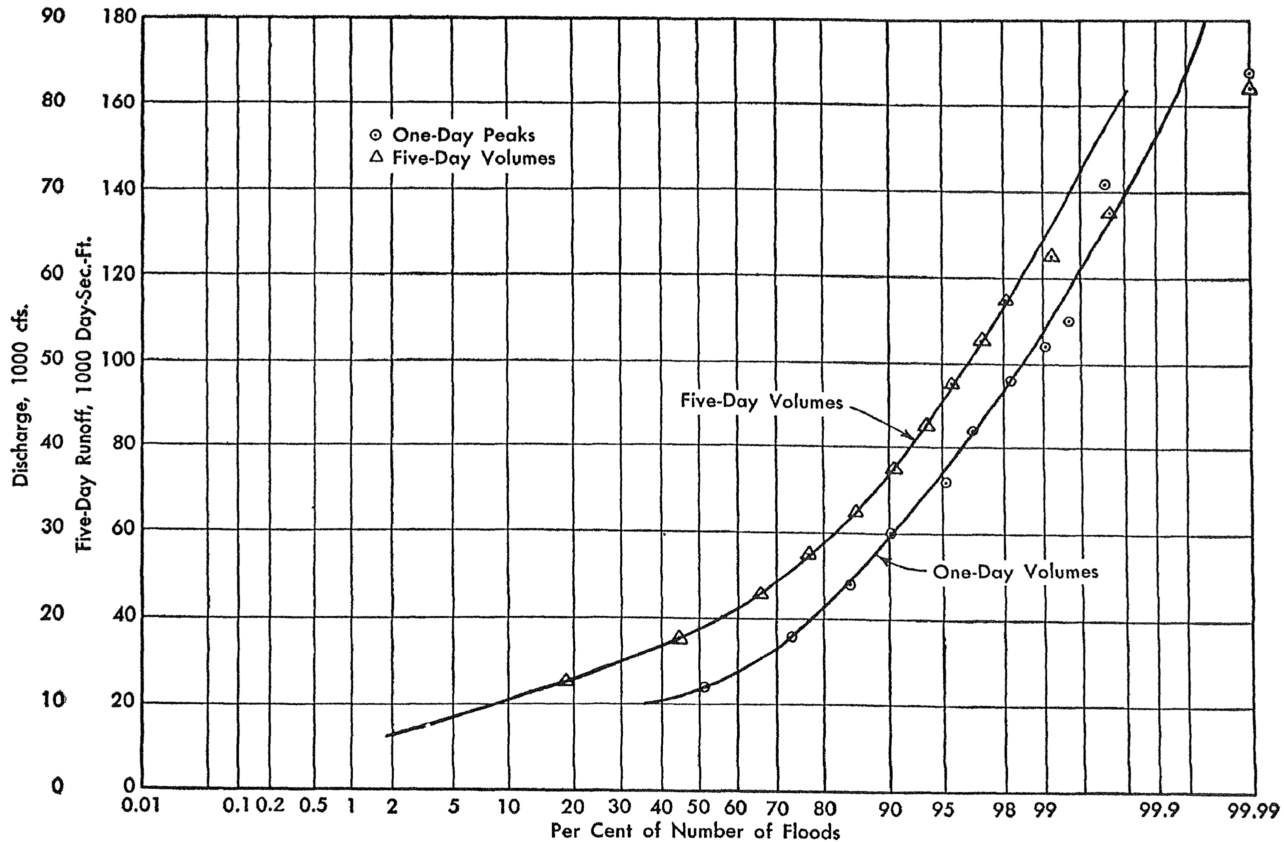



FIGURE 142. Distribution of Flood Data, James River, Buchanan, Va.

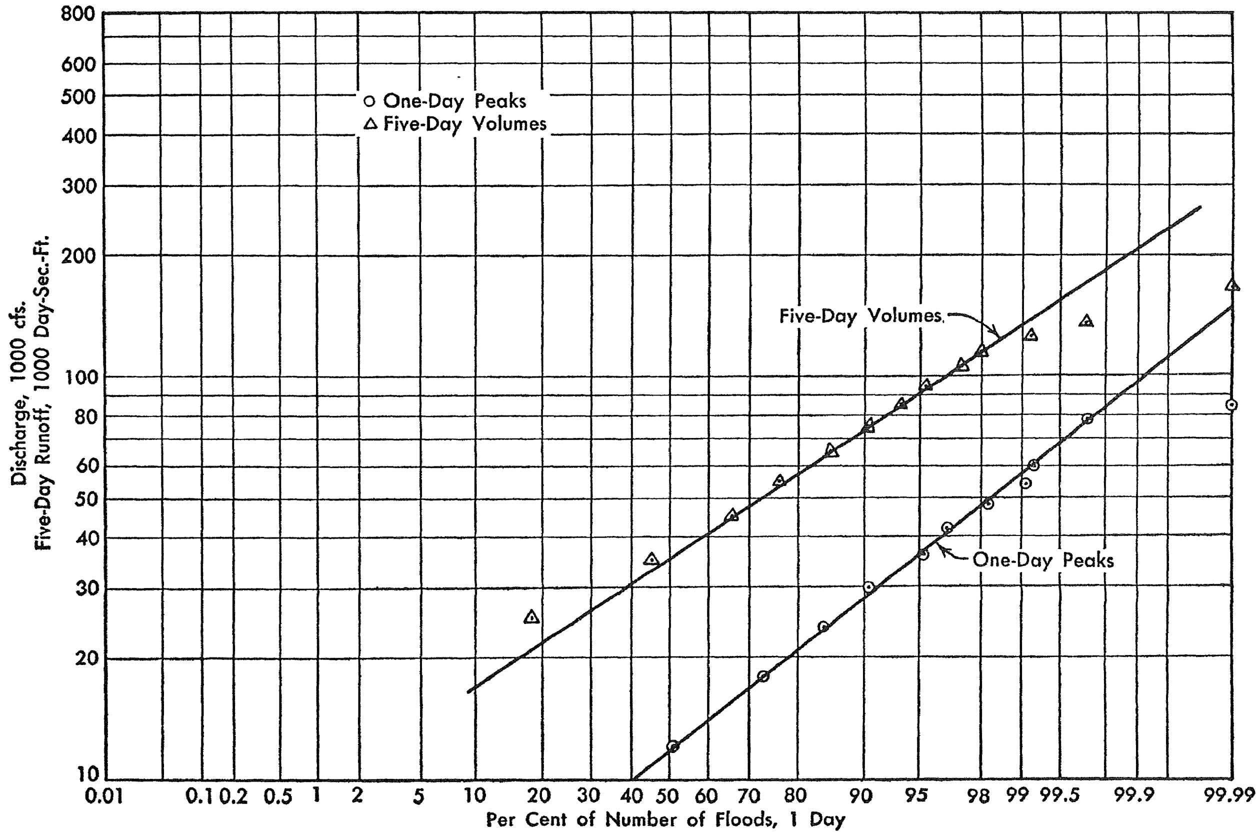



FIGURE 143. Distribution of Flood Data, James River, Buchanan, Va.

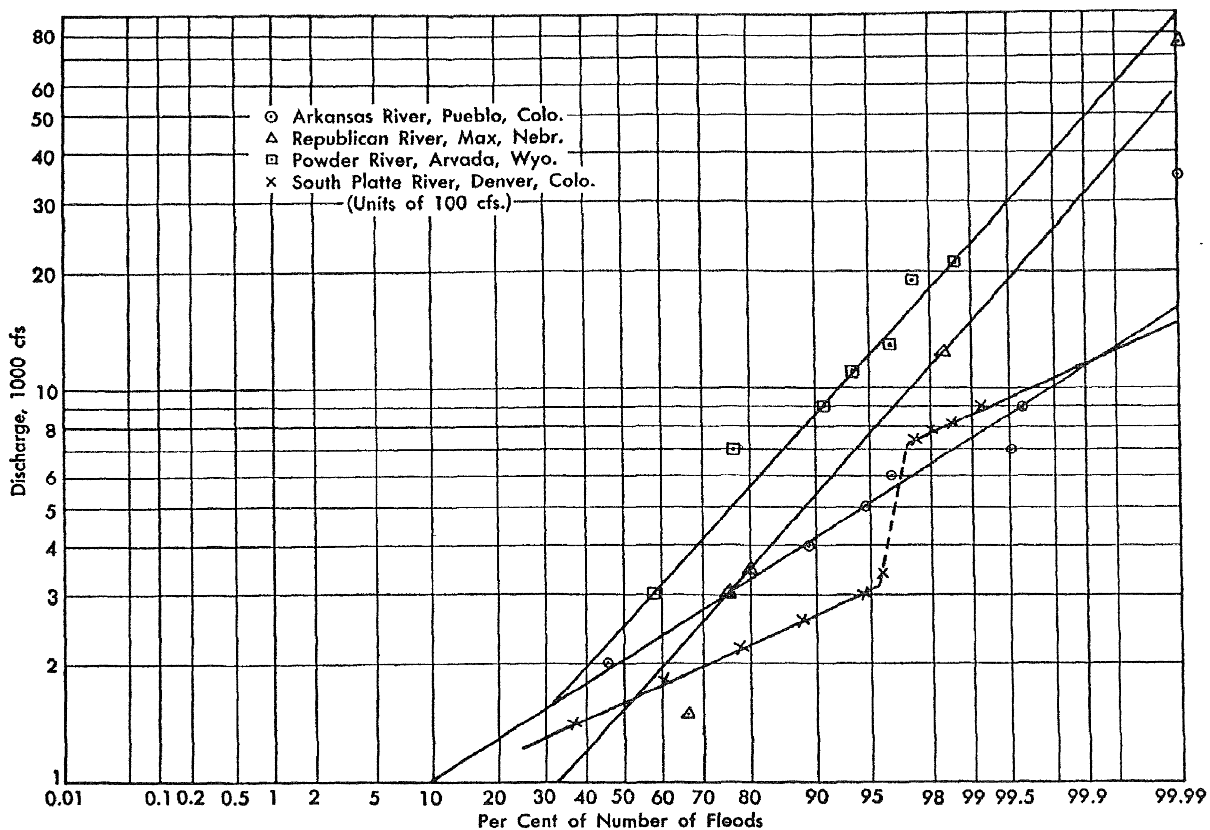



FIGURE 144. Distribution of Flood Data, Rivers on the Great Plains, One-Day Peaks

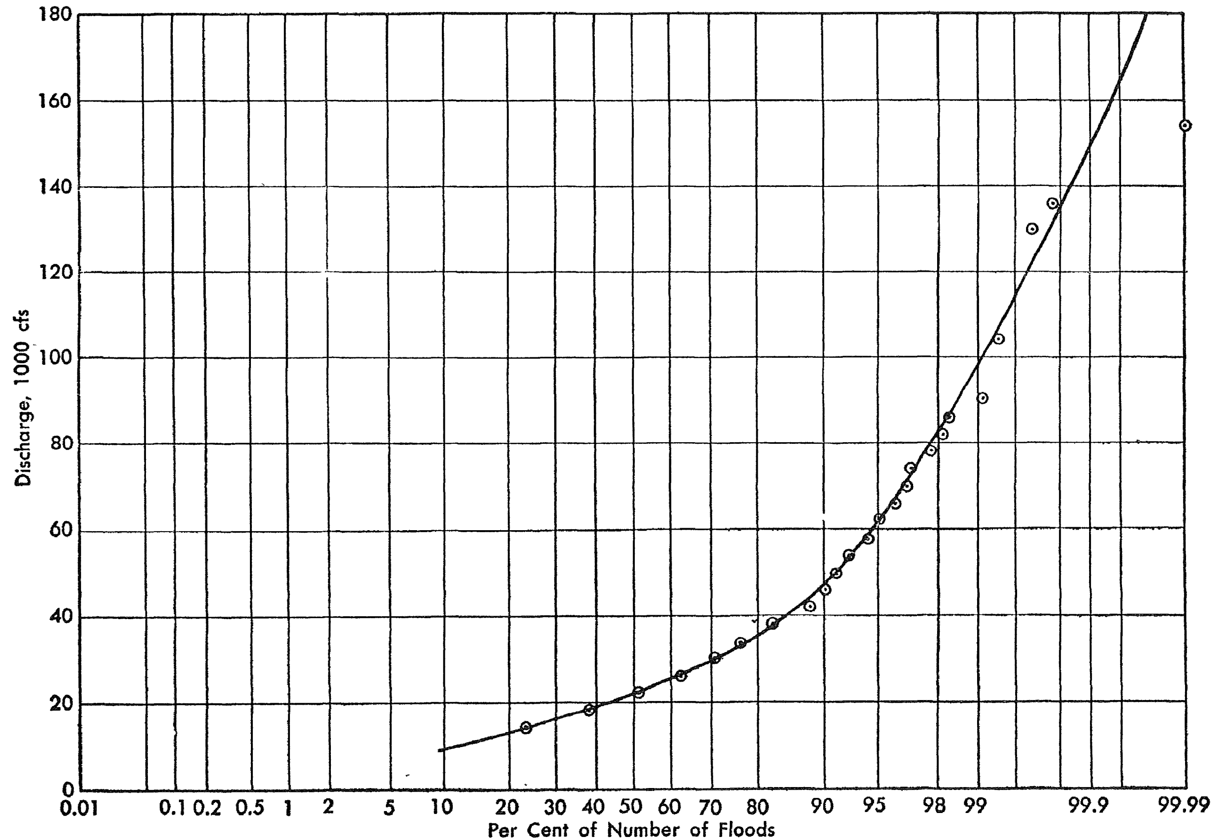



FIGURE 145. Distribution of Flood Data, Kennebec River, Waterville, Maine, One-Day Peaks

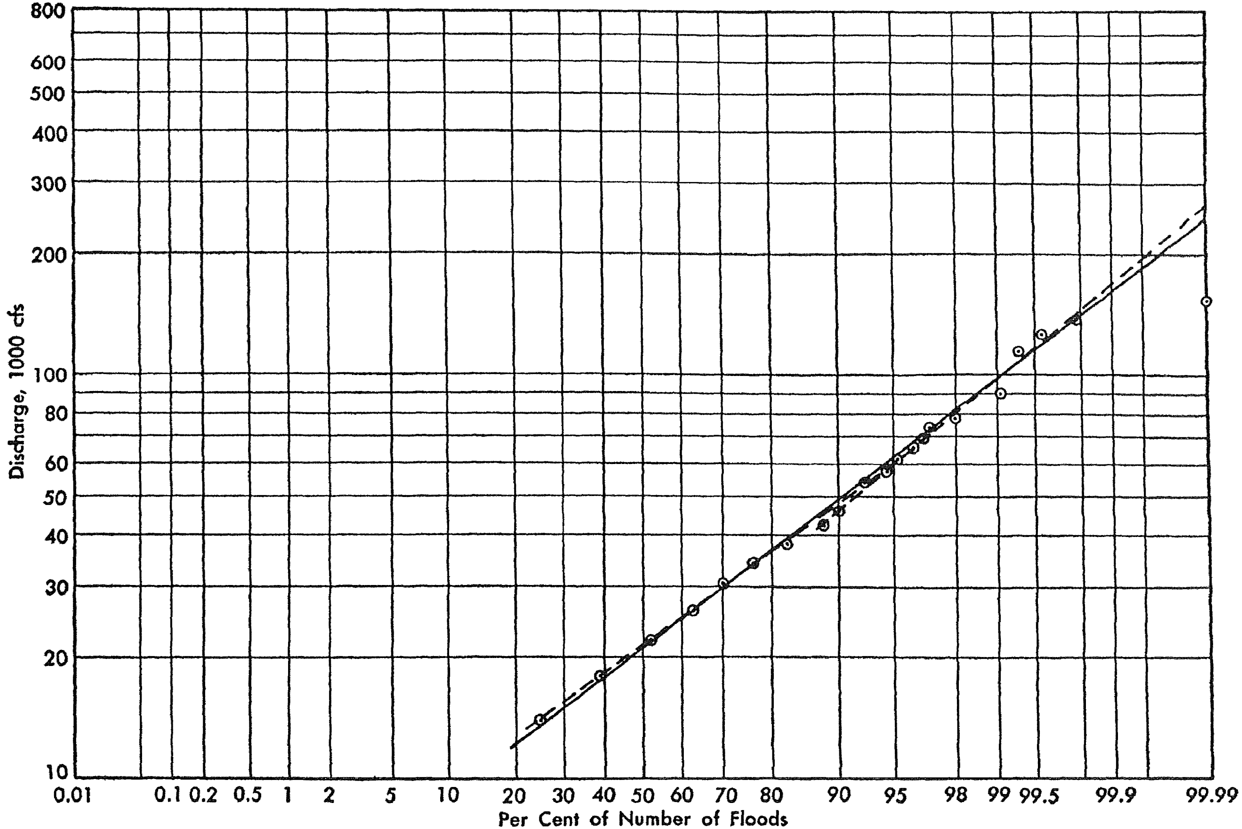



FIGURE 146. Distribution of Flood Data, Kennebec River, Waterville, Maine, One-Day Peaks

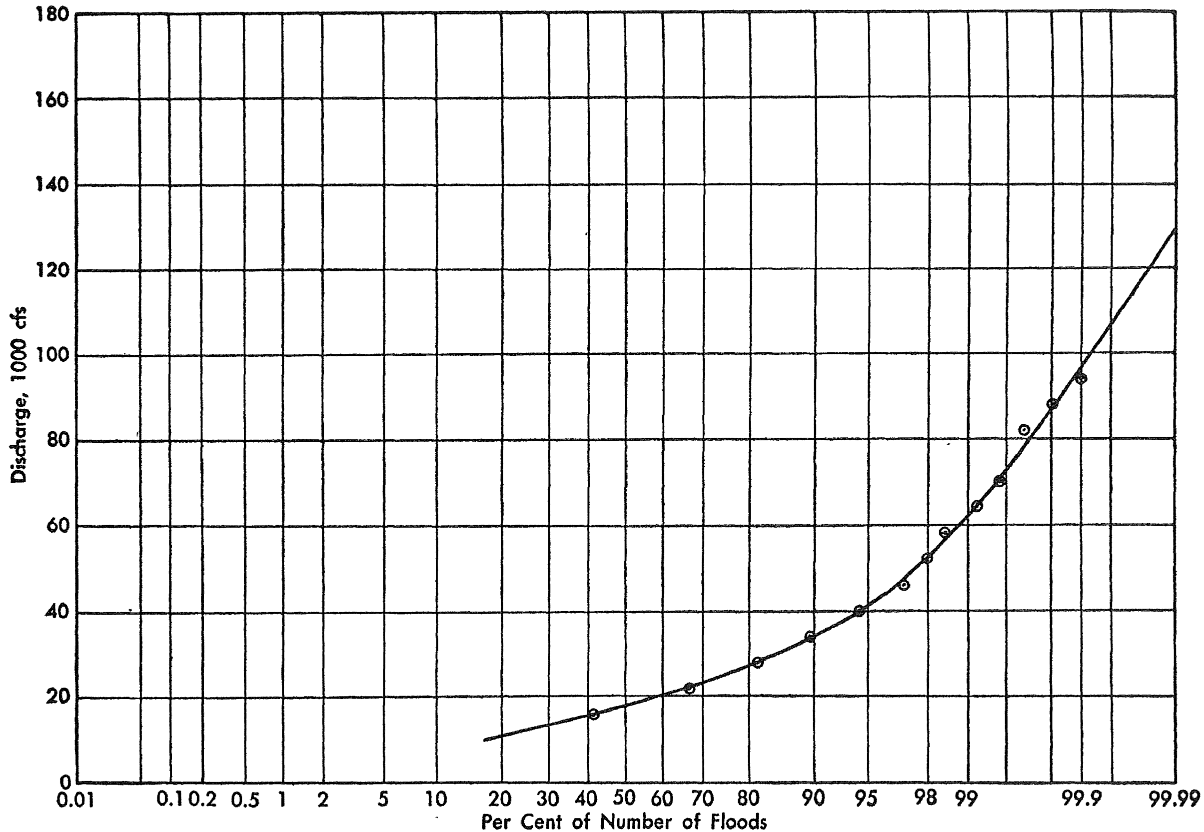



FIGURE 147. Distribution of Flood Data, Merrimack River, Lawrence, Mass., One-Day Peaks

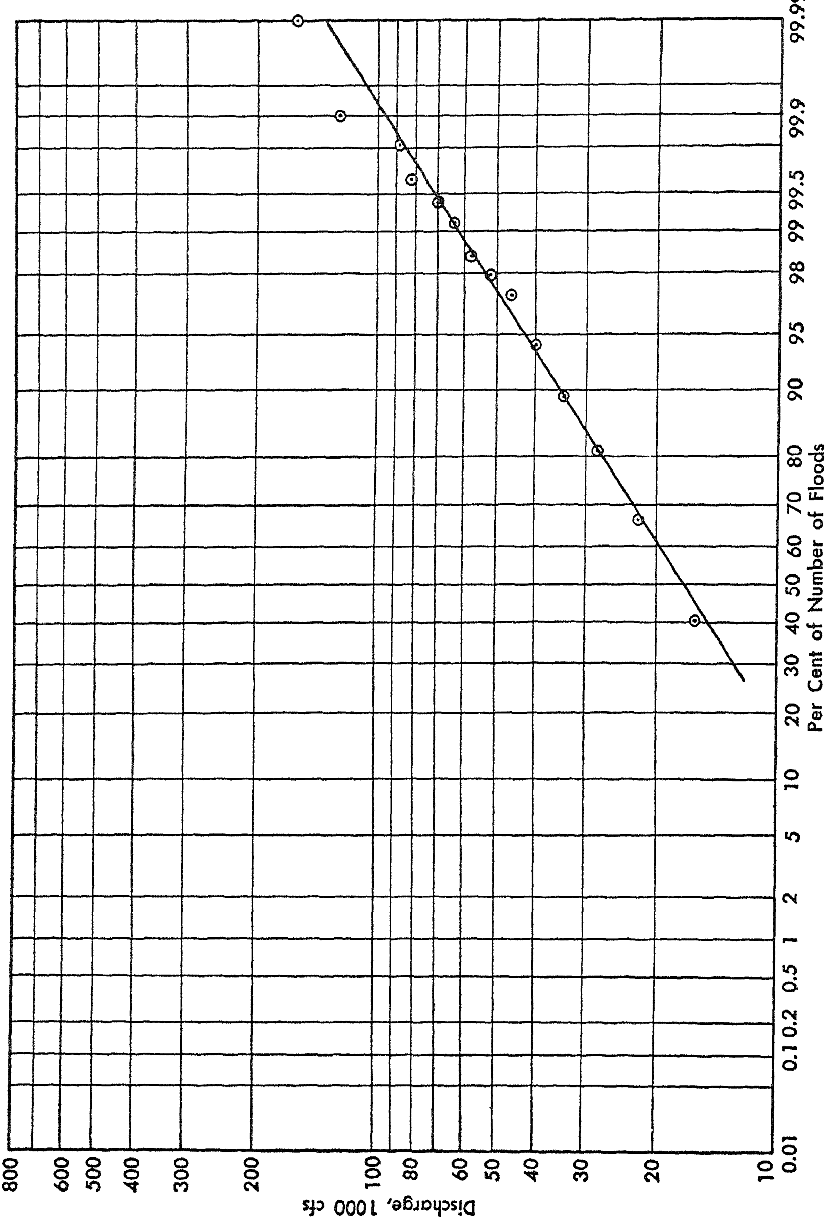



FIGURE 148. Distribution of Flood Data, Merrimack River, Lawrence, Mass., One-Day Peaks

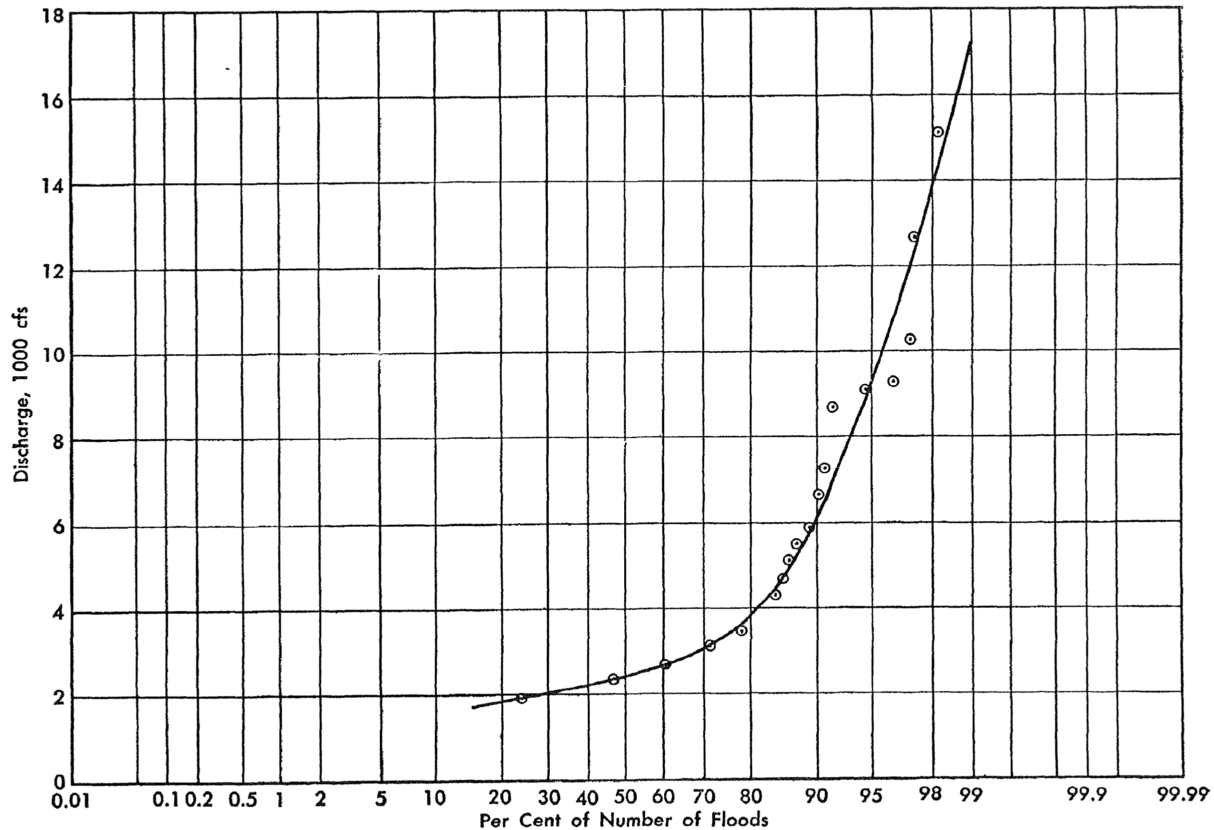



FIGURE 149. Distribution of Flood Data, South Platte River, Denver, Colo., Momentary Peaks

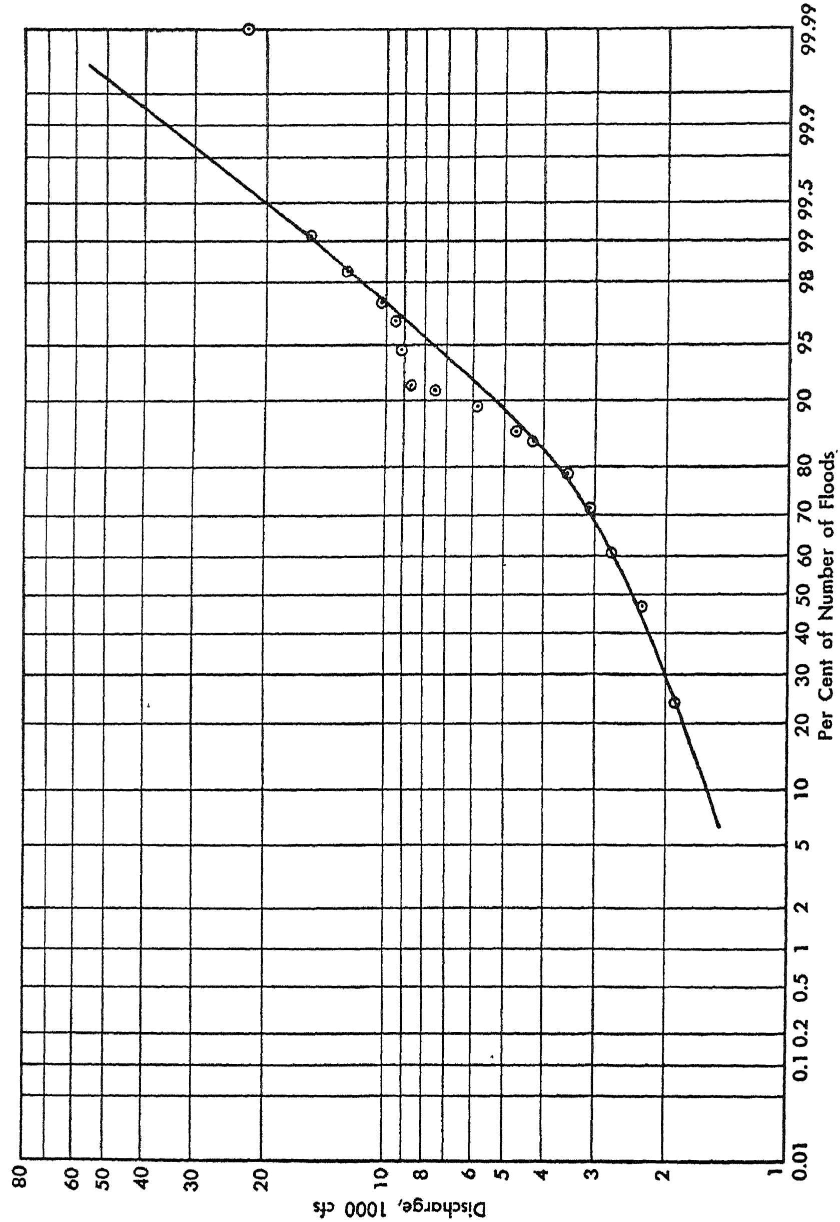



FIGURE 150. Distribution of Flood Data, South Platte River, Denver, Colo., Momentary Peaks

Floods in this region are caused most frequently by spring snowmelt or a combination of snowmelt and rain. The region is subject to invasion of tropical air masses and receives heavy rain which causes some floods. Rarely it suffers from a tropical hurricane. The curves on arithmetic probability paper are distinctly and sharply curved. For both streams a straight line on logarithmic probability paper does not quite fit the data, but two segments of straight lines would fit them well. These breaks in the lines of the data indicate that there are two causes for the floods; that is, while rainfall alone is the predominating cause for the smaller freshets, heavy snowmelt and rain together must occur to cause the higher peaks. However, since the discrepancy in this case is not great there is no serious objection to treating all data as one distribution.

The distribution of data consisting of momentary peak flows on the South Platte River is shown in Figures 149 and 150. As usual, the curve on arithmetic probability paper has strong curvature. On logarithmic probability coordinates two sharply divergent lines are shown indicating two incongruent causes of the peaks, as was seen in the data of one-day peaks shown in Figure 144. Data and other information are not sufficient to determine what the causes may be.

The evidence of the distributions indicates strongly that a probability function with a logarithmic transformation of the variate should fit the distributions of most flood data. On the basis of the evidence then, the frequency curves of the above mentioned streams and others have been computed by means of Slade's partly bounded function; this function does not utilize a predetermined limit of maximum flood, which is a matter that will be considered in detail later.

*Distribution of Data of Number of Floods per Year.* A distribution of flood data of a different sort is presented here for comparison with those in Figures 138 to 150. On Figure 151 is shown a distribution of data of the number of floods experienced in a year. This distribution is substantially a straight line when plotted as shown on arithmetic probability coordinates, indicating that it follows the law of the normal probability function.

*Preparation of Data.* The data for computing flood frequency consist of all flows exceeding a discharge selected as a base. This base is selected somewhat arbitrarily but nevertheless depends upon certain limits. It should be low enough to include all suitable floods both large and small, for the larger the number of data, the more accurate will be the computed parameters and curves. It should be high enough to exclude not only the base flow or ground water, but also small fluctuations caused by power plant operations or other minor artificial

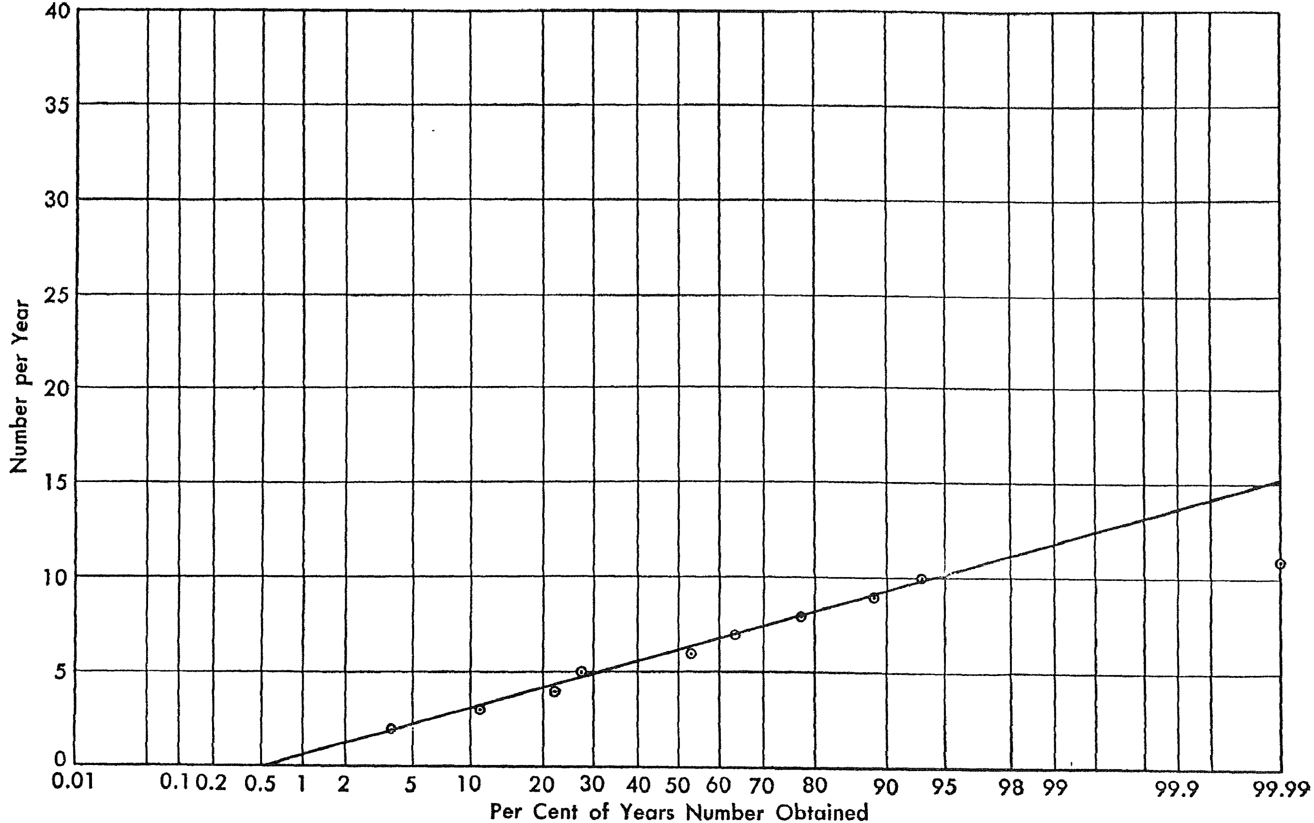



FIGURE 151. Distribution of Data of Number of Floods per Year, Chattahoochee River, West Point, Ga.

regulation. In winter and spring, melting snow in the colder regions causes a steady flow which is similar to ground water flow. The base should be above this steady spring flow, yet it must not be too high or many of the small floods occurring in late summer and fall when the ground water flow is at a minimum but otherwise suitable, will be lost. The proper base flow must be selected on the characteristics of each record and the magnitude varies in different watersheds. The author has used bases amounting to 2 to 10 cubic feet per second per square mile.

No good reason can be found for taking only one flood each year and discarding the remainder. All floods, large and small, which result from natural causes are proper data for a statistical study of frequency of floods. This conclusion is substantiated by the straight line plotting on logarithmic probability paper.

After selecting the base, all peak flows in excess of that value are taken from the records available. The records should be confined to full years in order to have a complete annual cycle. All distinct peaks should be taken that are clearly the result of separate natural causes, which are generally separate rain storms. It is not necessary or desirable to eliminate any portion of the base flow, even in those cases in which a small rain makes an acceptable peak on the receding waters of a previous flood. A small rain producing an appreciable peak on the tail of another flood is a natural chance combination of causes producing that flood peak.

Most available published discharge records are given as mean daily flow for the 24 hours midnight to midnight, or stages that can be converted only to mean daily flow. For that reason, peaks of mean daily flow must be used generally. The momentary peaks would be preferable but records of them are usually not numerous nor continuous enough for statistical study, so that one must be content with mean daily flow.

The calculation of a flood frequency curve consists of two distinct steps, namely, calculation of the probability of a given flood in the series of observations, and calculation of the number of floods to be expected annually. The first is achieved hereinafter by means of Slade's function

$$P = ae^{-c^2[\log d(x+b)]^2}.$$

The second step is made by multiplying the results of the first step by the average number of floods per annum. Both steps will be shown by an example in the computations of the frequencies of floods of the Merrimack River at Lawrence, Mass.

*Climatology of the Merrimack Valley.* The Merrimack watershed is located in central New England in the States of New Hampshire and Massachusetts. The total drainage area is 4,674 square miles but the net contributing area is reduced to 4,463 square miles by diversions for municipal water supply. The precipitation, amounting to 40.24 inches a year, occurs uniformly throughout the year. The average annual temperature is 45.6 F; the average monthly temperature ranges from 21.5 F in January to 69.9 F in July. Most of the precipitation in the months December to March is in the form of snow. In keeping with the latitude, most of the precipitation occurs during the passing of extra-tropical cyclones. The region is traversed by polar continental, polar Atlantic, tropical Atlantic, and transitional air masses. The heaviest precipitation occurs with an invasion of warm tropical air, which, when it comes in spring, also causes rapid melting of the snow cover. As a result of these climatological factors, spring is the principal season for floods on the Merrimack River, while a secondary flood season occurs in the fall. However, floods may occur at any time during the year except in midwinter. Floods may result from rainfall, from snow melting, or a combination of both, when surface conditions are favorable to high runoff.

*Computation of the Frequency Curves, Merrimack River, Lawrence, Massachusetts.* This computation is based on the record of the period 1879–1936, inclusive; the length of the record is 57 years. There were 587 flood peaks taken above the base of 10,000 cfs, which gives an average of 10.3 peaks per year. The base flow of 10,000 cfs was selected somewhat arbitrarily as being a value high enough to eliminate large base flow in the hydrograph and yet low enough to include appreciable peaks during the low water season. A histogram of the distribution is shown in Figure 135.

Slade's partly bounded probability function is used in the integral form, hence the first step is to compute the probability that a flood will exceed a given magnitude. The function expressed in the integral form is

$$P = \frac{N}{\sqrt{\pi}} \int_{-\infty}^{c \log_e \frac{d(x+b)}{t^2+1}} e^{-z^2} dz.$$

The quantity $c \log_e [d(x + b)/(t^2 + 1)]$ is the argument for entering the table of values of the probability integral,

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{z} e^{-z^2} dz.$$

The quantities $b$, $c$, $d$, and $t$ are parameters which are computed from

the first three statistical moments, $M$, $U_2$, and $U_3$. In order to facilitate computations another factor is used, $k = d/(t^2 + 1)$, which eliminates $d$.

The values of the parameters are as follows:

$$\sqrt{2}\,c = \frac{1}{\sqrt{\log_e (t^2 + 1)}}$$

$$b = \frac{(U_2)^{1/2}}{t}$$

$$k = \frac{d}{t^2 + 1} = \frac{t\,(t^2 + 1)^{1/2}}{(U_2)^{1/2}} = \frac{(t^2 + 1)^{1/2}}{b}.$$

The parameter $t$ is derived from the cubic equation $t^3 + 3t - D = 0$, where $D = U_3/(U_2)^{3/2}$.

The classes of peak magnitude, the distribution $f(x)$, and computation of the power sums are shown in Table 68. Following is the calculation of the statistical moments $M_1$, $U_2$, and $U_3$.

TABLE 68. COMPUTATION OF STATISTICAL PARAMETERS, FLOODS ON MERRIMACK RIVER

| CLASSES *1000 cfs* | $X$ | $f(X)$ | $Xf(X)$ | $X^2f(X)$ | $X^3f(X)$ |
|---|---|---|---|---|---|
| 10–16 | 13 | 250 | 3250 | 42250 | 549250 |
| 16–22 | 19 | 143 | 2717 | 51623 | 980837 |
| 22–28 | 25 | 84 | 2100 | 52500 | 1312500 |
| 28–34 | 31 | 47 | 1457 | 45167 | 1400177 |
| 34–40 | 37 | 37 | 1369 | 50653 | 1874161 |
| 40–46 | 43 | 12 | 516 | 22188 | 954084 |
| 46–52 | 49 | 3 | 147 | 7203 | 352947 |
| 52–58 | 55 | 2 | 110 | 6050 | 332750 |
| 58–64 | 61 | 4 | 244 | 14884 | 907924 |
| 64–70 | 67 | 2 | 134 | 8978 | 601526 |
| 76–82 | 79 | 1 | 79 | 6241 | 493039 |
| 82–88 | 85 | 1 | 85 | 7225 | 614125 |
| 154–160 | 157 | 1 | 157 | 24649 | 3869893 |
| Totals | | 587 | 12365 | 339611 | 14243213 |

From the totals of Table 68 above, the statistical moments are computed by the formulas given in Chapter 1.

$$M = \text{average} = \frac{12365}{587} = 21.064736$$

$$U_2 = \frac{339611}{587} - (21.064736)^2 = 134.83056$$

$$U_3 = \frac{14243213}{587} - \frac{3(339611)}{(587)}\,21.064736 + 2(21.064736)^3$$

$$= 6396.997.$$

Next the following values are computed:

$$(U_2)^{1/2} = 11.611656$$
$$(U_2)^{3/2} = 1565.606.$$

The first parameter to be computed is $t$, which is obtained from the cubic equation

$$t^3 + 3t - D = 0$$

in which $$D = \frac{U_3}{U_2^{3/2}}$$

Then $$D = \frac{6396.997}{1565.606} = 4.085956.$$

The value of $t$ may be obtained by any standard method of solving a cubic equation, but because of the type of the equation it can in this case be solved as follows:

$$t = A^{1/3} + B^{1/3}$$

where $$A = \frac{D}{2} + \sqrt{\left(\frac{D}{2}\right)^2 + 1} \text{ and } B = \frac{D}{2} - \sqrt{\left(\frac{D}{2}\right)^2 + 1}.$$

In the present example $t = 1.01422$. It is convenient to obtain next the following values: $t^2 + 1 = 2.02864$; $\sqrt{t^2 + 1} = 1.42430$; and $\sqrt{\log_e (t^2 + 1)} = 0.84105$.

Then $$b = \frac{11.611656}{1.01422} = 11.44880$$

$$k = \frac{1.01422 \times 1.42430}{11.611656} = 0.124406$$

$$2.30258c\sqrt{2} = \frac{2.30258}{.84105} = 2.73775.$$

As indicated by the upper limit of the integral, it becomes necessary at one stage of the computation of probability to find the logarithm of the upper limit, and since the logarithms must be natural and it is convenient to use tables of common logarithms, the conversion factor 2.30258 is introduced here in the value for the parameter $c$ to facilitate computation.

After computing the above parameters, the next step is calculation of the probabilities, as shown in Table 69.

TABLE 69. COMPUTATION OF PROBABILITY OF FLOODS, MERRIMACK RIVER

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $X$ | $x = (X - M + b)$ | $kx$ | $\log_{10}(kx)$ | $z = 2.7 \log (kx)$ | $f(z)$ | $F_t(x)$ | $F(x)$ |
| 16 | 6.38406 | 0.79421 | −0.100064 | −0.27395 | 0.39206 | 230 | 250 |
| 28 | 18.38406 | 2.28709 | .359283 | .98363 | .83735 | 492 | 477 |
| 40 | 30.38406 | 3.77996 | .577487 | 1.58102 | .94307 | 554 | 561 |
| 52 | 42.38406 | 5.27283 | .722044 | 1.97678 | .97597 | 573 | 576 |
| 64 | 54.38406 | 6.76570 | .830313 | 2.27319 | .98850 | 580 | 582 |
| 76 | 66.38406 | 8.25858 | .916905 | 2.51026 | .99396 | 583 | |
| 88 | 78.38406 | 9.75145 | .989069 | 2.70782 | .99662 | 585 | 586 |
| 100 | 90.38406 | 11.24432 | 1.050933 | 2.87719 | .9979936 | | |
| 120 | 110.38406 | 13.73244 | 1.137748 | 3.11487 | .9990798 | | |
| 140 | 130.38406 | 16.22056 | 1.210066 | 3.31286 | .9995382 | | |
| 160 | 150.38406 | 18.70868 | 1.272043 | 3.48254 | .9997516 | 586.85 | 587 |
| 180 | 170.38406 | 21.19680 | 1.326270 | 3.63100 | .9998588 | | |
| 200 | 190.38406 | 23.68492 | 1.374472 | 3.76296 | .9999160 | | |
| 230 | 220.38406 | 27.41710 | 1.438021 | 3.93694 | .9999587 | | |
| 260 | 250.38406 | 31.14928 | 1.493448 | 4.08869 | .9999783 | | |
| 290 | 280.38406 | 34.88146 | 1.542594 | 4.22324 | 0.9999880 | | |

In Table 69, $X$ in the first column is given in 1000 cubic feet per second. The second and third columns are computed as shown by the equation at the top of each; the values in column 4 are merely the common logarithms of the figures in column 3. In column 5, $z = 2.73775 \log_{10}(kx)$; $z$ is then the argument for entering the table of the probability function to obtain the values $f(z)$ in column 6. Column 7, $F_t(x)$, is the computed number of flood peaks to be expected for the classes given in column 1, $f(z)587 = F_t(x)$. Column 8, $F(x)$, is the summation of column 3, Table 68, and is placed in Table 69 for comparison with column 7.

In the above table the probability of the magnitude of a given peak $X$ has been computed. Column 6 in Table 69 gives the probability equal to or less than $X$. It is now to be expressed as equal to or greater

TABLE 70. FREQUENCY OF FLOODS, MERRIMACK RIVER

| $X$ | $1 - f(z)$ | $10.3 \times (2)$ | $X$ | $1 - f(z)$ | $10.3 \times (2)$ |
|---|---|---|---|---|---|
| 16 | 0.60794 | 6.261 | 120 | 0.0009202 | 0.009478 |
| 28 | 0.16265 | 1.675 | 140 | 0.0004618 | 0:004757 |
| 40 | 0.05693 | .5864 | 160 | 0.0002484 | 0.002559 |
| 52 | 0.02403 | .2475 | 180 | 0.0001412 | 0.001457 |
| 64 | 0.01150 | .1185 | 200 | 0.0000840 | 0.000865 |
| 76 | 0.00604 | .0622 | 230 | 0.0000413 | 0.000425 |
| 88 | 0.00338 | .0348 | 260 | 0.0000217 | 0.000224 |
| 100 | 0.002006 | 0.02067 | 290 | 0.0000120 | 0.000124 |

than $X$. After that must be introduced the factor of time to obtain the frequency. These steps are made in Table 70.

In Table 70 the second column gives the probability that any one flow peak, or flood, will equal or exceed the given values of $X$. This is still a probability of magnitude and time has not entered into it.

There was a total of 587 peaks in 57 years. The average number per year is 10.3. Therefore, $10.3[1 - f(z)]$ is the probability of obtaining a peak in any one year equal to or exceeding the given value of $X$, which is expressed in units of 1000 cubic feet per second. Then to determine how often a flood of a given magnitude, say 160,000 cfs, can be expected, 1.0 is divided by the probability of obtaining that flood in one year. Thus, $p$ for 160,000 cfs = 0.00256 and 1/0.00256 = 390 (very nearly) which is the average number of years for which a flood of 160,000 cfs may be expected. This period is sometimes called the "recurrence interval."

To check the assumption that the variations of magnitude and of frequency of occurrence of flood flows are amenable to the laws of probability, two tests are made by comparing computed results with the observations. The first test is made by applying the chi-squared test (Pearson's goodness-of-fit) to the computed results of column 7,

TABLE 71. GOODNESS-OF-FIT OF FREQUENCY OF FLOODS, MERRIMACK RIVER

| CLASS *1000 cfs* | NUMBER SUMMED | EXPECTED *By class* | NUMBER OBSERVED | DEVIATION | DIVERGENCE |
|---|---|---|---|---|---|
| 10 to 16 | 230 | 230 | 250 | 20 | 1.74 |
| 22 | 409 | 179 | 143 | 36 | 7.25 |
| 28 | 492 | 83 | 84 | 1 | .01 |
| 34 | 532 | 40 | 47 | 7 | 1.22 |
| 40 | 554 | 22 | 37 | 15 | 10.25 |
| 46 | 566 | 12 | 12 | 0 | .00 |
| 52 | 573 | 7 | 3 | 4 | 2.28 |
| 58 | 577 | 4 | 2 | 2 | 1.00 |
| 64 | 580 | 3 | 4 | 1 | .33 |
| 70 | 582 | 2 | 2 | 0 | 0.00 |
| 76 | 583 | 1 | 0 | 1 | 1.00 |
| 82 | 584 | 1 | 1 | 0 | .00 |
| 88 | 585 | 1 | 1 | 0 | .00 |
| 154 160 | 586.85 | 1.65 | 1 | 0.65 | 0.26 |
| Total, (Chi-squared) | | | | | 25.36 |
| Total in Classes 46–160 | | | | | 4.87 |

Table 69, with the observations, column 3, Table 68. The second test is made by computing by Poisson's formula the number of peak flows to be expected for various years.

Table 71 contains the comparison of expected number of floods in the classes of Table 68, and the corresponding number of observed peaks.

For 14 classes, and with chi-squared = 25.36, $P'$ (from Pearson's table) = 0.021. For 9 classes, 46 to 160, and with chi-squared = 4.87, $P' = 0.77$. The fit for 14 classes is rather poor, but it is largely caused by two classes and is probably due to chance sampling. The fit for the 9 classes above 40,000 cfs, is good, and these nine classes are the particularly important ones when dealing with flood damages.

*Variation in Yearly Number of Floods, Merrimack River, Lawrence, Massachusetts.* Before continuing with computation of frequency of floods, it seems desirable to discuss in detail the suitability of the average number of floods per annum as a factor in the computation of such frequency. Even a brief experience with the phenomena of floods is sufficient to show that they occur in variable magnitudes and at variable intervals of time. The variation in time results in a fluctuating yearly number of floods. If the number of floods per annum varies by chance, it should have a stable mean and the fluctuations therefrom should be amenable to a law of probability. The yearly number of floods on the Merrimack River at Lawrence is compared below with the probable numbers computed by Poisson's equation. There were altogether 587 peaks, which is an average of 10.3 a year over a period of 57 years. Poisson's equation is

$$P_{(n)} = \frac{e^{-m}m^n}{n!}$$

in which the notation is the same as given in Chapter 1. The factor $m$ is the average annual number of flood peaks and is 10.3. As before, the values for $P_{(n)}$ are taken directly from Pearson's tables. The computation follows in Table 72.

For 13 classes, $\chi^2 = 10.69$, goodness-of-fit from Pearson's tables is 0.56. For 12 classes, excluding the last, $\chi^2 = 7.21$, and goodness-of-fit from Pearson's tables is 0.78. This means that for 13 classes the probability of agreement of the computed number of years with the observed number of years with the same number of floods is good. Therefore it may be inferred that the average annual number of floods is at least as good as any other factor entering into the computation of any flood frequency.

*Frequency Curves, Merrimack River at Lawrence, Massachusetts.* The frequency curve plotted from Table 70 is shown in Figure 152, curve $C$. In this curve the ordinates show the number of floods to be expected annually to equal or exceed the selected abscissas. The circled points

TABLE 72. EXPECTED YEARLY NUMBER OF PEAKS, MERRIMACK RIVER

| NO. OF FLOODS $n$ | $P_n$ | COMPUTED NO. OF OCCURRENCES $57 \times P_{(n)}$ | | OBSERVED NO. OF OCCURRENCES | | CHI-SQUARED TEST *Deviation Comp.-Obs.* | CHI-SQUARED TEST *Divergence* |
|---|---|---|---|---|---|---|---|
| 0–3 | 0.00829 | 0.47 | | 1 | | | |
| 4 | .0158 | .90 | 6.41 | 2 | 8 | −1.59 | 0.40 |
| 5 | .0325 | 1.86 | | 4 | | | |
| 6 | .0558 | 3.18 | | 1 | | | |
| 7 | .0821 | 4.68 | | 8 | | −3.32 | 2.35 |
| 8 | .106 | 6.03 | | 6 | | .03 | 0 |
| 9 | .121 | 6.89 | | 5 | | 1.89 | .52 |
| 10 | .125 | 7.12 | | 4 | | 3.12 | 1.37 |
| 11 | .117 | 6.66 | | 6 | | .66 | .07 |
| 12 | .100 | 5.70 | | 6 | | −.30 | .02 |
| 13 | .0793 | 4.52 | | 3 | | 1.52 | .51 |
| 14 | .0584 | 3.33 | | 2 | | 1.33 | .53 |
| 15 | .0401 | 2.29 | | 4 | | −1.71 | 1.28 |
| 16 | .0258 | 1.47 | | 1 | | .47 | .15 |
| 17 | .0156 | .89 | | 1 | | −0.11 | 0.01 |
| 18 | .00894 | .52 | | 0 | | | |
| 19 | .00485 | .28 | | 0 | | | |
| 20 | .00250 | .14 | 1.07 | 1 | 3 | −1.93 | 3.48 |
| 21 | .00122 | .07 | | 1 | | | |
| 22–inf. | 0.001014 | 0.06 | | 1 | | | |
| | | | | | | $\chi^2 =$ | 10.69 |

along the curve are plotted from the observed floods, the number of which, given as $f(x)$ in Table 68, is accumulated from the maximum to minimum class. These points are plotted to show the observed number of floods which have exceeded the lower class limit during the period of observation. The observed points are therefore comparable to the curve. The curve itself is plotted entirely from the computations.

Four different curves have been computed for the station at Lawrence, Mass., each of which covers a somewhat different period of record. The periods of record and magnitude of peak for a given frequency are given in Table 73 to compare the effect of the changes produced by different lengths of record.

TABLE 73. FLOOD FREQUENCIES ON MERRIMACK RIVER, LAWRENCE, MASS.

| CURVE | PERIOD | FREQUENCY IN YEARS TO GIVE TABULATED DISCHARGE IN 1000 CFS *1 yr* | *10 yr* | *25 yr* | *100 yr* | *250 yr* |
|---|---|---|---|---|---|---|
| *A* | 1849–1938 | 34 | 69 | 86 | 117 | 143 |
| *B* | 1849–1936 | 36 | 66 | 83 | 114 | 136 |
| *C* | 1879–1936 | 34 | 68 | 85 | 118 | 147 |
| *D* | 1879–1935 | 36 | 55 | 63 | 82 | 93 |

These curves show the effects of big differences in length of record and the result of one or two floods relatively much larger than the

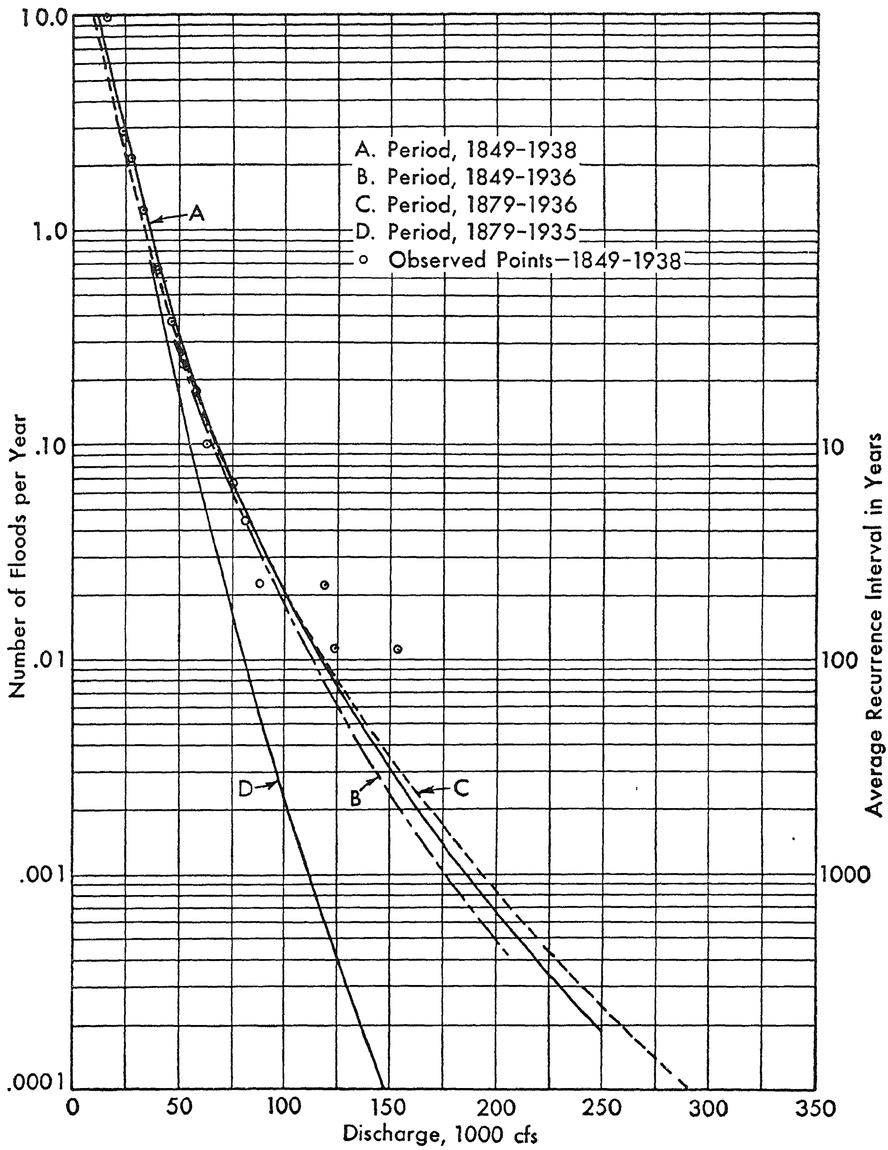



FIGURE 152. Flood Frequency, Merrimack River, Lawrence, Mass., One-Day Peaks

remainder of the data. The largest flood of record occurred in March, 1936; the second largest occurred in September, 1938. The effect of the occurrence of the second big flood is shown by curves *A* and *B*. The effect of a big difference in length of record is shown by curves *B* and *C*. The effect of one large flood in a record of small and moderate peaks is shown by curves *C* and *D*; this difference, it will be noted, is by far the largest. This series of curves indicates that once a frequency

curve based on representative distribution of an average proportion of low, moderate, and high floods is obtained, the occurrence of another large flood (or more years of record) has relatively small effect on a frequency curve computed by mathematically sound methods. Curve $D$ was not computed from a representative distribution because it did not contain a flood in the higher magnitudes.

Certain significant parameters from which the above curves were computed are given in Table 74 for comparison with each other in connection with the preceding table.

TABLE 74. CHARACTERISTIC FREQUENCY VALUES, MERRIMACK RIVER

| CURVE | MOMENTS | | | SKEWNESS | | |
|---|---|---|---|---|---|---|
| | $M$ | $U_2$ | $U$ | $D$ | $t$ | $b$ |
| $A$ | 21.27 | 134.8 | 5644.4 | 3.61 | 0.932 | 12.47 |
| $B$ | 21.18 | 127.6 | 4872.8 | 3.38 | 0.891 | 12.68 |
| $C$ | 21.06 | 134.8 | 6397.0 | 4.09 | 1.014 | 11.45 |

*Note:* Computed from flow units of 1000 cfs.

None of the items in the foregoing table differ widely from other values of the same type. Nevertheless, agreement between moments close enough to explain the similarity between curves $A$ and $C$ is found only in the moment $U_2$, which indicates the importance of the second moment in computed frequencies.

*Frequency Curves, Merrimack at Franklin Falls and Connecticut River.* In Figure 153 are two curves of the Merrimack River at Franklin, N. H., and one of the Connecticut River at Montague City, Mass. Curves $A$ and $B$ cover two periods of the Franklin station; the shorter, 1904–1936, includes the highest flood of record and the longer, 1904–1938, includes also the second largest. The frequencies are compared in Table 75.

TABLE 75. FLOOD FREQUENCIES, MERRIMACK RIVER, FRANKLIN FALLS, N. H.

| CURVE | PERIOD | FREQUENCY IN YEARS TO GIVE TABULATED DISCHARGE IN 1000 CFS | | | | |
|---|---|---|---|---|---|---|
| | | *1 yr* | *10 yr* | *25 yr* | *100 yr* | *250 yr* |
| $A$ | 1904–1936 | 20.0 | 38.0 | 48.0 | 67.0 | 81.0 |
| $B$ | 1904–1938 | 20.0 | 39.0 | 50.0 | 70.0 | 84.5 |

Fairly close agreement is shown here also, indicating again that a frequency curve based on a full range of observational data is not subject to much change.

An interesting coincidence is the closeness of the frequency of the flood of 1936 obtained from these curves for periods ending in 1936. For the Merrimack River, discharge 73,700 cfs, the frequency has a

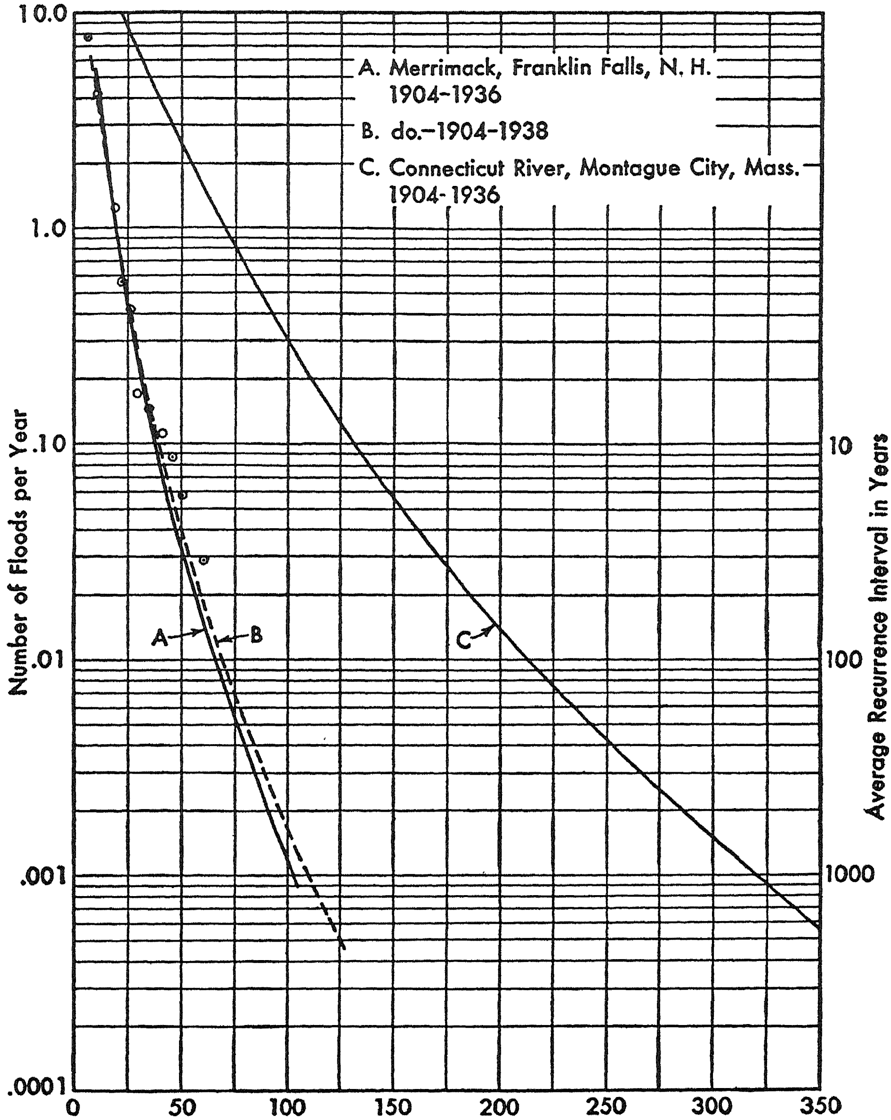



FIGURE 153. Flood Frequencies, New England Rivers, One-Day Peaks

period of 160 years (being read to the nearest 5 years), and for the Connecticut River, discharge 233,000 cfs, a frequency of 160 years. These two stations have adjacent drainage basins, have records of about the same length, and have similar flood histories in that both were affected strongly by the floods of 1927 and 1936. This indicates that given similar flood histories, similar floods on adjacent basins may be expected to have similar frequencies.

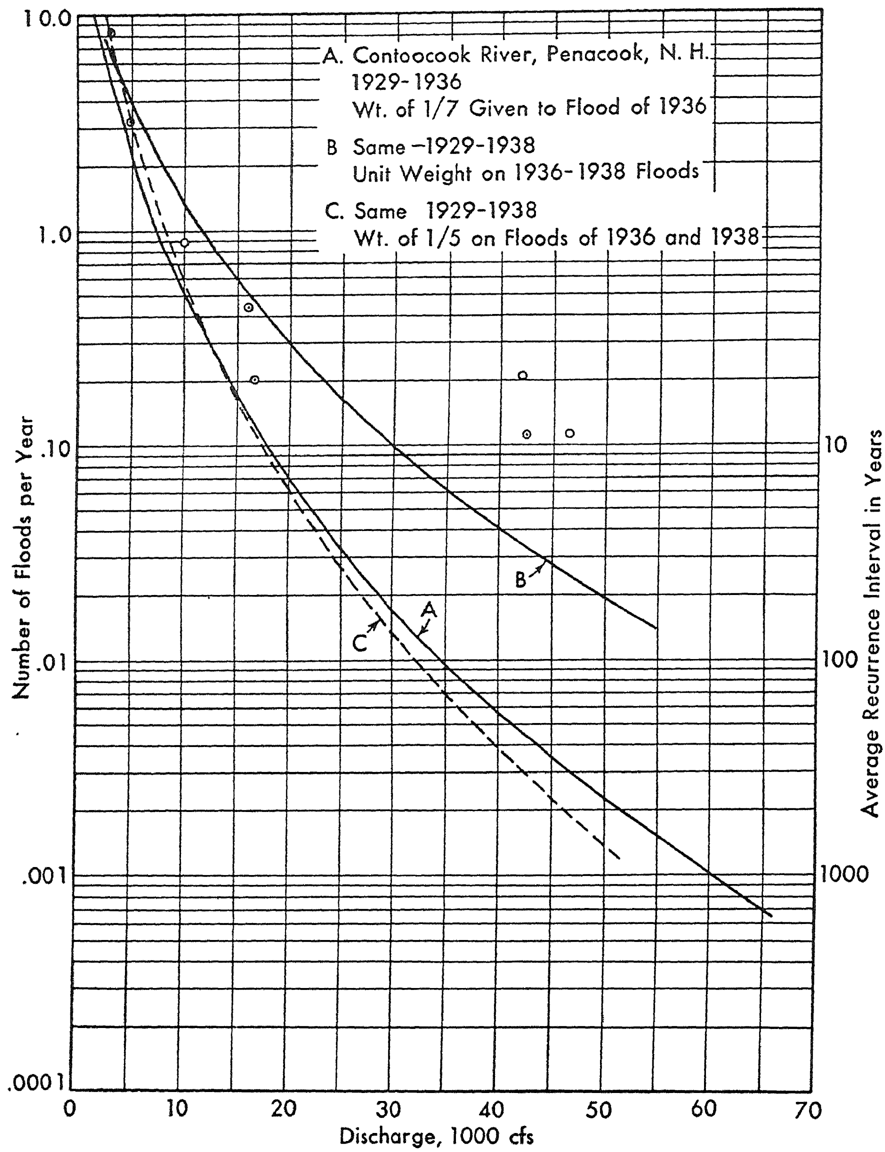



FIGURE 154. Flood Frequencies, a River in New Hampshire, One-Day Peaks

*Frequency Curves, Contoocook River, New Hampshire.* The effect of weighting a big flood occurring in a short record is shown in Figure 154, curves *A*, *B*, and *C* of the Contoocook River at Penacook, N. H. The shorter record covers the years 1929–36; the frequency curve which includes the flood of 1936 with a unit weight is shown by curve *B*. In curve *A*, the same flood has a weight of one-seventh. Curve *C* contains two notably high floods, 1936 and 1938, each with a weight of

one-fifth. These weights were selected because the periods of record to 1936 and 1938 were respectively one-seventh and one-fifth of that of the Pemigewasset River near Plymouth, N. H., a station in the region and with a similar flood history, but a longer record. The significant parameters of the Contoocook River frequency curves are given below:

TABLE 76. COMPARISON OF MOMENTS, CONTOOCOOK RIVER, N. H.

| CURVE | $M$ | $U_2$ | $U_3$ | $D$ |
|---|---|---|---|---|
| $A$ | 4.68 | 30.99 | 980.7 | 5.68 |
| $C$ | 4.18 | 12.20 | 244.7 | 5.80 |

*Note:* Computed from flow units of 1000 cfs.

Since $D$ in each is practically equal to the other, it may be seen that the difference in the two curves is caused to a large extent by the difference in the second moments $U_2$, the square root of which enters through the parameters $b$ and $k$.

*Frequency Curves, Pemigewasset River, New Hampshire.* The two curves of the Pemigewasset River in Figure 155 illustrate again the effect of extending the record, curve $B$ being computed from a record 23 years shorter than that for curve $A$. The moments are as follows:

TABLE 77. COMPARISON OF MOMENTS, PEMIGEWASSET RIVER, N. H.

| CURVE | $M$ | $U_2$ | $U_3$ |
|---|---|---|---|
| $A$ | 11.23 | 34.83 | 678.8 |
| $B$ | 10.61 | 35.11 | 961.3 |

*Comparison of Frequency Moments.* In the following paragraphs there are given for comparison the moments of a number of flood frequency curves computed for various streams in the United States east of the Rocky Mountains. These moments of course must be put on a uniform basis in order to permit such comparison, but because of the nature of these frequency curves with their extreme skew, comparison by the usual statistical parameters such as the standard deviation and the coefficient of variation cannot properly be used. Furthermore, each curve is for floods of an area different in size and flood-producing conditions. Therefore, in order to avoid these difficulties, the comparison of the moments is made on the basis of the computed values of $M$, $U_2$, and $U_3$ reduced to a value per unit area by dividing each by the drainage area.

This procedure is similar to the comparison of flood peaks of different watersheds, such as is made by dividing the peak flow by the drainage area to obtain the discharge per unit area in the manner discussed in this chapter. The effect of the area is thus eliminated from the mo-

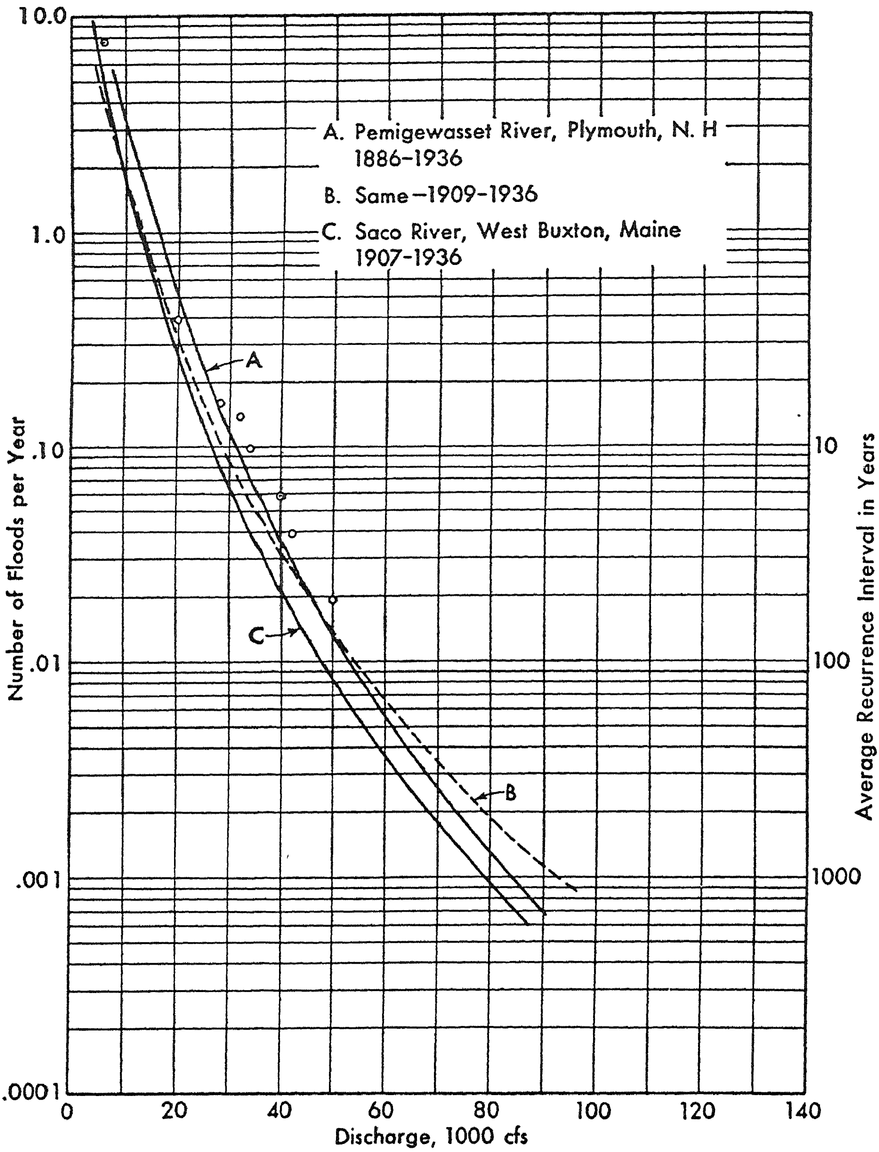



FIGURE 155. Flood Frequencies, Two Rivers in New Hampshire, One-Day Peaks

ments to show the net results for the basin characteristics and the storms and other flood-causing factors of the region producing the given distributions of flood peaks.

Usually the frequency curves were computed by using units of flow of 1000 cfs, as in the example of the Merrimack River. In a few instances the curves were computed in other units of flow, either 100 or 10,000 cfs, being respectively 1/10 or 10 times the usual unit. These

curves were converted to a basis of 1,000 cfs units. In the case of the smaller units the moments $M$, $U_2$, and $U_3$ were divided by 10, 100, and 1,000, respectively, and in the case of the larger units they were multiplied by 10, 100, and 1,000, respectively. The reason for this can be seen easily by inspection of Table 68, where $f(x)$ is multiplied by the powers of the value $X$ of the mid-points of the respective classes, which powers are $X$, $X^2$, and $X^3$. Therefore, to make the conversion, the factor 10 must be applied in the same powers.

*Comparison of Moments, Saco and Pemigewasset Rivers.* The first comparison of this sort is made of the Saco and Pemigewasset Rivers, in Table 78.

TABLE 78. FREQUENCY MOMENTS, SACO AND PEMIGEWASSET RIVERS

| RIVER | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| | *Square Miles* | $M$ | $U_2$ | $U_3$ |
| Saco, W. Buxton, Maine | 1,572. | 0.00426 | 0.0136 | 0.254 |
| Pemigewasset, Plymouth, N. H. | 622. | 0.0181 | 0.560 | 1.090 |
| Ratio: Pemigewasset to Saco | 0.396 | 4.25 | 4.12 | 4.29 |

*Note:* Computed from flow units of 1000 cfs.

Comparison of the frequency curves of the Saco and Pemigewasset Rivers shows that they are not greatly different for comparable periods of record, but they illustrate the effect of channel storage on the frequency of flood peaks. The upper portion of the watersheds of each are adjacent and both streams originate in the White Mountains, hence have approximately the same source conditions. However, before reaching the gaging station at West Buxton, the Saco River passes through some lakes and broad flat valleys which provide much channel storage, and thereby reduce the flood peaks. This situation does not occur on the Pemigewasset River so that peaks of comparable size for a given frequency are obtained on each stream. However, a comparison of the moments given in Table 78 shows that differences in the characteristics of the drainage basins result in a substantial balancing of the larger drainage basin and flat peaks of the Saco River to give peak flows for similar frequencies that differ but little from those produced by widely different characteristics of the Pemigewasset River. It can be noted from the table that while the drainage area of the Pemigewasset River is 0.40 of the Saco, its mean and moments on the basis of unit area range from 4.12 to 4.28 times as big.

*Comparison of Moments, Rivers in Maine.* In Figure 156 are shown a group of frequency curves of streams in Maine. Their moments, reduced to values per square mile, are listed in Table 79.

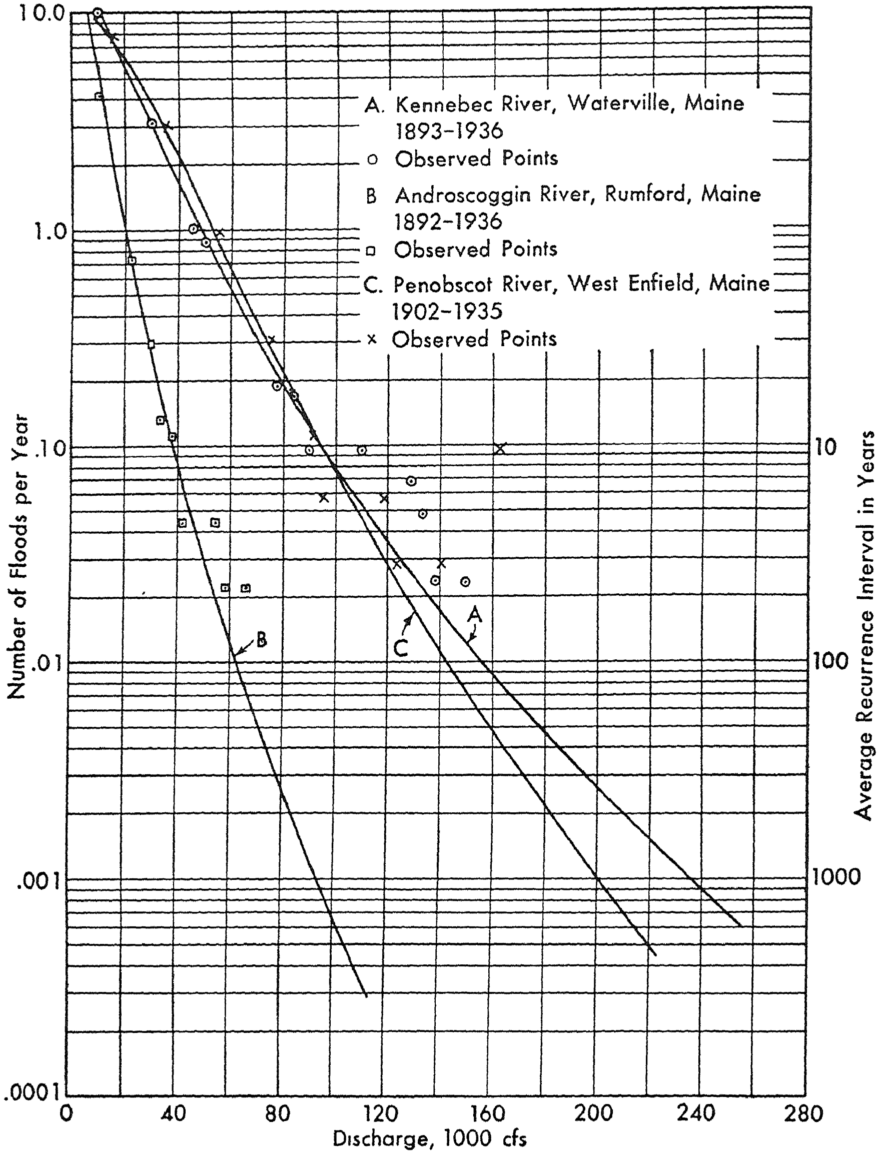



FIGURE 156. Flood Frequencies, Maine Rivers, One-Day Peaks

TABLE 79. FREQUENCY MOMENTS, RIVERS IN MAINE

| RIVER | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| | *Square Miles* | *M* | $U_2$ | $U_3$ |
| Kennebec, Waterville, Maine | 4,270* | 0.00881 | 0.109 | 5.390 |
| Androscoggin, Rumford, Maine | 2,090** | 0.0131 | 0.0514 | 1.050 |
| Penobscot, West Enfield, Maine | 6,600 | 0.00524 | 0.0525 | 1.905 |

*Note:* Computed from flow in units of 1000 cfs.
* 1240 square miles under complete control.
** 1095 square miles under complete control.
There is considerable diversity in the watersheds of these streams, which is reflected in the variation of the moments. Storage in artificial and natural lakes is conspicuous in all the above streams.

*Comparison of Moments, Rivers in Middle Atlantic States.* The frequency curves in Figure 157 belong to a group of Middle Atlantic streams. The statistical moments, reduced to a basis of discharge per square mile, are as shown in Table 80.

TABLE 80. FREQUENCY MOMENTS, RIVERS IN MIDDLE ATLANTIC STATES

| RIVER | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| | *Square Miles* | *M* | $U_2$ | $U_3$ |
| Potomac at Point of Rocks, Md. | 9,651 | 0.00538 | 0.168 | 26.10 |
| James at Buchanan, Va. | 2,080 | 0.00757 | 0.0509 | 1.290 |
| Shenandoah, at Millville, W. Va. | 3,040 | 0.00578 | 0.0813 | 4.135 |

*Note:* Computed from flow in units of 1000 cfs.

These three streams have their headwaters in adjacent regions within the Appalachian Mountains. Part of the region is underlain with cavernous limestone formations which in some degree affect the runoff in a manner similar to surface lakes. Since the Shenandoah River, a tributary of the Potomac, is strongly affected by this condition, its flood peaks tend to be broad and flat, thus showing lower peak values than would normally be expected for the drainage area.

*Comparison of Moments, Rivers in Southeastern United States.* The two frequency curves in Figure 158 belong to two streams, the Tennessee and Chattachoochee Rivers in the southern Appalachian Mountain region; the headwaters of the latter drain territory adjoining some of the tributary headwaters of the former, but otherwise the two watersheds are not contiguous. The flood frequency of the Chattahoochee River, which is closer to the source region of tropical maritime air masses and more openly exposed to storms from the

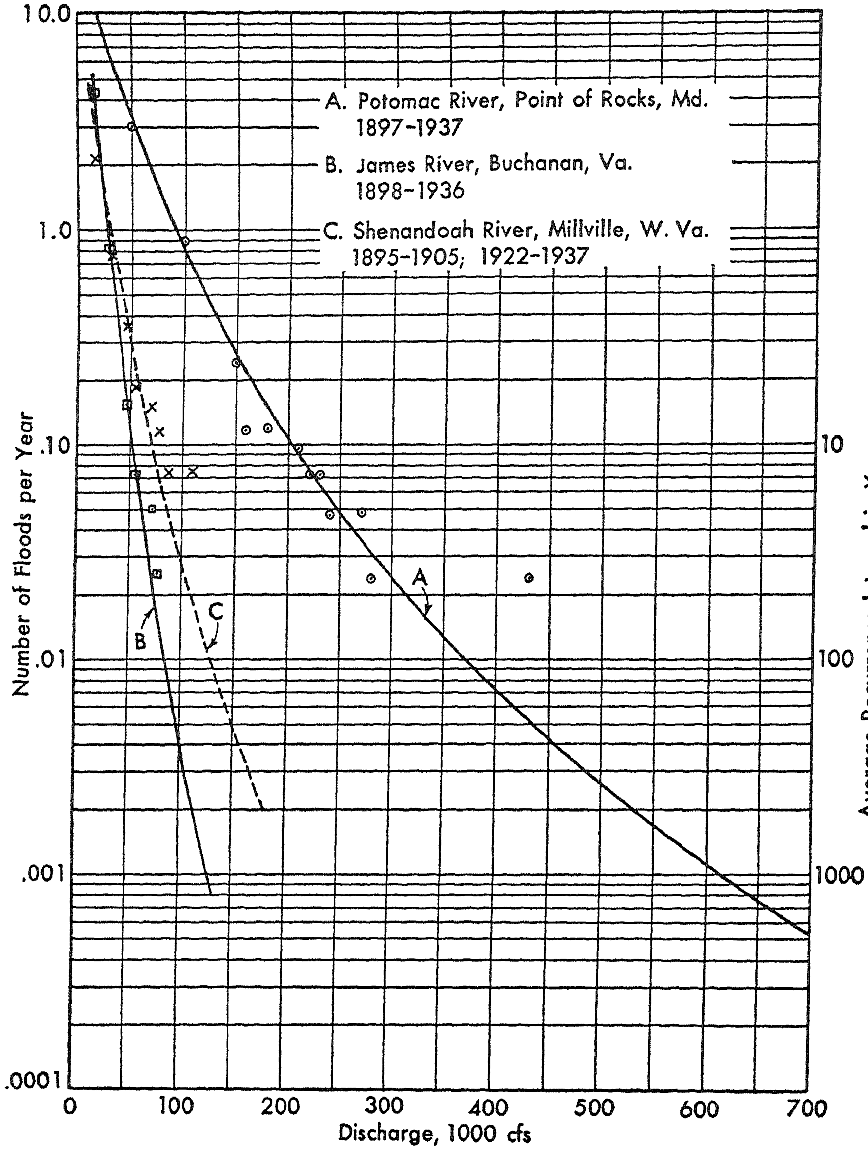



FIGURE 157. Flood Frequencies, Mid-Atlantic States, One-Day Peaks

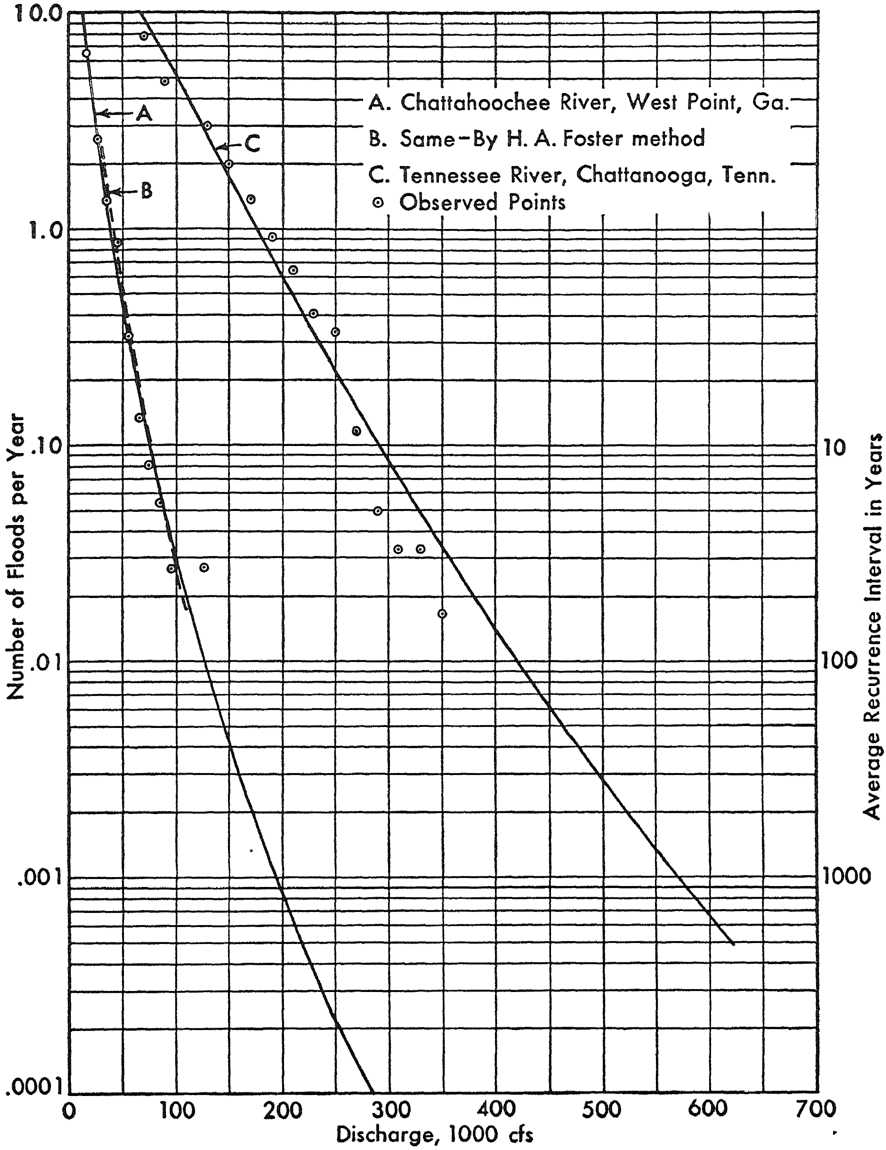



FIGURE 158. Flood Frequencies, Southeastern States, One-Day Peaks

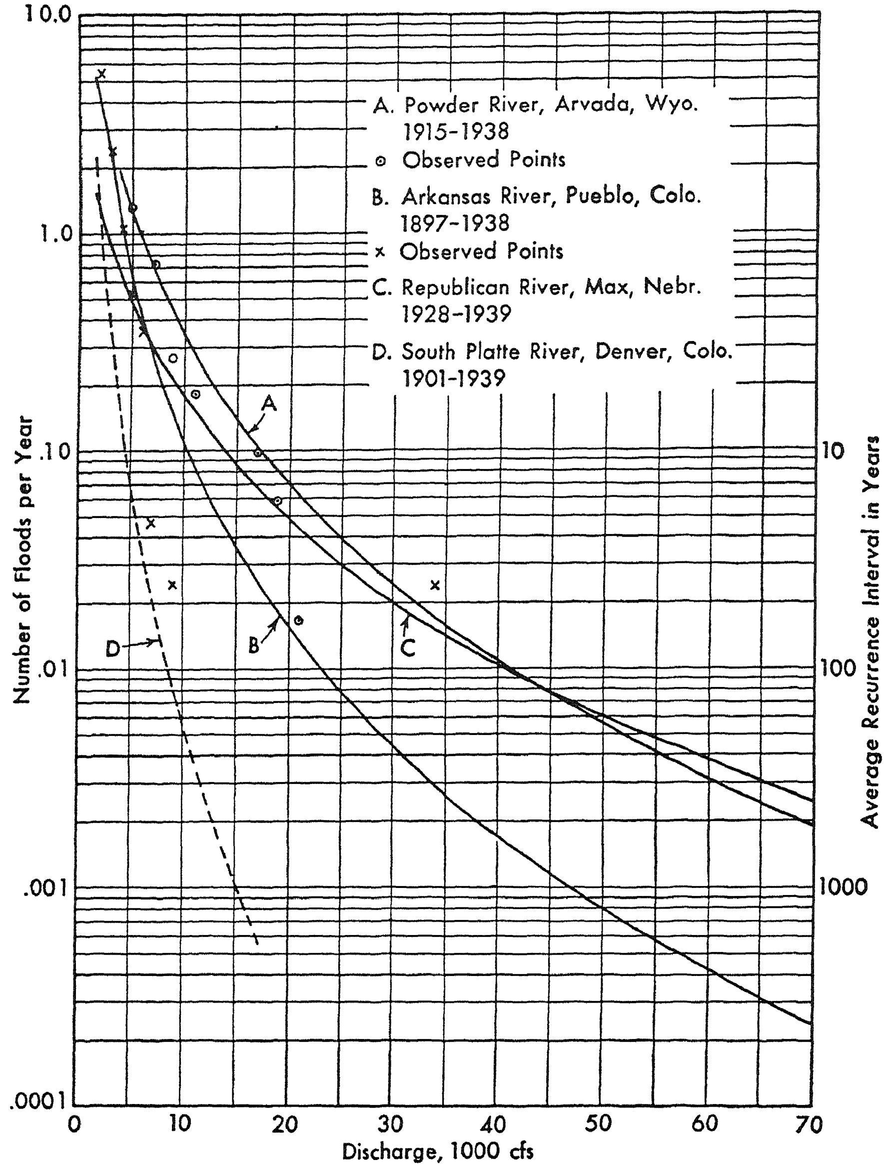



FIGURE 159. Flood Frequencies of Rivers on the Great Plains, One-Day Peaks

south, should be expected to have different characteristics from those of the Tennessee River. The moments, reduced on a basis of unit area (square miles), are listed in Table 81.

TABLE 81. FREQUENCY MOMENTS, RIVERS IN SOUTHEASTERN STATES

| STREAM | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| | *Square Miles* | $M$ | $U_2$ | $U_3$ |
| Chattahoochee, West Point, Ga. | 3,550 | 0.00493 | 0.0425 | 1.640 |
| Tennessee, Chattanooga, Tenn. | 21,400 | 0.0039 | 0.129 | 11.35 |

*Note:* Computed from flow in units of 1000 cfs.

*Comparison Moments, Rivers on the Great Plains.* The four frequency curves shown in Figure 159 are computed from streams on the Great Plains. Except for the Republican, all have their headwaters in the Rocky Mountains, and except for the South Platte River, all have experienced a great flood caused by rare, intense storms of interacting air masses. The watersheds cover a rather widely spread region, yet they are subject to floods of similar types, for all have high water from spring snow runoff and all are similarly subject to the chance of intense rainfall from the rapid lifting of warm moist air masses. The moments, reduced on a basis of unit drainage area, are listed in Table 82.

TABLE 82. FREQUENCY MOMENTS, RIVERS ON THE GREAT PLAINS

| STREAM | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| | *Square Miles* | $M$ | $U_2$ | $U_3$ |
| Powder, Arvada, Mont. | 6,050* | 0.000112 | 0.00928 | 0.452 |
| Arkansas, Pueblo, Colo. | 4,820 | 0.000528 | 0.000830 | 0.0169 |
| Republican, Max, Nebr. | 5,840** | 0.000319 | 0.00404 | 0.279 |
| South Platte, Denver, Colo. | 3,855 | 0.00510 | 0.000488 | 0.00248 |

*Note:* Computed from flow in units of 1000 cfs.

* The maximum flood was given a weight of 0.4 because it was the maximum observed flood in 57 years.

** The actual area is 7740 square miles but 1900 square miles do not contribute runoff. The maximum flood of record was given a weight of 0.15 because it was known that no other flood had approached its magnitude over a much longer period of non-recorded flood history.

*Comparison of Moments, Missouri River.* The four curves in Figures 160 and 161 are of four stations on the Missouri River, extending from Bismarck, N. Dak., to Omaha, Nebr. Floods on this river, particularly on the portion covered by the curves, arise from two distinct causes which do not intermingle to an appreciable extent. In March and April the principal causes of floods are melting snow from the plains,

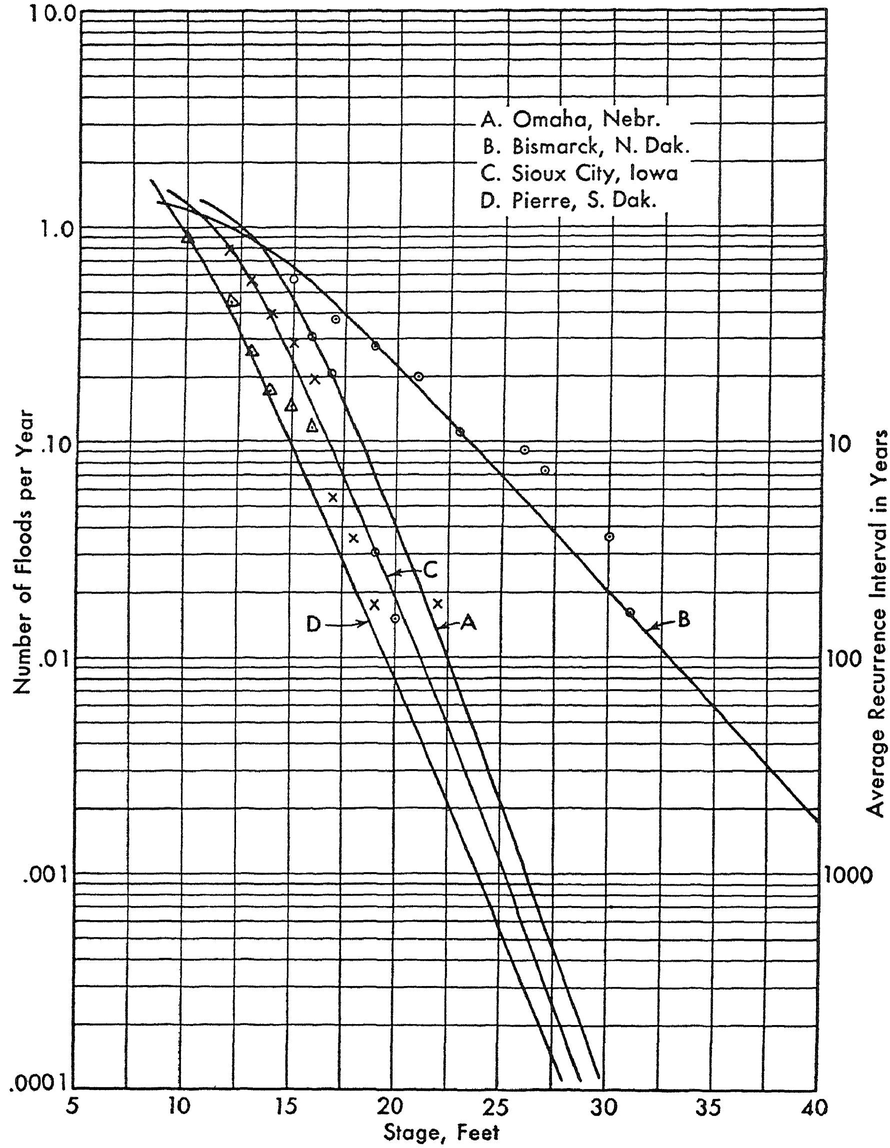



FIGURE 160. Flood Frequencies, Missouri River, Stages of Peaks in March and April

and ice jams during the spring break-up. Although warm air masses occasionally invade the watershed and accelerate the melting of snow, they usually do not bring a great amount of rain in that season. During the months of May, June, and July, the floods are composed of runoff from melting snow of the mountains, with or without additional runoff from storms produced by interaction of air masses. For those reasons, separate curves were computed for the two seasons.

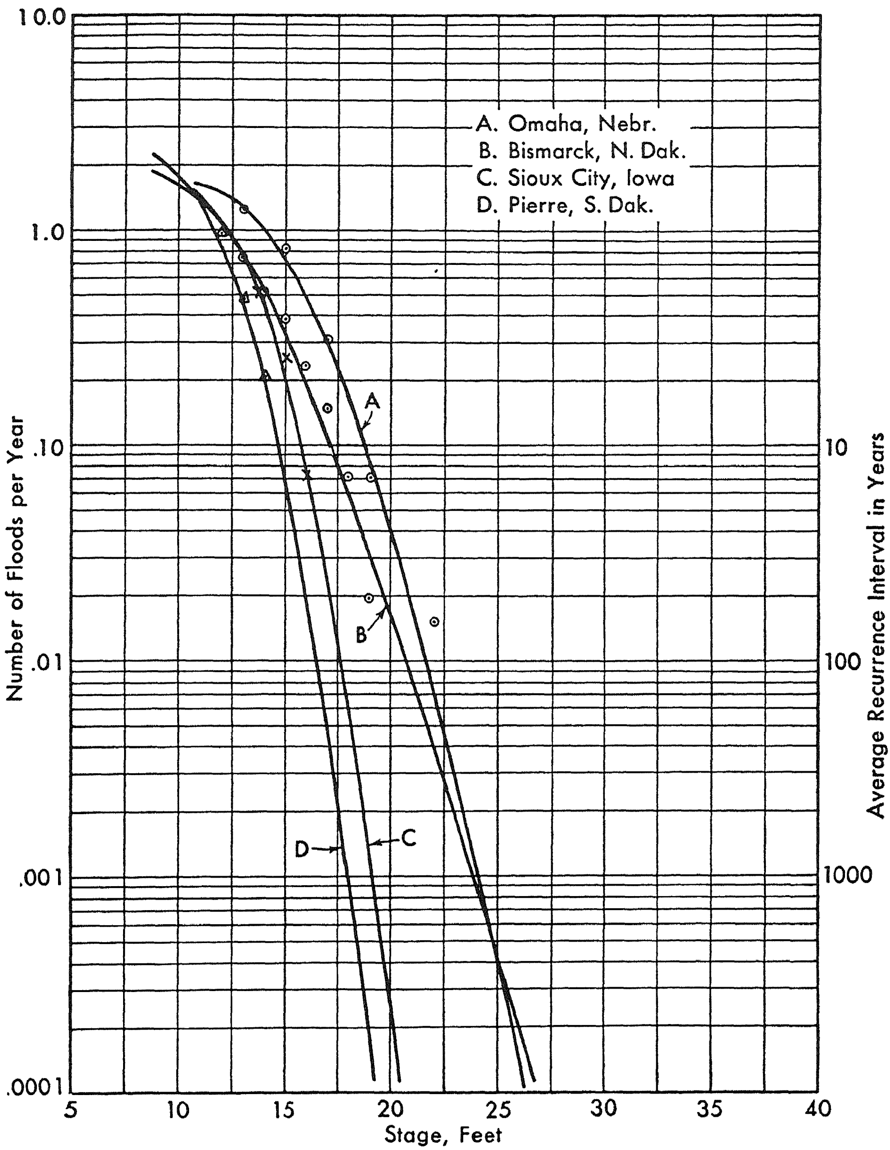



FIGURE 161. Flood Frequencies, Missouri River, Stages of Peaks in May, June, and July

The frequency curves of both seasons are computed from data of stages. Since the stage of a stream is dependent upon local conditions of channel capacity or control, it follows that the curves may or may not follow in the normal order with flood frequency of increasing values of discharge for downstream location of the gages. Moreover, it is usually not convenient to select a base stage for the corresponding

values of low flow at all stations. These considerations mean that when dealing with stage frequencies one should not expect the individual frequency curves of stages derived from a number of stations on a given stream always to lie in the regular downstream order of the stations.

It happens, however, that three of the four curves of the Missouri River fall in order from left to right, thus showing an increase in stage value for a given frequency in the same order as the drainage area increases downstream from Pierre, S. Dak. The curves for Pierre, Sioux City, and Omaha are practically parallel to each other, while the Bismarck curve is at variance for both seasons of the year. This variance is directly attributable to local channel conditions. Some distance below the gage at Bismarck there is a tortuous channel containing many islands; this condition will partially explain the high curve for March and April, since many of the spring floods are caused by ice jams forming in this irregular reach. The Bismarck curve for summer is still not in agreement with the other summer curves, but it is close enough to be explainable by known differences in the characteristics of the channel control alone, without ice jams. In both cases obvious physical conditions can explain the divergences shown both by comparison of the moments and the actual curves.

The moments for curves computed from data of stages cannot be expressed in terms of unit drainage area because of the nature of the data, so they are comparable only on the basis of the moments as computed for the entire drainage area. The parameters for the four curves are given in Table 83.

TABLE 83. FREQUENCY MOMENTS, MISSOURI RIVER

| STATION | DRAINAGE AREA *Square Miles* | $M$ | $U_2$ | $U_3$ |
|---|---|---|---|---|
| SEASON OF MARCH AND APRIL | | | | |
| Bismarck, N. Dak. | 186,360 | 15.45 | 26.48 | 157.0 |
| Pierre, S. Dak. | 243,530 | 10.88 | 5.84 | 16.9 |
| Sioux City, Iowa | 314,600 | 12.33 | 7.22 | 17.3 |
| Omaha, Nebr. | 322,800 | 14.09 | 7.38 | 14.4 |
| SEASON OF MAY TO JULY | | | | |
| Bismarck, N. Dak. | 186,360 | 12.68 | 6.74 | 10.00 |
| Pierre, S. Dak. | 243,530 | 11.46 | 3.43 | 0.88 |
| Sioux City, Iowa | 314,600 | 13.07 | 2.99 | 1.11 |
| Omaha, Nebr. | 322,800 | 14.53 | 6.39 | 5.00 |

*Comparison of Moments, Red River.* In Figure 162 is shown a group of frequency curves of stations on the Red River between Texas, Arkansas, and Oklahoma. The data of these curves consist of discharges obtained by applying rating curves of comparatively recent

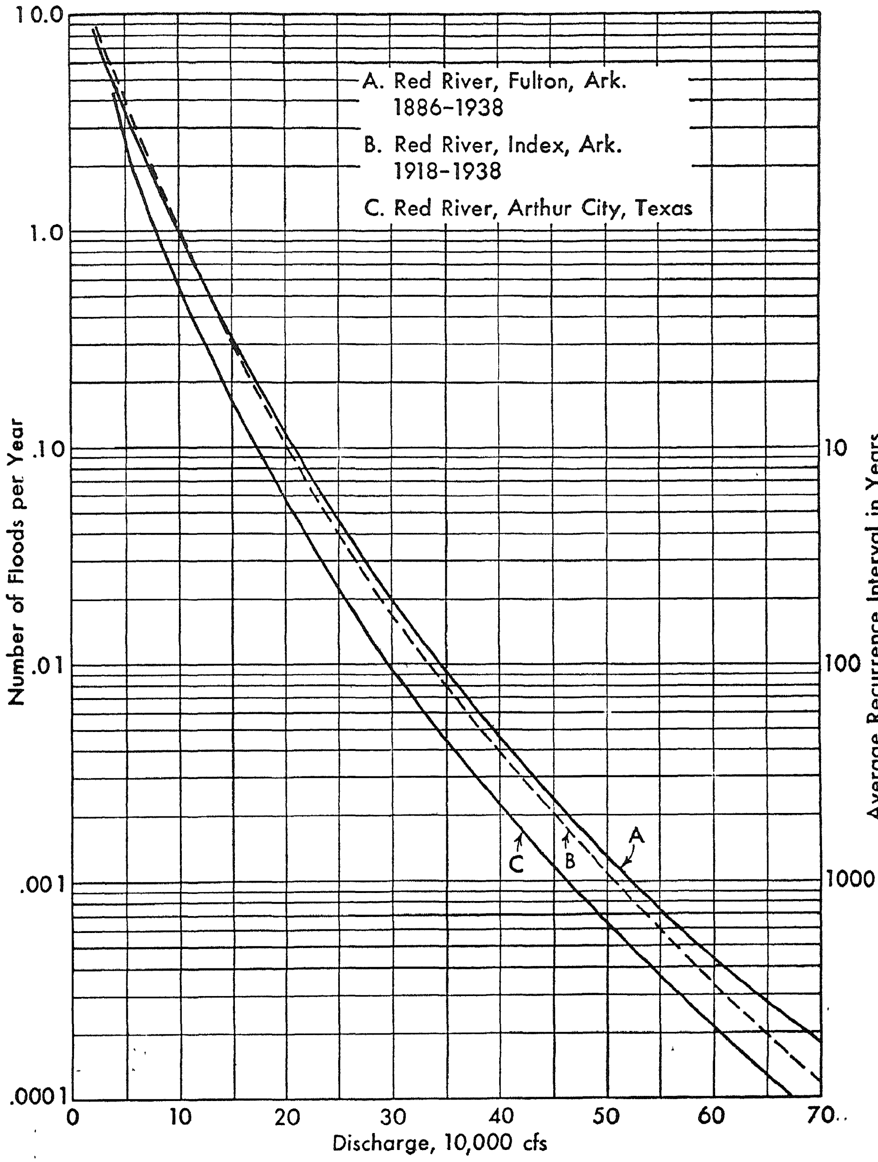



FIGURE 162. Flood Frequencies, Red River, One-Day Peaks

origin to a much longer record of river stages gathered by the U. S. Weather Bureau. Conditions for all these stations are quite similar and hence, as would be expected, the curves are comparable. Because of changing channel conditions, especially shifting of the stream bed during high water, the stage discharge relationship is not constant. No corrections could be applied, since no discharge measurements had been made during the early years of the record of the Weather Bureau.

For these reasons, the discharge data used in the frequency curves of Figure 162 are only roughly approximate, but since all three stations are on the same basis, the curves are comparable. The similarity of the curves is reflected in the comparison of their moments as given in Table 84.

TABLE 84. FREQUENCY MOMENTS, RED RIVER

| STATION | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| | *Square Miles* | $M$ | $U_2$ | $U_3$ |
| Arthur City, Texas | 43,170 | 0.00123 | 0.0329 | 3.55 |
| Index, Ark. | 46,560 | 0.00114 | 0.0272 | 2.55 |
| Fulton, Ark. | 50,720 | 0.00100 | 0.0351 | 3.84 |

*Note:* Computed from flow in units of 10,000 cfs and reduced to values for units of flow of 1000 cfs.

The decreasing average flood per unit drainage area given under the column $M$ is a normal feature of streams and is attributed to the effect of channel and valley storage, which operate to reduce the peak value per square mile on geologically mature streams. (See Figures 133 and 134.)

*Comparison of Moments, Yellowstone River.* Wherever it is possible, data from a long record of flow should be used for computing flood frequencies, though even then it is not always possible to secure a satisfactory array of data. The frequency curves of the Yellowstone River, shown in Figure 163, were computed from fairly long records but because floods are relatively infrequent, few data were available for the curves; the number ranged from 61 flood events at Livingston to 126 at Intake, Montana. Nevertheless, the curves appear to be fairly consistent, as a comparison of their moments given in Table 85 indicates.

TABLE 85. FREQUENCY MOMENTS, YELLOWSTONE RIVER

| STATION | DRAINAGE AREA | VALUE PER SQUARE MILE | | |
|---|---|---|---|---|
| *Montana* | *Square Miles* | $M$ | $U_2$ | $U_3$ |
| Livingston | 3,580 | 0.00558 | 0.00615 | 0.0226 |
| Billings | 11,870 | 0.00278 | 0.00760 | 0.0526 |
| Miles City | 48,440 | 0.001215 | 0.00694 | 0.176 |
| Intake | 67,000 | 0.00103 | 0.00590 | 0.111 |

*Volume Frequency Curves.* On Figures 142 and 143 are shown the distributions of data of the five-day volumes of the surface flood runoff of the James River, together with the one-day peak distribution; this distribution is typically representative. The data consist of the total volume of surface runoff for the same floods that are shown in the one-day peak distribution; the unit of runoff was the day-second-foot.

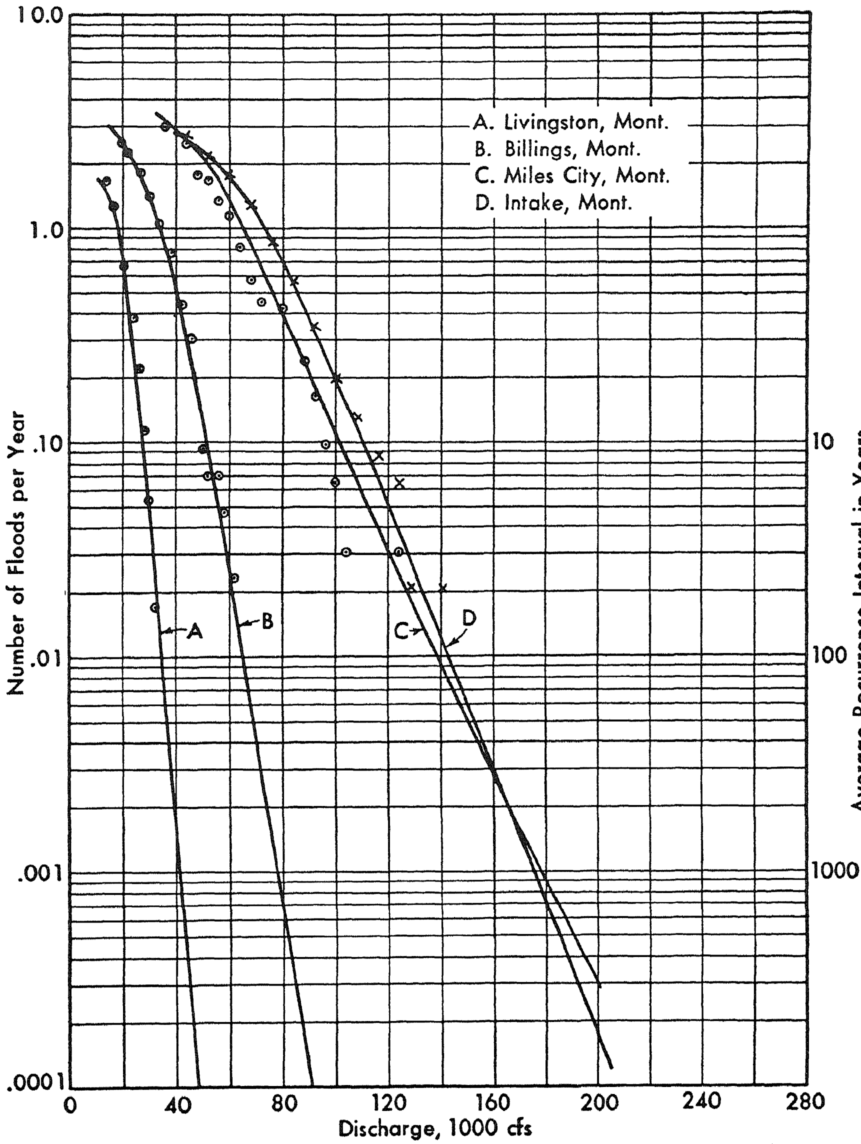



FIGURE 163. Flood Frequencies on the Yellowstone River, One-Day Peaks

On Figure 164 are shown three frequency curves of six-day volumes of surface flood runoff of three stations on the Merrimack River in New England. As would be expected in curves having volumes as the variant, these show the effect of the increasing drainage area by their position relative to the ordinate axis, in which increasing volumes for a given frequency correspond to successive downstream points of the stations. Aside from the slightly greater tendency to leftward curvature

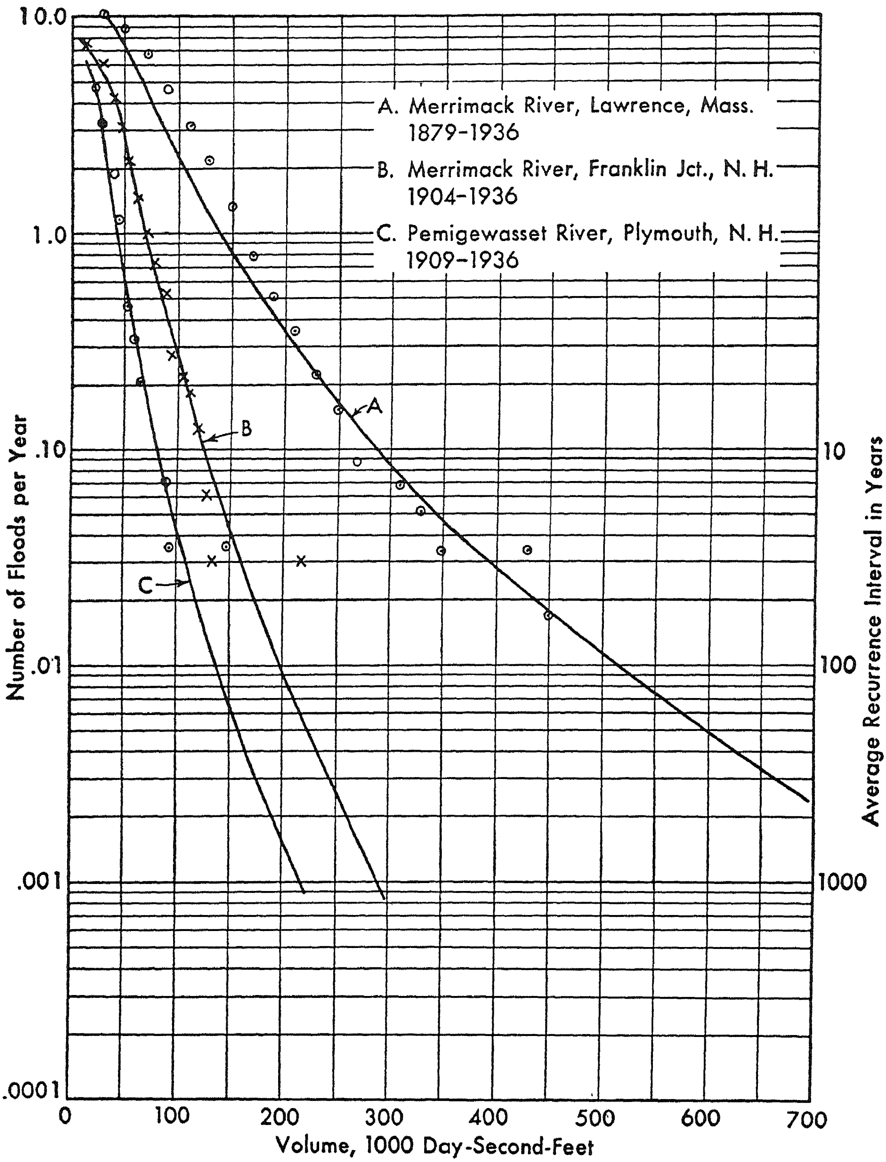



FIGURE 164. Frequencies of Volumes of Floods, Merrimack River Basin, Six-Day Volumes

at the upper ends of the curves there is no material difference in the appearance of volume frequency curves from those of peak flow at these stations. There is no difference in the method of computation after the data have been compiled.

*The Probable Maximum Flood.* During past two decades (1920–40) of study of floods there has been developed a concept of a probable

maximum flood, by which is meant the greatest flood that ever could be experienced on a given stream by reason of the limitations of drainage area, channel (or rather valley capacity), rates of inflow of warm air, precipitation, snowmelt, and other meteorological, hydrologic, and hydraulic elements that operate to produce a flood. This concept has had a considerable bearing in advocating different functions for computing flood frequencies and has caused a definite tendency in some quarters toward favoring a function which includes a factor to represent the predetermined or estimated upper limiting value of flood magnitude. Because of this trend of thought it is desirable to study the relation of the function used to compute the frequencies presented in this chapter with respect to the value obtained at its upper limit.

The data of floods as defined herein range from some value of a few hundred or a few thousand cubic feet per second to the maximum observed occurrence. The Slade partly bounded function, which has been used to compute the frequencies shown heretofore, allows the computed frequencies to vary from $-b$ to plus infinity. According to the concept of probable maximum flood, this function would give a definite probability to an impossibly large flood, and therefore is not a suitable function with which to compute flood frequencies. In a sense this criticism is correct but the same objection is applicable to the use of a function with an unlimited range to determine the frequency of any observed distribution, including those of errors. Infinite magnitudes are conceived but never observed in any field of physical science, except perhaps astronomy. The objection to a function, such as the partly bounded one used here, can be raised against almost any array of magnitudes. Nevertheless, it is well to examine the respective rates at which the partly bounded function approaches the limits of zero probability and therefore the maximum flood.

From the equation in Section *C*, Chapter 1, and the computations shown heretofore, it may be seen that the magnitude $X$ of floods at a given point on a stream varies inversely as the probability. It can be seen through the computations in Tables 69 and 70 that the period of the frequency (sometimes called "recurrence interval") varies as the quantity

$$\int_{-b}^{c} \left(\log d \frac{(x+b)}{(t^2+1)}\right) e^{-u^2}\, du.$$

For any given set of observations, the values, $c$, $b$, $d$, and $t$ are constant and the variate in the integral is $x$. But the logarithms of $x$ are used with the fast decreasing function $e^{-u^2}\, du$, so that it is evident that zero probability (and hence an infinitely long recurrence interval) will be

approached at a rate many times faster than $x$ approaches the probable maximum flood.

A few concrete examples will emphasize this disparity of rate in approaching the limits referred to. The tabulation of Table 86 is taken from the frequency curves of the Merrimack River at Lawrence, Mass., Figure 152, and for the Chattahoochee River at West Point, Ga., Figure 158. The probability is shown by the time in which the given discharge may be expected.

TABLE 86. FREQUENCY OF POSSIBLE MAXIMUM FLOODS

| PERIOD | MEAN DAILY DISCHARGE TO BE EXPECTED, CFS | | REMARKS |
|---|---|---|---|
| *Years* | *Merrimack River* | *Chattahoochee River* | |
| 100 | 118,000 | 125,000 | |
| 140 | ...... | 134,000 | Maximum recorded |
| 400 | 160,000 | ...... | Maximum recorded |
| 500 | 168,000 | 170,000 | |
| 1,000 | 194,000 | 196,000 | |
| 5,000 | 265,000 | 256,000 | |
| 10,000 | 310,000 | 280,000 | |

For a period 100 times as great there is to be expected on the Merrimack River a flood only 2.6 times as great, and for the Chattahoochee River a flood 2.2 times as great as the respective 100-year floods. For the Merrimack River, the 10,000-year flood is less than twice as great as the maximum observed flood, while the period is 25 times as long. For the Chattahoochee River the 10,000-year flood is only slightly greater than twice as large as the maximum recorded, while the period is over 71 times as long. It is evident from the foregoing considerations that there is little danger of giving an appreciable value to the probability of any flood the magnitude of which approaches the maximum possible. Since time increases so much faster than magnitude, the possibility of a limiting value on the size of flood is a matter of little concern in the computation of frequency curves.

*The Transference of Flood Frequencies to Adjacent Watersheds.* The method of transferring flood frequencies is based on the hypothesis that on watersheds with homogeneous climatic conditions and comparable drainage areas, analogous storms would have similar volumes of runoff per unit of area, subject only to differences caused by the runoff characteristics of the basin. In regions subjected to such climatic conditions, precipitation in the large general storms such as result from frontal action of air masses would be of similar depths for equal frequencies. Smaller storms of the thunderstorm type would occur with similar frequency. Snowfall and the conditions of its melting would likewise be similar. And finally, if the region were not too large,

the conditions of ground surface would tend to have comparable average rates of infiltration.

Similarity in hydrological conditions also requires a small difference in number and extent of lakes and swamps on the areas considered. The geological features of the watersheds should not differ greatly, so that the ratio of rainfall to runoff would be approximately uniform on the area under consideration. A large diversity in the matter of lakes or swamps or texture of soil and bedrock would invalidate the assumption of similar unit volumes of storm runoff, and would require a correction before the application of the following method of transferring frequencies.

Two stations are assumed, either on the same stream with different drainage areas, or on streams of adjacent drainage basins. In either case the meteorological and hydrological conditions are similar, so that there is every reason to believe that frequencies of comparable floods are practically the same at both stations. One station is designated $M$ and it is assumed that a record of daily discharge has been obtained over a sufficiently long period with satisfactory accuracy and regularity, so that a frequency curve can be computed. It is assumed also that reasonably good distribution graphs and daily factors have been derived. The second station is designated $N$ and it is assumed that distribution graphs and factors are available, but not a frequency curve. It is now desired to obtain a frequency curve for station $N$ from the one of station $M$.

Let $A_1$ = drainage area above $M$

$A_2$ = drainage area above $N$

$d_1$ = peak-day distribution factor at $M$ from one-day storms

$d_2$ = peak-day distribution factor at $N$ from one-day storms

$F_1$ = the magnitude of a one-day peak at $M$ for any given frequency

$F_2$ = the magnitude of a one-day peak at $N$ for any given frequency

$V_1$ = the total volume of flood in day-second-feet at $M$ with frequency of $F_1$

$V_2$ = the total volume of flood in day-second-feet at $N$ with frequency of $F_2$.

In the application of the method, the $A$'s, the $d$'s, and $F_1$ are known; $F_2$ is to be found; and the $V$'s are not needed in the final equation and hence need not be evaluated.

Now on the assumption that the meteorological and hydrological

factors are similar and the difference in size of drainage area is not too great,

$$\frac{V_1}{A_1} = \frac{V_2}{A_2}.$$

If the difference in drainage area is appreciable, a correction must be introduced to adjust for the diminishing average intensity of storms over greater than about 200 square miles.

It is desired to compute a series of values for $F_2$ in order to construct a frequency curve for $N$.

Now,
$$F_1 = V_1 d_1$$
$$F_2 = V_2 d_2.$$

From the preceding paragraph, and with volumes roughly proportional to a drainage area,

$$V_1 = \frac{F_1}{d_1} \quad \text{and} \quad V_2 = \frac{A_2 V_1}{A_1}.$$

Then substituting, $F_2 = \dfrac{d_2 A_2 F_1}{A_1 d_1}$.

Furthermore, for any pair of stations, $M$ and $N$, the terms

$$\frac{d_2 A_2}{d_1 A_1} = \text{a constant} = k.$$

Then for the purposes of computation, $F_2 = kF_1$.

In order to construct a frequency curve for a station, it is necessary only to take off the curve the values for $F_1$ for the selected frequencies, multiply them by $k$, and plot the curve for $F_2$.

*Frequency by H. A. Foster's Method.* It was stated earlier in this chapter that the frequency of floods on the Chattahoochee River at West Point, Ga., would be computed by the method derived from Type III of Pearson's curves and presented by H. Alden Foster (67). In this method the frequency is computed from two coefficients (or parameters) which are derived from the observed data and a set of tables prepared and published by Mr. H. A. Foster. The first is the coefficient of variation which has been defined previously, and the second, which he called the "coefficient of skew," is identical with the skew $D$ derived for use in the Slade function.

Some slight variations in the method were used in the computation shown in Table 87. In the procedure in the original paper the data were

arranged in order of magnitude, the mean computed, and then each item divided by the mean. Next the residuals obtained by subtracting 1.0 from the ratios of the data to the mean were used to compute the coefficient of variation. Then to find the expected floods of various magnitude (fifth column of Table 87) the skew factors were multiplied by the coefficient of variation; these products were then added or subtracted to or from 1.0 and multiplied by the mean to obtain the desired flood. These procedures appeared unnecessary, particularly so since the mean and standard variations had been computed to construct the frequency curve by Slade's function. The same results were obtained by multiplying the skew factors by the standard deviation $\sqrt{U_2}$, which for the Chattahoochee River was 12.256. To these products were added, or subtracted, the mean flood $M$. Except the calculation of the standard deviation and mean, which may be obtained from Table 81, the entire computation is shown in Table 87.

TABLE 87. FREQUENCY BY H. ALDEN FOSTER'S METHOD

| PER CENT OF NUMBER OF FLOODS | FACTORS FOR SKEW OF 3.162 | STANDARD DEVIATION $\sqrt{U_2}$ | AVERAGE FLOOD *1000 cfs* | EXPECTED FLOOD *1000 cfs* | NUMBER OF FLOODS *per yr* |
|---|---|---|---|---|---|
| .1 | 7.46 | 12.256 | 17.49 | 108.9 | .0185 |
| 1.0 | 4.07 | | | 67.4 | .185 |
| 5.0 | 2.02 | | | 42.3 | .92 |
| 10.0 | 1.16 | | | 31.7 | 1.85 |
| 20.0 | .39 | | | 22.3 | 3.7 |
| 30.0 | .01 | | | 17.6 | 5.55 |
| 40.0 | − .25 | | | 14.4 | 7.4 |
| 50.0 | − .41 | | | 12.5 | 9.2 |
| 60.0 | − .50 | | | 11.4 | 11.1 |
| 70.0 | − .58 | | | 10.4 | 13.0 |
| 80.0 | − .60 | | | 10.15 | 14.8 |
| 90.0 | − .62 | | | 9.90 | 16.65 |
| 95.0 | − .63 | | | 9.77 | 17.5 |
| 99.0 | − .64 | | | 9.65 | 18.25 |
| 99.9 | −0.64 | 12.256 | 17.49 | 9.65 | 18.5 |

Instead of the designation "Per cent of Number of Floods" used for the first column, the original paper used "Per cent of Time," which was appropriate because the data consisted of only the maximum annual flood. The skew factors in the second column were obtained by extrapolation of Mr. Foster's table, which is limited to skews of 3.0. The values of the last column were obtained by multiplying percentages in the first column by the average number of floods per year, which was 18.5. The last two columns were used to plot Curve $B$, Figure 158.

The magnitudes obtained by this method do not cover the full range of flood data that were used, as Slade's function does. It is probable

that if the lower classes of data had been discarded the computation would have reached the maximum flood. However, the computed frequencies agree very well with those of Slade's equation for the range from 11,400 to 108,800 cubic feet per second.

*Frequency by Fisher's Function.* In some studies the author has used Fisher's function to compute flood frequencies, in which a better computed fit with the data was obtained than by Slade's partly bounded function. Fisher's function as published is in the distribution form; the results are computed as number of floods in the class groups, rather than a number above or below a given magnitude. It was necessary to use the function as developed into a series, the best results being obtained by using four terms similar to the Gram-Charlier series. The coefficients had to be obtained by the method of least squares, an operation that entails much arithmetical work. This is one objection to using the function in its present form. In spite of its better theoretical fit the use of the function developed other objectionable aspects that have not been satisfactorily solved. The author has attempted to put the function in the integral form, but has not carried the study far enough to use it to calculate flood frequencies.

*Past Records or Future Probabilities.* In closing this chapter it may not be out of place to point out the difference between the past record and future probable floods. It is true, of course, that predictions can be made for the future only on the basis of experience and trends indicated by past record. While duplication of a past record item by item, or even an approximation of such repetition, is not to be expected, occurrence of similar floods at similar intervals may be predicted; the series will be different, but similar means, standard deviations, and third moments may be anticipated, provided the record consists of a representative series. Since no closer agreement may be foreseen than is indicated by the various measures of precision, it is not important to fit constants to frequency functions by the method of least squares. It appears more logical to depend upon laws of probability to form a guide to future flood events, even though this course may not provide a close fit between a theoretical distribution and the observed data.

# 11 GROUND WATER

*Economic Value of Ground Water.* The economic value of ground water is great; it is so far beyond price that the value can be indicated only by giving instances of its utilization. Ground water has been used from unrecorded antiquity in many countries for all domestic purposes. Many small cities, towns, and rural homesteads depend entirely upon ground water as a source of public and private water supply. It has been and still is used extensively for irrigation. Meinzer (127) estimates that 215,000,000 gallons per day were used in 1940 on the coastal plain of Texas for irrigation, and requirements may amount to 610,000,000 gallons per day in Louisiana and Arkansas for irrigation of rice in some years. On the Pacific coast the Santa Ana River Valley, for one example, uses 300,000,000 gallons per day of ground water. Irrigation from ground water is extensively practised in the Platte River Valley in Nebraska and in the valley of the South Platte in Colorado. These few examples show that the economic value of ground water justifies much expenditure of time and effort to develop and to determine its best utilization.

*Definition and Division of Subterranean Water.* Subterranean water, which is also referred to as "subsurface water," is defined as all water existing in the voids or interstices of the soil and rock beneath the surface of the earth. Subsurface water is divided into two zones with respect to the water and the manner in which it is held within the rock and soil. The lower zone is called the "zone of saturation" or "saturated zone"; in this zone the voids or interstices of the soil or rock are filled with water, which is the true ground water. The upper zone is called the "zone of aeration," because it is only partially filled with water which is held there by various agencies against the force of gravity. Both of these zones are important. The position of these divisions is illustrated diagrammatically in Figure 165.

*Zone of Aeration.* The zone of aeration, also referred to as the "vadose zone," is the soil or rock above the zone of saturation and

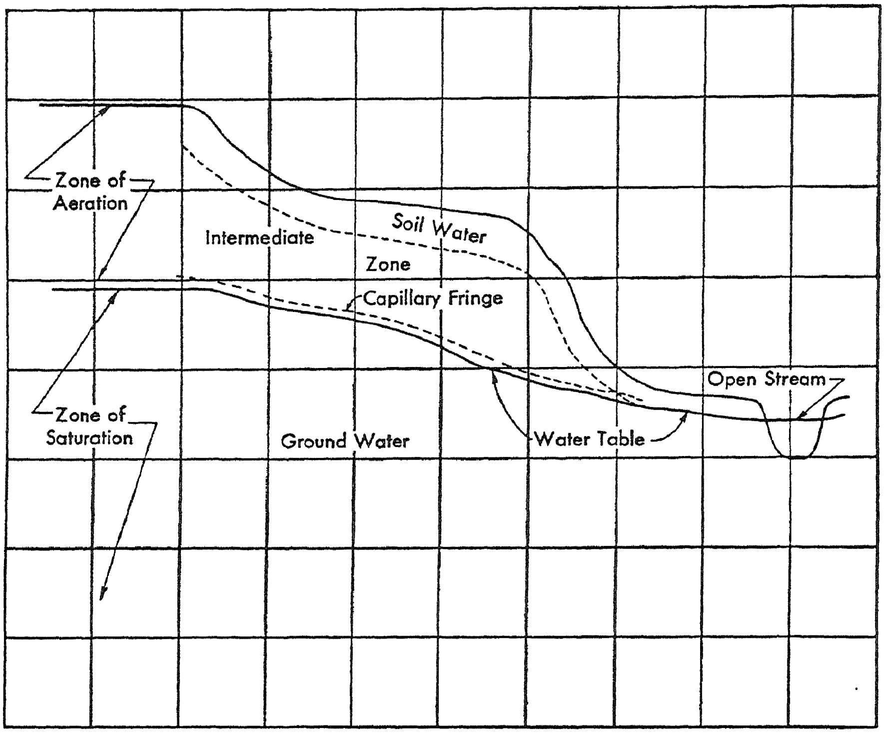



FIGURE 165. Zones of Subterranean Water

extending to the ground surface. The water in it is known as "vadose water," or "suspended water," because it is held in place against gravity by other forces such as adhesion, cohesion, and capillarity. The zone of aeration is divided into three parts called "belts."

The uppermost is the belt of soil water which is limited in depth to the thickness of soil, as distinguished from the bed or mantle rock, by the presence of roots of vegetation. This belt receives the water of infiltration and holds a certain amount by adhesion to the surfaces of the soil particles and in the interstices and joints between the grains and rock fragments. The water so held in the soil belt is subject to depletion by evaporation and transpiration through plants. Water in excess of this holding capacity (referred to by the term "field capacity") and requirements of vegetation, percolates downward toward the saturated zone.

Just above the saturated zone is the lowest belt of suspended water, called the "capillary fringe." This belt is distinguished by the presence of water which is held against gravity by capillarity. The height of lift is independent of the shape of the openings between the grains provided no opening exists that will limit the rise by interposing a space too

large to be spanned by capillary force. Because of this, the height of the capillary fringe is dependent upon the type of soil or rock or earth material. Tolman (187) cites experiments to show that the maximum capillary lift is about 10 feet for fine silt, and from 1 to 5 feet for sand.

Between the belt of soil water and capillary fringe there is the third belt, or intermediate belt of the zone of aeration. Water is held in this belt by adhesion to the soil particles.

The importance of the zone of aeration should not be overlooked, even though it contains no water available for extraction and utilization elsewhere. The soil belt contains all the water available for plant use, except in occasional places where the roots tap the capillary fringe or ground water itself. Through infiltration it is the collecting zone for practically all the water in the saturated zone. The field capacity of the soil belt must be filled before water can percolate to the lower depths containing the ground water, and therefore the degree of saturation of the soil water belt has a bearing on the amount of water available from precipitation to the zone of saturation. Furthermore, some studies have indicated also that the amount of soil water present is an important factor in the rate of infiltration, the smaller the amount of soil water, the greater is the infiltration capacity.

*The Zone of Saturation.* The zone of saturation is that portion of the earth crust in which all voids, interstices, and openings (other than perhaps large caverns or solution channels) are entirely filled with water. This is the ground water; it is also known as "underground water" or "phreatic water." Ground water is under the force of gravity and it moves in the manner of surface water from point to point only as a result of a difference of head between the points. The upper surface of the ground water is called the "water table." It is not, however, a sharply defined surface because the lower portion of the capillary fringe approaches saturation, the completeness of which diminishes as the distance above the water table increases. The exact surface of the water table within the granular material is therefore difficult to locate. However, ground water fills an open hole or well up to the level of the water table with reasonable exactness, since there is no capillary fringe above the free water surface in the hole. There is no water table in impermeable rock formations.

*Relation of Ground Water to Geology.* The existence of ground water is intimately related to the geological conditions of the earth. The kind of rock determines the possibility of ground water, the amount, if any, and its availability. The stratification, slope, and outcrops of the various rock strata determine the exposure to precipitation for

replenishment, the depths to the water table, flow, pressures, and other pertinent characteristics of ground water.

Not all rocks contain water because not all have the necessary pore space, since the first requirement for the existence of ground water is the voids permitting its entrance into the stratum, and its storage there. This space may exist as interstices, pores, or voids between the grains of soil and rock or as cracks, joints, seams, cavities, or other openings. The units of this space must be interconnected, for otherwise there could be no movement of water through the rock, and therefore no water within the body of the stratum except that which may be contained within the formation of the rock.

*Porosity.* The property of having voids is called "porosity." It is considered as a ratio between the volume of voids and the total volume of a given unit of rock, and is expressed as a percentage. Practically all rocks and soils have some porosity. Some, like granite, have so little, that they are for all practical purposes impermeable. Others, like shale or clay, have high porosity but the voids are so small that water moves through them with such extreme difficulty that they also are virtually impervious.

*Aquifers.* A formation of porous rock holding ground water under the force of gravity only, is called an "aquifer." This term is generally applied to a geologic formation or stratum with sufficient porosity and of such a character as to make the water it contains readily available for extraction.

*Classes of Rocks for Aquifers.* Although many kinds of rocks are known, they may be grouped into three general classes on the basis of immediate origin. Each class has good aquifers and each has types of rocks that are too dense or impervious to yield appreciable quantities of water. It should be noted in passing that all deposits of earth materials are designated "rock" regardless of hardness, even though some, like sand, gravel, and alluvial deposits are not consolidated into firm masses.

*Igneous Rocks.* Igneous rocks are all those that have been derived or formed by the internal heat of the earth or volcanoes. This class includes such rocks as granite, basalt, rhyolite, lava, obsidian, and many others grouped under the name "trap rock." Igneous rocks are dense and heavy with little porosity within the rock. They become aquifers only because of extensive jointing, fracturing, faulting, and other processes that permit water to enter voids surrounding fragments of the stratum. Some small wells have been successful in rhyolite that had been thoroughly fractured. In some cases the strata may be fractured to the extent that they permit water to percolate through them to

enter a porous aquifer below. Lava flows are likely to present this condition.

*Metamorphic Rocks.* The second class of rocks consists of metamorphic rocks which are formed by the alteration of the other classes, igneous or sedimentary, through application of heat and pressure of tectonic and diastrophic processes and by weathering and other chemical processes of the earth. Gneiss, slate, marble, and quartzite are examples of metamorphic rocks. These metamorphic rocks are similar to igneous rocks in so far as their water-bearing characteristics are concerned; that is, their capacity depends entirely upon how badly they are fractured and shattered by the weathering or diastrophic movements which, however, is a condition that has been achieved not infrequently in the metamorphosis of the rock. Crosby (43) reports, for example, that some small wells in slate have been successful in southern New England.

In the literature of ground water, both igneous and metamorphic rocks are sometimes considered together under the designation of "crystalline rocks." There is some justification for this since the two classes are similar in their water-bearing capacities. The laws of hydraulic flow do not apply to crystalline rocks in the same manner as to sedimentary or granular rocks because of indefinite boundary conditions and uncertain nature of flow. Water from crystallines may come from a single fault zone or a few large joints. In other cases, such as the wells in rhyolite referred to above, it may come from extensive and fine fracturing, in which the hydraulic laws would hold approximately.

*Sedimentary Rocks.* Sedimentary rocks are those that have been formed by the deposition of material obtained by weathering or erosion of land surfaces elsewhere. This class of rocks includes a wide variety, both in regard to hardness or consolidation and to water-bearing capacities. It is desirable to discuss in detail some of the more important kinds of rock individually, since sedimentary rocks include some of the most important aquifers.

1. LIMESTONE. Limestone is composed primarily of calcium carbonate, and is typically a dense and impervious rock. It is, however, somewhat soluble in percolating ground water so that cracks, fractures, or joints are enlarged by solution to form underground channels and caverns, sometimes of enormous size. The value of limestone as an aquifer is derived from these solution channels which may hold large quantities of water. Dolomite is a similar rock, being composed of calcium and magnesium carbonate, but it is not so soluble as limestone.

2. GYPSUM. Gypsum is another soluble material and its value as an aquifer depends upon its solution cavities and channels.

3. CLAY AND SHALE. Clay and shale are similar materials, the latter differing chiefly by being harder because of compaction. Both may be porous, but the pores are so fine that water can percolate through them only with extreme difficulty, if at all. Neither clays nor shales are useful as aquifers, but occasionally an old strongly compacted shale yields enough water through joints and fractures to supply household or farmstead requirements of a gallon or two a minute.

4. SANDSTONE. Sandstones that have not been altered to quartzites are granular materials and form some of the best aquifers. The porosity of sandstone ranges from 25 to 40 per cent. The pores are large as compared to those of clay, so that they yield water readily.

5. SAND AND GRAVEL DEPOSITS. Sand and gravel deposits are unconsolidated formations of variable areal extent and depth. They are located in a variety of places dependent upon their origin, such as stream valleys, alluvial cones in mountains, and outwash plains among glacial deposits. In places such as central Nebraska, sand dunes occupy extensive areas. Although these deposits are porous and yield water readily, their usefulness as aquifers is dependent upon their location with respect to the water table. While the most important deposits are located in river valleys or relatively low elevations, so that all or part of the formation is below the water table, other deposits are located above the water table and are consequently useless as aquifers.

6. TILL. The term "till" designates material that has been deposited by glacial action, and is applied particularly to the residual ground cover that has been left by the ablation of the glacier. Till is a characteristically heterogeneous mixture of clay, loam, sand with gravel, and boulders. Usually it is not a prolific aquifer because the unsorted material containing clay and loam does not readily permit the movement of water. However, wells yield enough for farmstead and similar small requirements. Occasionally in a deposit of till, there is an extensive accumulation of sand and gravel located below the ground water; such a deposit forms an excellent aquifer with an abundant supply of water.

7. LOESS. Loess is a fine-grained porous material that is generally considered to have been deposited by wind. Although porous, it is not a good aquifer because of the slow movement of water through the body of the material. It is, however, used as a source of water for farmstead and similar small requirements.

The foregoing enumeration of types of rocks should not be taken as

the complete list of sedimentary rocks. There are many other types and likewise many variations and gradations of those enumerated, as one is replaced by another in natural deposition.

In plains regions the geological conditions may be similar over extensive areas. Under such conditions ground water may be expected to occur in a similar manner except for minor local variations. In mountain regions, on the other hand, geological conditions and consequently ground water are extremely variable. In such regions too, precipitation is likely to be variable from place to place, a condition that adds to the variability of ground water.

Despite the variability of ground water, geologists have established typical areas, referred to as "provinces," of similar ground water conditions. Meinzer (125) has outlined 21 provinces of ground water in the United States. An adequate discussion of these provinces is outside the scope of this book.

*Relation of Ground Water to Topography.* In general the surface of the ground water table conforms to the surface of the ground, but with appreciably subdued topography having fewer extreme peaks and hollows. This general conformity is explainable by the facts that rains fall on high land as well as low land, and that the infiltrating water is held at least temporarily in the soil or rock into which it is absorbed. This action would cause the water table to conform closely to the land surface. But in a porous medium not too fine-grained, ground water moves under action of gravity whenever there is a difference of head between adjacent points on the surface of the water table. This movement takes water out of the higher land areas into the lower, the only restriction being the resistance to flow offered by the passages through the rock strata. This flow then produces a more subdued topography for the water table than that of the ground surface. Ground water is at relatively higher elevations under hills and uplands and slopes down to stream channels, lakes, or other outlets and thus is found at greater depths under the higher surface areas. This characteristic feature is illustrated by Figure 165.

*Springs.* Due to irregularities of ground surface and water table and to outcropping of an underlying impervious geological formation, the water table may intersect the ground surface in favorable spots, and water flows out on the surface. This flow is known as a spring. Many springs are of sufficient size to be of value as a source of water supply; a number in the United States are reported to yield 100 to 700 cubic feet per second and one in France is said to yield approximately 4000 cubic feet per second in periods of high discharge. Springs occur under

many conditions. Tolman (187) gives a classification of them, based on type of water bearing-formation or type of opening. His classification is as follows:

"I. Springs issuing from pervious veneer formations." Water issues from an aquifer underlain by an exposed impervious formation.

"II. Springs issuing from a thick pervious formation." The water table outcrops on the ground surface so that water flows on the surface.

"III. Springs issuing from interstratified pervious and impervious formations."

"IV. Springs issuing from solution openings." This type of spring issues from solution channels outcropping in limestone.

"V. Springs issuing from lava."

"VI. Springs issuing from fractures." This type of spring issues from either pervious or impervious formations, the fractures of which outcrop on the ground surface.

*Ground Water Basins.* While surface runoff is derived from basins definitely limited to topographic features, ground water exists in similar basins but the boundaries are not so definite. The uplands, known as "interfluves," between surface basins frequently fix the limits of the ground water basins also, since the water table follows the general topography. However, this situation does not exist in all cases, for in flat topography the water table may extend from one surface basin across the interfluve and into the adjacent watershed. Cady (28) presents a good example of this situation in the area between the Platte and Niobrara Rivers in Nebraska; his map is reproduced in Figure 166.

In mountainous regions, however, there are many basins surrounded by impervious interfluves; they may be entirely enclosed by rock ridges or may have only one opening at the lower end for drainage. These basins can appropriately be called closed basins, because they are closed to the inflowing movement of ground water from outside areas. Examples of these closed basins are the watersheds of the Pomperaug River in Connecticut, described by Meinzer and Stearns (129), and the Escalante Valley in Utah, described by White (192).

River valleys provide ground water basins of special interest to engineers. It sometimes happens that during past ages the stream has eroded a considerable valley that has subsequently been wholly or partially filled with debris such as sand, gravel, and in mountainous areas boulders, the whole of which constitutes an excellent aquifer. The filled valleys may become important sources of supply, as the ground water flows down the valley (if there be sufficient slope) as well as from adjacent uplands. In an area selected for a source of water

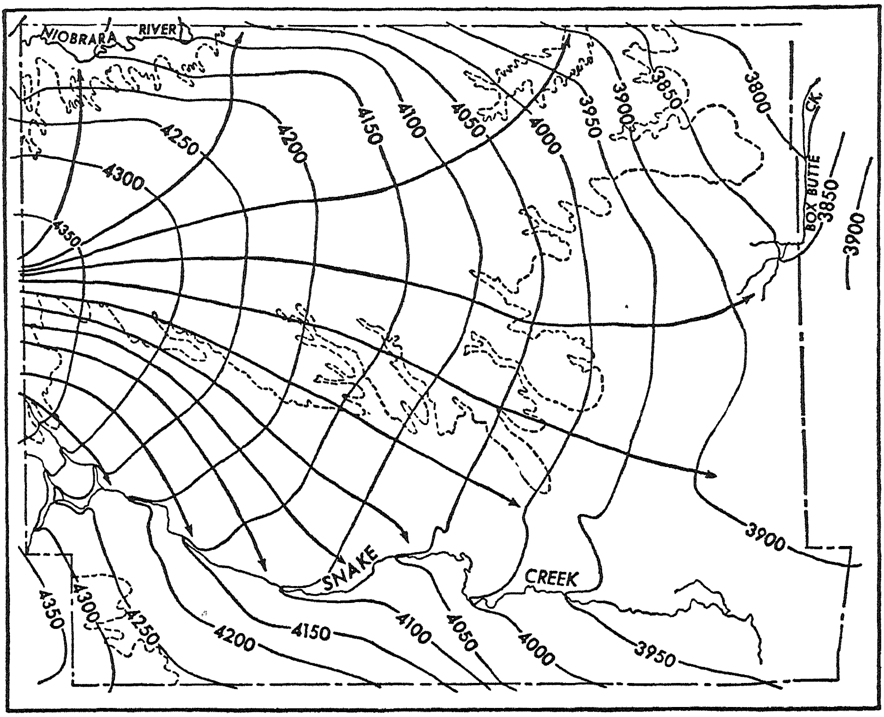



FIGURE 166. Contours and Gradients of the Water Table, Box Butte County, Nebr.

supply in such a valley, there is influent ground water at the upstream end and underground outflow at the downstream end.

*Confined Ground Water.* Confined ground water has not heretofore been differentiated from that in which the water table is under atmospheric pressure only. Confined ground water is that in an aquifer whose upper surface beyond the inflow area is formed by an impervious stratum, and whose lower end is wholly or partially closed by some means to prevent the outflow of the confined water.

*Artesian Basins.* An artesian basin is one with a confined aquifer from which the water will rise in a well to an elevation above the bottom surface of the upper confining stratum. The distinguishing feature of an artesian basin lies in the hydraulic head or pressure on the ground water, so that it is forced to rise where there is an opening in the confining stratum.

Stephenson and Veatch (172) give the following conditions which were stated by Chamberlin as necessary conditions for the existence for artesian wells:

A pervious stratum to permit the entrance and the passage of the water.
A watertight bed below to prevent the escape of the water downward.

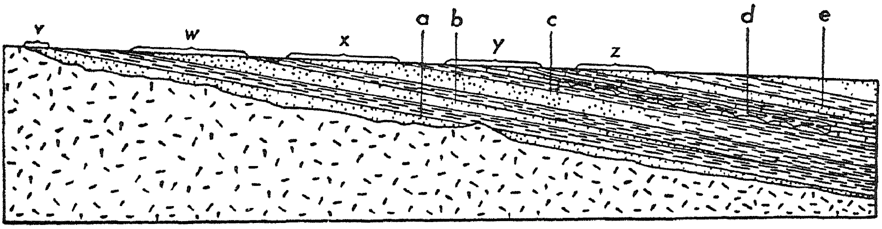



FIGURE 167. Conditions Governing Artesian Pressure, Coastal Plain of Georgia

A like impervious bed above to prevent the escape upward, for the water, being under pressure from the fountain head, would otherwise find relief in that direction.

An inclination in these beds so that the edge at which the waters enter will be higher than the surface at the well.

A suitable exposure of the edge of the porous stratum, so that it may take in a sufficient supply of water.

An adequate supply of rainfall to furnish this supply.

An absence of any escape for the water at a lower level than the surface at the well.

These conditions have generally been recognized as necessary for the existence of artesian wells, even though the details of their application may vary. These conditions are illustrated by Figure 167 adapted also from Stephenson and Veatch (172).

Figure 167 represents the artesian conditions under the coastal plain of Georgia, and since it illustrates several cases of artesian flow, a more detailed description of the figure is desirable. Stephenson and Veatch discuss it as follows:

Layer *a* is a bed of porous sand underlain by crystalline rocks and overlain by clay, both relatively impervious, and cut off down the dip by an unconformity. Water enters the bed in the catchment area and passes down the dip through the porous sand to the point where the bed is cut off; the weight of the water in the higher part of the layer produces hydrostatic pressure in the water confined in the lower part. Layer *b* is a bed of porous sand confined above and below by relatively impervious clay and pinching out down the dip between the beds of clay; hydrostatic pressure is produced in the same manner as in layer *a*. Layer *c* presents the same conditions except that the downward circulation of water is prevented by the merging of the porous sand down the dip into relatively impervious clay. Layer *d* is a cavernous, water-bearing limestone confined above and below by relatively impervious layers and becoming noncavernous and compact down the dip. Layer *e* is a bed of porous sand in which the only hindrance to the passage of water down the dip is friction, which causes a certain amount of hydrostatic pressure to be developed.

The slope of the ground surface of the coastal plain should also be noted, as it is the means of producing the hydrostatic head at the wells in the vicinity of *a*, *b*, *c*, *d*, and *e*. Replenishment of ground water is obtained by rainfall on the intake areas *v*, *w*, *x*, *y*, and *z* for the strata *a*, *b*, *c*, *d*, and *e*, respectively.

When released from the confining stratum by means of a well or similar opening, the water under artesian pressure rises to its hydrostatic head. This point for a number of wells defines a surface which is called the "piezometric surface." Lines of equal elevation on the piezometric surface are called "isopiestic lines."

The hydraulics of artesian wells is an extensive subject beyond the scope of this book. Books on water supply (11, 60) or ground water investigations furnish ample information. Nevertheless, it is desirable to discuss here briefly a few concepts necessary to pursue further study of the general hydrology of ground water.

*Specific Retention and Yield.* Specific retention of rock or soil is defined as the ratio of the volume of water in an aquifer that will be retained after saturation against the force of gravity, to the volume of rock or soil. It is expressed as a percentage, thus:

$$S_r = 100\left(\frac{r}{V}\right)$$

where $S_r$ is the specific retention; $r$, the volume of water; and $V$, the volume of rock or soil containing $r$. The values of $S_r$ vary according to the methods of derivation so that the values should be derived in a manner comparable to a proposed use.

Specific yield is defined as the ratio of the volume of water in an aquifer yielded after saturation under action of gravity, to the volume of rock or soil. It is also expressed as a percentage, thus:

$$S_y = 100\left(\frac{y}{V}\right)$$

where $S_y$ is the specific yield; $y$, the volume of water yielded; and $V$, the volume of rock or soil containing the water.

*Permeability.* Permeability of a soil or rock is defined as the capacity to transmit water through its pores. It is measured by the quantity of water passing through a cross section of unit area in unit time and under a unit gradient. The U. S. Geological Survey (124) has defined the coefficient of permeability as the rate of flow in gallons per day through a cross section of one square foot at a temperature of 60 F, and under a hydraulic gradient of 1.0. A unit value of this coefficient has not been

given a name. Muskat (141) has proposed that the name, "darcy" (in honor of the French engineer, Darcy) be given to a unit of permeability which he defines as one cubic centimeter per square centimeter under a pressure of one atmosphere per square centimeter with unit viscosity. There are also other units in use (141).

It should be pointed out that permeability depends upon other factors than simple porosity. For example, clay is a porous material but its permeability is so low that it makes one of the best confining materials. The permeability of sand and gravel depends much upon the sorting or grading of the particles composing the materials, those having grains of uniform size being the most permeable. Density likewise affects permeability, the more densely packed material having the smaller permeability. Some curves of the grading obtained by the mechanical analysis of material from two wells in the Platte River valley are shown in Figure 168, together with a tabulation of the values of porosity and coefficients of permeability. The tabulation on Figure 168 shows a great variation of permeability in the water-bearing material, while the values of porosity do not differ enough to account for that variation. The variation shown by the two wells in permeability at different depths is a common characteristic in alluvial deposits which are usually highly stratified.

Muskat (141) lists some typical unconsolidated sands, the grains of which vary in size from 30 to 140 to the inch as measured by the meshes in a sieve. For all these sands, the porosity is 40 per cent, but the permeability ranges from 9.26 to 345 darcys, approximately.

Permeability is the property that denotes the fluid-carrying capacity of an aquifer. Porosity, on the other hand, is the property that measures its holding or reservoir capacity, and as shown above, the recoverable portion of its capacity depends upon its specific yield. The remainder or the unrecoverable portion is its specific retention.

The permeability of an aquifer may be determined by a number of standard methods in the laboratory or by methods applicable in the field. Except the field method mentioned later, the determination of coefficients of permeability, specific retention, and specific yield are primarily matters of soil mechanics and outside the scope of this book.

*Movement of Ground Water Through an Aquifer.* Water moves through an aquifer under the force of gravity just as surface water does in its channels, but its rate of movement is dependent upon the head, and the permeability of the material through which it passes. In channels of cavernous limestone and gypsum, and perhaps the interstices of the largest gravel, it flows in accordance with the same laws as surface water. However, in granular materials which form most aquifers, it

| Well | Curve | Depth Feet | Porosity Per Cent | Coef. Perm U.S.G.S. Units |
|---|---|---|---|---|
| 1 | A | 30-39 | 30.6 | 4350. |
| 1 | B | 39-40 | 40.3 | 2. |
| 2 | C | – | 27.0 | 815. |
| 2 | D | – | 27.5 | 2515. |

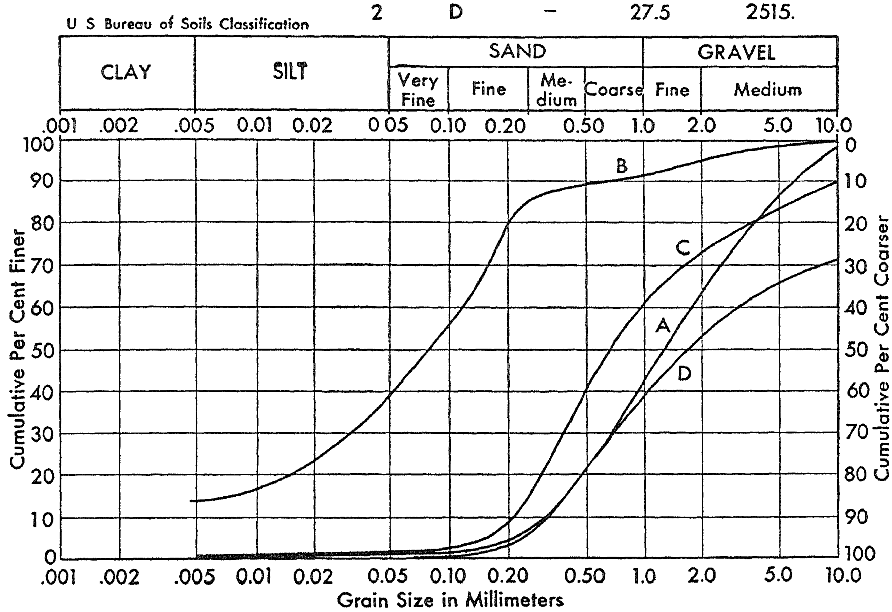



FIGURE 168. Mechanical Analysis of Material from Wells, Platte River, Nebr.

moves by different laws because of the fineness of the interstices and relatively greater effects of other factors such as adhesion and viscosity, which are negligible in surface flow. No special discussion is given here to flow in large subterranean channels, but some consideration is given to the passage through granular aquifers.

The law of flow of ground water was first stated in 1856 by Darcy, an eminent French engineer and investigator of hydraulic phenomena, who expressed it in the following formula:

$$Q = \frac{kAH}{L}$$

where $Q$ is the rate of flow; $A$, unit area cross section normal to the flow; $H$, the difference in head elevation, or pressure; $L$, the distance between the two points taken to obtain the head $H$; and $k$, the coefficient of permeability.

Many investigators have spent much time and effort in determining whether or not Darcy's formula holds for all flow through porous media; the general agreement is that it does hold. One question raised was whether or not it is valid for flow under small gradients. Meinzer

and Fishel (128) found in tests which they made that Darcy's formula held for gradients of 0.5 foot per mile. Fishel (58) later found that the law held for gradients as low as 2 or 3 inches per mile, which is probably as low as can be determined accurately in the field on account of the capillary fringe.

Darcy's formula, when used with a coefficient of permeability such as the darcy or that defined by the U. S. Geological Survey, is a complete mathematical description of the rates of flow or discharge through porous media under conditions fixed by the coefficient. For a given flow condition, the data needed are those for head (or gradient), cross sectional dimensions, an average coefficient of permeability of the entire area of flow, and temperature of the water to determine the viscosity. Unless the temperature of the water is equal to that defined in the coefficient of permeability, it is necessary to correct the latter proportionately to difference in temperature.

Darcy's formula has been accepted universally for the analysis of problems of ground water. Although simple in itself, its application to many problems becomes complicated because of boundary and other conditions of a particular application. Muskat (141) has discussed many of these problems. The many applications of Darcy's formula are beyond the scope of this book, except a brief discussion of two hydraulic problems of ground water: wells, and flow in a filled valley.

*Wells.* The flow into a well under a steady state of pumping is represented in Figure 169. The initial pumping draws the water down in the well until there is produced a sufficient head $(D - d)$ to cause the water to flow into the well as fast as it is pumped out, so that the elevation of the water is held constant. The rate of flow has been derived for this condition and is given by writers on water supply and ground water flow (11, 141, 187). The formula is as follows:

$$Q = \frac{\pi k(D^2 - d^2)}{2.3 \log_{10} (R/r)}$$

where $Q$ is the rate of discharge in gallons per day; $D$, the depth at the bottom of the well below the water table; $d$, the depth of water in the well under steady pumping; $R$, the radius of influence from the center of the well to the point where the water surface under pumping is tangent to the initial water table; $r$, the radius of the well; and $k$, the coefficient of permeability as defined in terms of gallons per day.

The formula is used also to determine the coefficient of permeability of a water-bearing stratum. A slight modification to meet field conditions is made by substituting in place of the two radii as defined above, the radii of two circles of observation wells within the cone of draw-

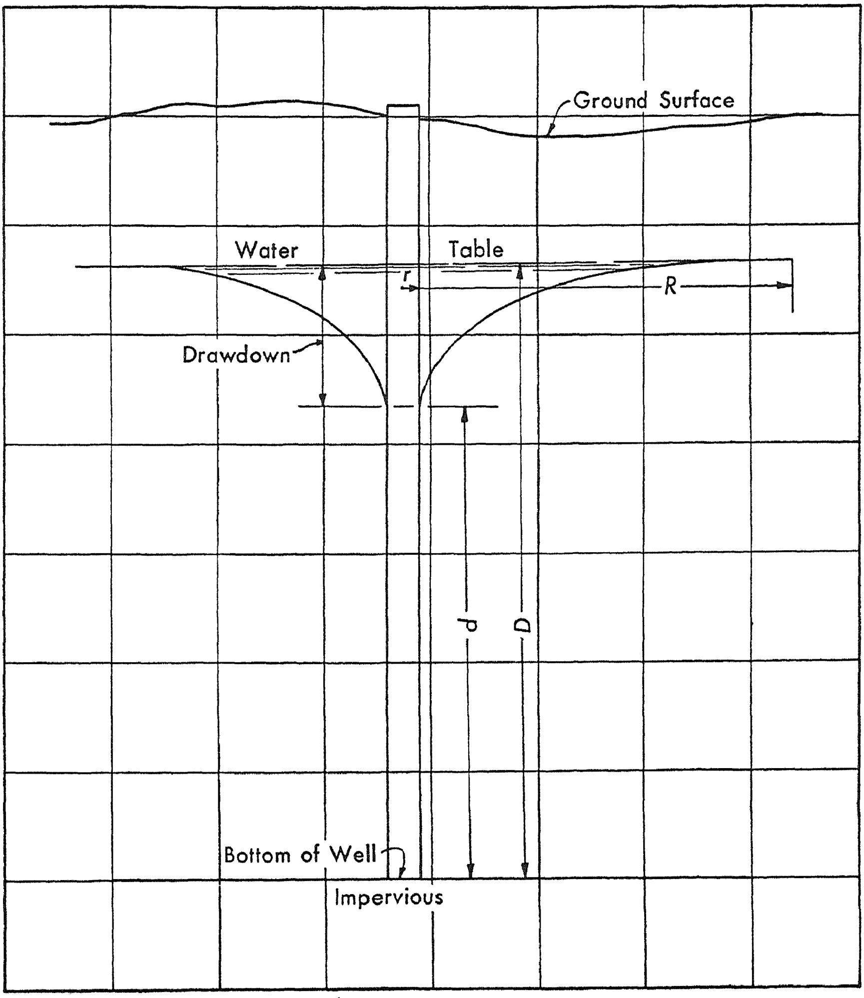



FIGURE 169. Representation of a Well and Relation to Ground Water

down and in line with the well. The observation wells are placed on one line up and down the gradient of the initial water table, and in another line perpendicular to the gradient; the drawdowns of the observation wells of the same radii are then averaged. After transposing terms to set $k$ as the dependent variable, the formula becomes

$$k = \frac{Q\,2.3\log_{10}(r_2/r_1)}{2\pi m(s_1 - s_2)}$$

where $r_1$ and $r_2$ are the radii of the circles of the observation wells; $s_1$ and $s_2$, the drawdown of the observation wells; and $m$, the thickness of the ground water stratum.

*Underground Channels.* The second hydraulic problem concerns the rate of flow through an aquifer in a debris-filled valley or underground channel. The measurement of this flow is a field problem, since it involves the dimensions of the channel and the velocity of the ground water. The dimensions of the channel can be obtained only by a number of borings along a cross section perpendicular to the gradient of flow, in order to measure the width and depths of the aquifer.

Direct measurement of the velocity of ground water in channels can be made by one of a number of methods: by the use of salt or other chemical that can be detected by analysis of water, the use of dyes, or by an electrolytic method. The fundamental principle of application of all these methods is the same: two observation wells are sunk into the aquifer parallel to the direction of flow and about 4 feet apart for shallow wells. The salt, dye, or electrolyte is introduced into the upper well and the time to arrive at or affect the lower well is noted. The velocity is obtained by dividing the distance between the wells by the time.

The chemical method was developed first for the determination of ground water velocities by European hydrologists who used salt for the chemical. The salt was injected in the upper well and its progress through the soil was noted by making tests for chloride.

The electrolytic method was developed in the United States by C. S. Slichter (126) who used salt as an electrolyte. An electrode was set up in each of the wells, these were placed in a circuit with a battery and an ammeter to measure the changes in the current. When the salt reached the electrode in the downstream well, there was an abrupt increase in the current, due to the greater conductivity of the salt water. Although the electrolytic method has been developed further and standardized for use in general ground water investigations, care must be taken in its use to prevent the introduction of large errors.

In the dye method the dye is injected into the upstream well and its progress determined by the change in color in the water from the downstream well. Uranin (a sodium salt of fluorescein) is considered the best dye for the purpose, as it is easily detected by its bright green color when dissolved in water. The dye method has been used largely in investigations for sanitation, being especially useful for detecting flow along joints or fractures in crystalline rocks.

Except the dye method, these methods are seldom used now but any one may be useful in determining velocity in filled underground channels. The velocity of ground water, particularly in aquifers without definite boundaries, may be determined better by tests to determine the mean coefficient of permeability and slope of the water table.

However, any method must consider the great variability of permeability of soil.

As may be inferred from the application, this method for determining velocities is limited to unconfined waterbearing underground channels. The measurement of velocities in confined or artesian aquifers is attended with greater difficulties, such as the necessity for deep and consequently expensive observation wells. Furthermore, the velocities are usually small, since the flow is limited to leakage and withdrawals from wells.

*Ground Water Use and Replenishment.* The use of unconfined ground water involves changes in volume, as evidenced by fluctuating depths in wells extending into the aquifer. Greater depths to the water table result from extractions whether by vegetation, drainage, or pumping from wells. Smaller depths mean a greater volume of water in the aquifer. Where ground water is used extensively for economic purposes, the changes in depths and the volume of water involved are factors of great importance. Consequently, replenishment after any extraction is also a matter of vital concern. Some means of analyzing the relationships between depths and volume are necessary to determine available supplies.

*Replenishment of Ground Water.* The term "replenishment of ground water" designates replacement of ground water abstracted by natural and artificial withdrawals as well as losses by drainage. The term "recharge" is used to convey the same meaning but the word "replenishment" denotes more accurately the idea of refilling an aquifer, hence it is retained in this book. Replenishment can come from two immediate sources, namely, flow from another body of ground water, or directly from precipitation. Ultimately, of course, all replenishment comes from precipitation.

As was seen previously, only a portion of the water of precipitation goes into the ground water, that is, the portion which infiltrates the soil in excess of the field capacity of the zone of aeration. The division of precipitation may be set up as follows to show the portion going into ground water.

$$\text{Precipitation} = \begin{cases} \text{Evaporation} \\ \text{Infiltration} \\ \text{Surface runoff} \end{cases} = \begin{cases} \text{Replenishment of field capacity.} \\ \text{Ground water.} \end{cases}$$

The portion of precipitation going into ground water may be replenishment of withdrawals or additions to storage, which would be obtained by raising the elevation of the water table.

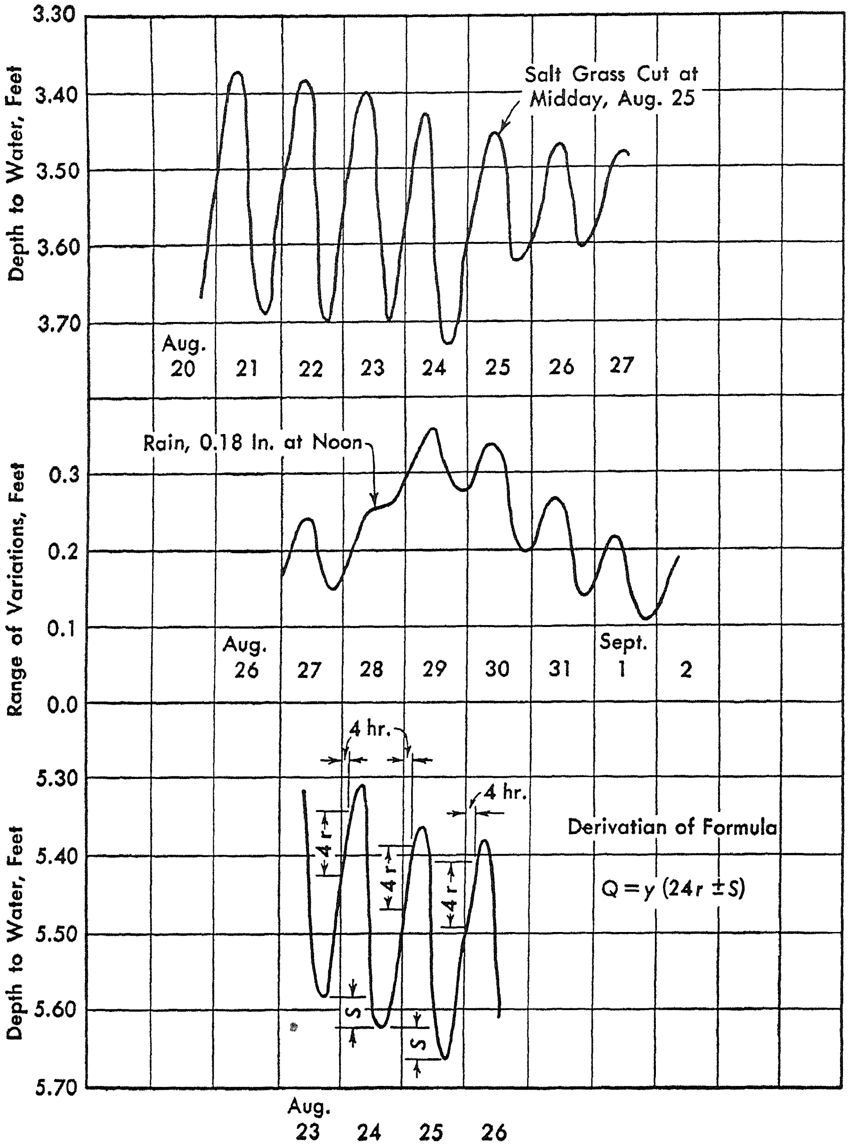



FIGURE 170. Daily Fluctuations of Ground Water, Escalante Valley, Utah

*Effect of Precipitation on the Water Table.* Because of the variability of precipitation and soil conditions, it is not certain what effect a given amount of precipitation will have on the water table. One can, in general, speak only of tendencies. During the warm growing season, light rains prevent a draft on soil water by vegetation and thus preserve the ground water, or permit a larger proportion of the heavier rains to percolate down to the water table. This has been shown con-

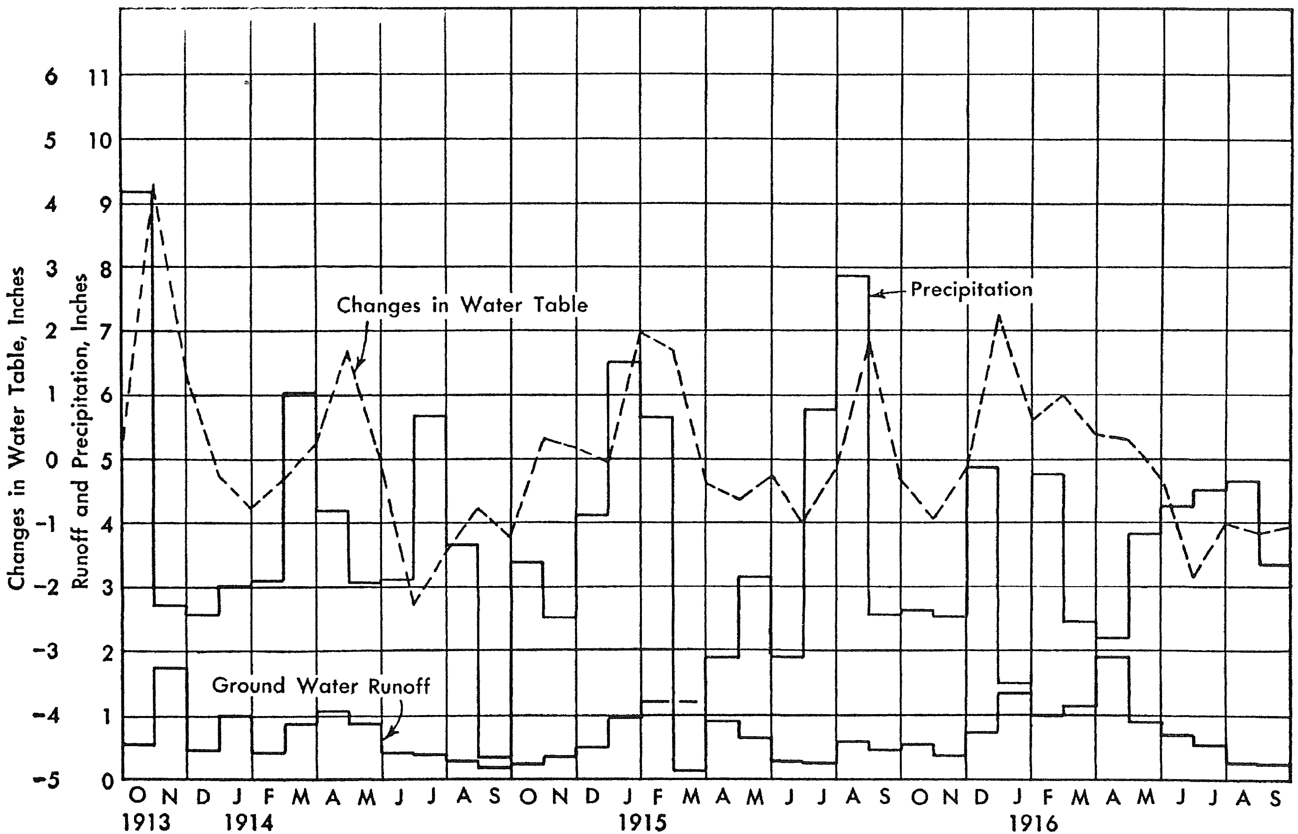



FIGURE 171. Ground Water Operations, Pomperaug River, Conn.

clusively by the investigations of White (192) in the Escalante Valley, in Utah. The ground water in Escalante Valley was within 2 to 10 feet of the ground surface so that vegetation extracted water from the capillary fringe, and the daily withdrawals were reflected in the elevations of the ground water table. In those investigations it was revealed that even light rains of one- or two-tenths of an inch prevented a diurnal draft on the ground water, but did not otherwise affect the water table. These conditions are illustrated in Figure 170. On the other hand, investigations in the Pomperaug River basin by Meinzer and Stearns (129) showed that spring rain combined with snowmelt raised the ground water materially, while precipitation through the summer did not prevent a general lowering of the water table. Replenishment was obtained in the season of no growth of vegetation and of heavy precipitation and snowmelt. These conditions are shown in Figure 171. This situation may be taken to be the rule in any region, except that where there is no snowfall, replenishment depends upon moderately heavy and sustained rainfall.

*Volume Changes in Ground Water.* The effect of that portion of precipitation which reaches the ground water on the free water table depends upon the nature of the soil or rock which fixes the holding capacity of the aquifer. Assuming that the capillary fringe is far enough below the ground surface and that the porosity of the soil or rock does not change with increasing elevation, the capillary fringe may be expected to rise as the water table does, keeping its original thickness; then the only space available for the ground water storage will be the space not filled by capillarity, the volume of which space amounts approximately to the specific yield. Therefore, if $P_w$ is the portion of the precipitation reaching the free ground water, and $S_y$ is the specific yield, the increase in elevation $W_s$ of the water table is

$$W_s = \frac{P_w}{S_y}.$$

*Areal Distribution of Changes in the Water Table.* Any consideration of volumetric change in the ground water must consider three dimensions, two of the area and one of the depth. All data of depths to the water table below the ground surface and changes in ground water are obtained from wells or point measurement of one sort or another. These wells provide point or isolated observations which must represent relatively extensive areas. Usually wells are spaced irregularly over a ground water basin, so that some type of weighting must be resorted to in computing average changes over the basin. This weighting should be based on soil and rock conditions unless it is known that they are

uniform, although the type of soil or rock and its texture vary so much that it is not likely that they can be found uniform over any large area. However, it is seldom that adequate information is available for suitable weighting on the basis of underground conditions, so simple areal weights must be resorted to. For this purpose the Thiessen method can well be employed.

*Changes in Storage.* Changes in storage can be computed by putting the foregoing considerations into mathematical form. The first step is, of course, to obtain the average change in elevation over the area of the basin. Because of the irregular spacing of observation wells, the observations must be weighted by a series of values, $a_1$ to $a_n$, which are percentages of the total area which are represented by the individual wells. The change in stage for any given well is expressed above in terms of the specific yield. Then for $n$ wells on the area $A$, the change in volume $V_s$, can be expressed as follows:

$$V_s = A[a_1(W_sS_y)_1 + a_2(W_sS_y)_2 + a_3(W_sS_y)_3 + \cdots + a_n(W_sS_y)_n]$$

where the notation is as previously given.

It must be emphasized that these computations of changes in volume can be applied only to wells in a water table under atmospheric pressure. Wells which reach a confined aquifer define changes in the piezometric surface, but do not indicate changes in volume of water in the confined or free water-bearing stratum which may exist under the same area.

*The Ground Water Equation.* For any given ground water basin the gains and losses of water can be expressed as an equation. Conkling (35) has set up one such equation which is as follows:

$$\left.\begin{array}{l}\text{Surface outflow plus}\\ \text{Underflow out plus}\\ \text{Consumptive use plus}\\ \text{Change in storage plus}\\ \text{Exportation}\end{array}\right\}\text{ equals }\left\{\begin{array}{l}\text{Surface inflow plus}\\ \text{Underflow in plus}\\ \text{Precipitation}\end{array}\right.$$

The terms used in the above equation are defined as follows:

1. SURFACE OUTFLOW. Surface outflow includes all water leaving at the lower end of the area by natural surface channels; it includes both surface and ground water runoff as defined in Chapter 9. It also includes water diverted from the surface channels for irrigation, public water supply, or use in other areas than the basin.

2. UNDERFLOW OUT. Underflow out consists of the outflow from the ground water basin by percolation across the lower boundary of the aquifer. Underflow is a difficult item to measure. The basin should be so

delineated that the underflow is at a minimum, and if possible of negligible proportions. The velocity of underground flow is usually small (except in cavernous channels) and the volume of water so conveyed is likely to be small as compared to surface flow. Any flow, however, is reflected in the elevations of the water table, so that errors in estimating the underflow out do not result in serious errors in estimate of the total water supply.

3. CONSUMPTIVE USE. Consumptive use is that water which is transpired through the growth of vegetation and evaporated from the soil.

4. CHANGES IN STORAGE. The changes in storage are obtained by raising or lowering the elevation of the water table as discussed above.

5. EXPORTATION. Water exported includes all water pumped from the aquifer and conveyed for use off the area of the basin.

6. SURFACE INFLOW. Surface inflow is that water brought to the boundaries of the basin by natural channels, by ditches or canals for irrigation, by pipelines for public water supply, or by other means for similar purposes.

7. UNDERFLOW IN. Underflow in is water brought to the basin by percolation through the aquifer.

8. PRECIPITATION. Precipitation includes rainfall and snow as measured on the basin.

Some data taken from the report of the ground water investigations on the Pomperaug River basin (129) may be substituted in the above formula to indicate its operation. The ground water basin included the entire surface watershed, so that, since the underlying bedrock was impervious, ground water underflow was so small that it was considered negligible. The ground water runoff was computed from hydrographs such as those illustrated in Figure 111 showing the ground water flow. There were no exportations of water from the basin.

Then for the water year 1913 to 1914, the equation can be set up as follows:

| | | | | |
|---|---|---|---|---|
| Surface outflow | | = | 21.04 inches | |
| Underflow out | | = | 0 | |
| Consumptive use* | | = | 25.17 | |
| Change in storage | | = | +0.45 | |
| Exportations | | = | 0 | |
| | Total | | | 46.66 inches |
| Surface inflow | | = | 0 | |
| Underflow in | | = | 0 | |
| Precipitation | | = | 46.66 | |
| | Total | | | 46.66 inches |

For the water year 1915 to 1916, similar data are as follows:

| | | | |
|---|---|---|---|
| Surface outflow | = | 24.15 | |
| Underflow out | = | 0 | |
| Consumptive use* | = | 19.18 | |
| Change in storage | = | −1.83 | |
| Exportation | = | 0 | |
| Total | | | 41.50 inches |
| Surface inflow | = | 0 | |
| Underflow in | = | 0 | |
| Precipitation | = | 41.50 | |
| Total | | | 41.50 inches |

*Composition of Ground Water.* The composition of ground water is usually markedly different from that of surface water because of the larger and more varied content of dissolved minerals. There is, however, a great variation in kind and amount of minerals in the ground water of different regions, a variation that depends upon the geologic formations through which the water passes. Some ground water may have little dissolved minerals; for example, the water from wells used at Pensacola, Fla., for public water supply, is reported to have only 22 parts per million of total dissolved solids. This case is exceptional, however, for ground water commonly has several hundred and even thousands of parts per million of total solids. The water from the Dakota sandstone, a deep-lying formation supplying artesian water to large parts of the states of North and South Dakota, is reported by Abbott and Voedisch (1) to carry usually more than 2500 and frequently more than 3000 parts per million.

In the same regions there may be also a wide variation in the dissolved minerals of ground water under a relatively small area. In one case, Abbott and Voedisch (1) give the results of analyses of water from 12 wells in alluvial deposit in an area of one square mile. The amount of total dissolved solids ranges from 706 to 2899 parts per million; the average is 1587 with a standard deviation of 717 parts per million. Such variation indicates little mixing, and therefore the movement of ground water is small despite the fact that all wells in the foregoing example were located in alluvium. This example is not extreme for the region (North Dakota), for the authors reporting the observations give even more extreme cases.

It is to be noted that methods of statistics should not be applied

* Consumptive use in this instance includes also changes in storage in the belt of soil water and in the surface storage. The latter item would be small; the value of the first item is unknown but it is probably small also.

indiscriminately to data such as given in the preceding paragraph. The means and standard deviations can, of course, be found with propriety, but it would not be permissible to discard any observation, even if it exceeded the limit of three times the standard deviation. The rejection of an observation for exceeding such a limit is based upon the laws of chance variation, while the variations in the data used above are the result of actual differences in material. Such variations cannot be judged by the laws of chance.

# 12 THE UTILIZATION OF HYDROLOGIC DATA

*Purpose of Data.* The purpose of collecting and compiling hydrologic data is the development and use of the water resources of a region. The first objective requires an inventory of the resources, or a determination of what water is available. It includes collecting and compiling the data of precipitation, runoff, and all pertinent related elements, including particularly those of the climate, which is the fundamental control of the amount and availability of water resources. The second objective requires analysis of the data to determine the best utilization of this water supply for the satisfaction of the requirements that have been found to exist.

*General Information Required for Basic Study.* The information to be obtained from an inventory of water resources should not be limited to data of rainfall and runoff but should be broad enough to give knowledge of climatic conditions. Studies pertaining to water resources must be based on a broad knowledge of local conditions. The compilation and analysis of most of this information is the particular province of the hydrologist or hydraulic engineer. Such a person should be equipped by thorough training to cope with the complex problems of water supply and its utilization.

Precipitation, being of primary importance, should be investigated thoroughly and in detail. Stations for obtaining precipitation data should be established as required so that there will be coverage adequate to obtain reliable data of the fall over the entire area under study, and this requires that they be established and continued over a long and uninterrupted period. The data to be obtained should include depths of precipitation for suitable intervals of time, duration of storm, and corresponding intensities which are best obtained by continuous records. Annual distribution is obtained from the daily values. Areal distribution can be secured by proper distribution of the stations. Where snow occurs particular attention should be given

to collecting data of depths, water equivalent, and areal distribution.

The collection of data of runoff is determined largely by the uses to which the data will be put. Nevertheless, some thought should also be given to the comprehensive picture of basin runoff, with some stations being established to investigate that aspect. With a few basic stations established as guides, others can be placed as the specific needs may justify them for the proposed investigation or development. It is advisable to locate stations on principal tributaries, as well as at suitable intervals along the main stream. The selection of sites is guided by the needs of the data and location of the important tributaries, which may govern to a large degree the yield of the watershed. Likewise, stations should be placed to gage those subareas of a basin where a change of climate may be discernible and where storage areas such as lakes produce a marked change in the regimen of the stream.

Other elements of climate should also be studied, but not necessarily as thoroughly as rainfall and runoff. Temperatures expressed as mean monthly, annual means, and the range of values, should be obtained; while the details are not necessary, enough data should be obtained to determine stable values. Prevailing winds and types of air masses should be determined, as these indicate the source and probable quantity of moisture. Data of the frost-free growing season are important where agriculture exists as a basic industry. The general aspects of climate can be compared by Koeppen's or Thornthwaite's system of climate classification, or by the construction of a few climatic charts as described in Chapter 2.

*Particular Use of Water Resources.* Only the broadest aspects of the uses to which water can be put by the various human activities can be covered in this brief discussion. Every instance of use is a problem by itself and must be solved by individual analysis, even though it may be similar to other instances. There are also many small problems arising in every large development, that vary from time to time and place to place. These problems involve the application of the principles of hydrology to the individual needs of the development. No attempt is made here to list all these problems, but a tabulation of some of the more important requirements in different types of development is made.

*Urban Water Supply.* The primary hydrologic problem of urban water supply is determination of what amount of water is available. This is an inventory problem. Other factors to be determined for successful operation are the seasonal distributions available to meet

demands, losses by evaporation, and minimum safe supply, which may govern storage requirements. If dams are to be constructed for storage there is a question not only of capacity, but of spillway and outlet design required for operation and safety of the structure. Storage problems are more likely to be acute on small basins that are utilized for water supply than for large ones, because the latter usually have an excess of water over the available storage capacity, so that the supply is not questioned. In smaller drainage basins, because of greater concentration of rainfall, spillway capacity is a more acute and serious problem.

*Irrigation.* Irrigation is a consumer of water in the sense that a large proportion of the water diverted from a stream for this use is lost or consumed in so far as the downstream interests are concerned. It is concerned primarily with the available water supply, next with consumptive use, and third with seasonal distribution. The minimum annual water supply available is important since it determines the area of land that can be put under irrigation. Consumptive use, which varies with the climatic temperature or heat of locality and type of crop, determines the area to which a given water supply can be applied. Problems of distribution may arise from a dry summer growing season or a short wet season. Runoff outside the crop-growing season is not available for irrigation, and is lost unless held over in storage. If dams are constructed a knowledge of the probable maximum flood for spillway design and frequency of floods for filling the reservoir is required. Flood frequency may also be a matter for investigation in connection with valuation of the irrigable land lying in the flood plain. Data of frequency and intensity of rainfall may be required for the design of the irrigating distribution system to insure its adequacy and safety, unless the climate is such that no rain falls on the irrigable land.

*Water Power.* The chief hydrologic interest for water power is also one of supply, that is, how much water is available through the year. The question of minimum flow is important because it determines the prime flow available for firm power. The annual distribution of runoff should be accurately known for purposes of storage, pondage, and regulation, all of which must fit the generation of energy demanded by the power system load. Usually some sort of dam must be built, which again requires a knowledge of the probable maximum flood for the spillway design and frequency of floods for storage operation. Floods are a menace to dams and related structures, and hence knowledge of flood frequency is needed for estimating the risk of such losses and making financial provision for them. Losses of water from evaporation must be considered in connection with assumed yields.

*Navigation.* Navigation on natural streams is confined generally to the larger rivers in humid countries where there is a reasonably good supply of water. The principal interest of navigation is in having sufficient depth of channel, a condition that requires a steady flow of water through the navigation season. Where the natural flow does not provide sufficient depth it may be augmented by storage, which involves an inventory of supply and evaporation, and consideration of the problem of regulation. In event of canalization by locks and dams, frequency and magnitude of floods are involved in designing structures of adequate capacity, as well as design floods for normal operation.

*Flood Control.* Strictly speaking, flood control is not a use of water or water resources, being rather protection from excess water, but it entails much the same type of investigations, and not infrequently is combined in a multiple-purpose reservoir with real utilization of water resources. It may therefore be considered with other types of stream development. The primary interest here is the magnitude and frequency of floods, since upon these factors depend the extent of the damages, and consequently by their elimination or reduction, the benefits to be expected from proposed remedial works. Since many types of flood protection are possible either singly or in combination there are as many variations of hydrologic investigations to be made as there are flood control projects. These investigations include, in addition to magnitude and frequency, the source of floods or the flood-producing areas of a basin, the frequency and intensity of precipitation, with the resulting flood runoff, regulation of storage reservoirs for flood reduction, and the probable maximum and other design floods.

*Drainage and Storm Sewers.* Because the hydrologic problems of drainage and storm sewers are similar, they may conveniently be considered together. In both, the primary objective is to remove the large quantities of water from intense rainfall as rapidly as it is economically justifiable. The principal hydrologic problem is therefore one of intensity and frequency of heavy rainfall and the resulting flood runoff. There is some difference, however, in the rates of removal demanded by storm sewers and drainage. The former is concerned with quick removal of runoff of the frequent as well as rarer storms; the latter can usually tolerate more flooding, and the added expenditure for rapid removal is less easily justified from an economic standpoint. Nevertheless, the problems in both cases are those of intensity and frequency of rainfall or rapid snowmelt, which cause excessive runoff.

*Problems in Other Types of Development.* Many other types of economic development encounter hydrologic problems of one sort or

another. Only a few can be mentioned here. Agriculture is interested in the supply and distribution of precipitation, and intensity and frequency of rainfall. Erosion of surface soil from farm land as well as other areas is influenced greatly by the frequency and intensity of precipitation. Transportation facilities such as highways and railroads are concerned with problems of intensity of precipitation in the adequate designing of culverts, waterways under bridges, and similar drainage works for passing excess water. Outdoor sport events are affected by rainfall, that which falls during the day being of particular interest. Finally all developments existing in the flood plains of rivers are affected adversely by floods, and hence are concerned with frequency and magnitude of floods in order to evaluate the risk of damage.

*Major Hydrologic Problems.* The foregoing review of the needs of the various types of utilization of water resources reveals that there are a number of major hydrologic problems which are of interest to one or more lines of development. These, of course, do not include many smaller but still important questions that must be answered as they arise. The major problems may be listed as follows:

1. The supply problem, or inventory of water resources.
2. Evaporation losses, which are deductions from available supply.
3. Distribution problem, which involves seasonal adjustment to fit water requirements. This may also be called a regulation problem.
4. Flood frequency problem.
5. Rain frequency problem.
6. Intensity-frequency problem.
7. Design flood problem for both spillway design and reservoir operation.

*The Supply of Water.* The question of supply of water is an inventory problem. It is a problem of determining and keeping on record the amount of water that can be obtained from a given surface basin for whatever development is contemplated. The inventory starts with precipitation because it is the ultimate source of all runoff. Upon initiating an investigation, precipitation stations should be located over the basin or area to determine the rain and snowfall within the required accuracy. To the extent that it is justified by the need for accurate results, stations should be located on high lands and mountains as well as in the valleys. In some regions it may be possible to use annual totalizers, but generally it is desirable because of the problems of distribution to install stations to obtain data on the basis of months, or preferably daily periods. In regions where snowfall is of economic

value, snow surveys should be made to obtain needed information of water from that source.

The objective of an inventory of water is to determine the amount of stream flow, and without doubt the best way to get that information is to measure the flow directly. This may sound like a silly statement but in view of the all too frequent attempts made by various agencies to determine stream flow from too few gaging stations, or to use without good correlation those located on adjacent streams, or to use fragmentary records, it is a simple fact that must be constantly reiterated and emphasized. For any stream development, the best records of stream flow are those obtained for the stream under consideration at the point in question. However, flow records from a station not too far distant up or down stream may readily be adjusted satisfactorily to the site to be developed proportionately to the drainage area or mean annual flow.

If records are not available on the stream or are from a station too distantly located, other steps must be taken to provide a record at the site. This estimate of flow may be made by one or more of several methods.

First, the yield may be estimated from records of adjacent basins by taking into consideration as far as possible the climatic, topographic, and other differences in the watersheds. Particularly any change in rates of precipitation should be given full consideration. For a comparison of the precipitation of the various areas, graphs such as those in Figures 17 to 21 should be prepared to show the yearly distribution. Monthly means and standard deviations should be computed from annual records of precipitation and it may be advisable to determine the respective trends; and, for estimating the closeness of the relationship of two areas, the coefficients of correlation should be computed. A qualitative estimate of differences in the characteristics of the basins should be made.

In a second approach, the data of stream flow may be compiled from stations on tributaries of the stream, care being taken to include the yield from all areas that can contribute to runoff, and to secure for the entire basin a record that is consistent, or to account for inconsistencies. Changes in climate and topography should likewise be considered in compiling data of this sort. Although in this case the climates of different parts of the basin are likely to be similar, investigations should be made to ascertain actual conditions.

A third source of runoff data is the records of precipitation and other meteorological elements. Stream flow data from this source, however, can be only roughly approximate, except in special instances where it

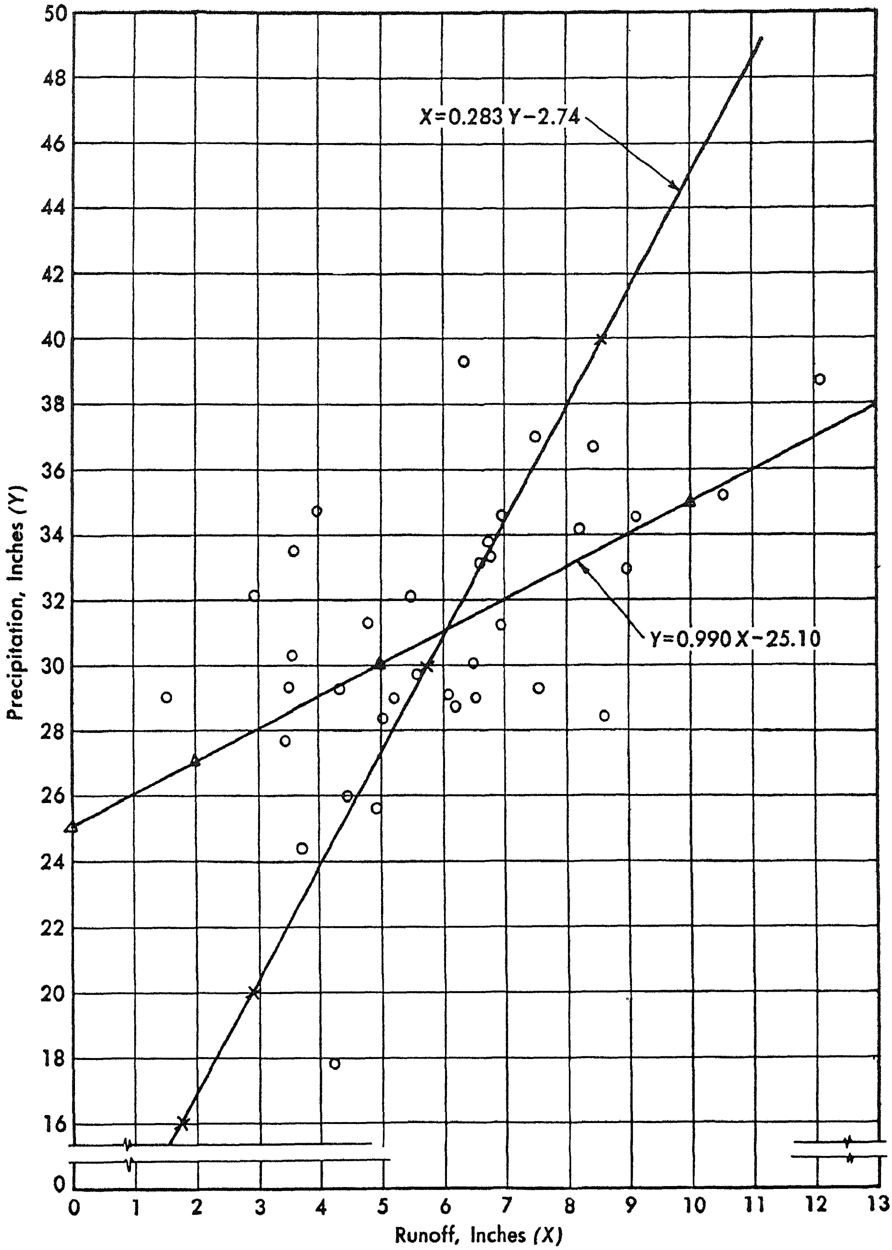



FIGURE 172. Scatter Diagram of Runoff and Precipitation, Cedar River, Cedar Rapids, Iowa

may be desirable to expend much labor to secure the greatest possible accuracy by utilizing distribution factors. Nevertheless, this method of estimating stream flow data merits further consideration, since there are occasions on which it must be used.

The method to be used for computing runoff from precipitation depends upon the use to which the results are put, the length of record computed, and the accuracy desired. For roughly approximate results where some suitable discharge records are available for a basis, runoff may be computed by deriving a relationship between precipitation and stream flow for concurrent periods of record and using this relationship to compute additional records of runoff from the records of precipitation. The relationship between rainfall and runoff should be found by seasons, which should be fixed by the hydrologic characteristics of the region instead of the calendar. For example, on the northern Great Plains the seasons may be winter from December to March inclusive, spring in April and May, summer from June to August, and fall from September to November. Other regions will have different seasonal periods, as for example, a climate like that of Red Bluff, Calif. (Figure 17), without snow, may be divided into two seasons to the year. The controlling factors for selecting the seasons are the amounts and types of precipitation, temperatures, and evaporation.

The methods of correlation illustrated heretofore can be used to advantage by utilizing the equation of the regression lines to express the relationship between precipitation and runoff. To illustrate the method, the following example is given from data furnished by Mr. L. C. Crawford of the U. S. Geological Survey; the data were prepared from the records of the Cedar River above Cedar Rapids, Iowa, and are for the calender year. The pairs of data consisting of depths of precipitation and runoff are plotted one against the other on the scatter diagram shown in Figure 172. The correlation coefficient was computed and found to be 0.53. The lines of the regression and the equations shown on the figure were computed from the following values obtained in the computation of the correlation coefficient:

For runoff — the mean, $\bar{X} = 6.06$ inches; the standard deviation, $\sigma_x = 2.22$ inches.

For precipitation — the mean, $\bar{Y} = 31.10$ inches; the standard deviation, $\sigma_y = 4.15$ inches.

Then, by substituting in the type equations given in Chapter 1, the following equations of regression are obtained:

For runoff on precipitation,

$$X = 0.283Y - 2.74.$$

For precipitation on runoff,

$$Y = 0.990X + 25.10.$$

Figure 172, showing the regression lines in relation to the plotted points, indicates clearly the need for deriving the equations for the relationship between runoff and precipitation. Frequently one straight line is drawn by eye through the plotted points, and through it the values of runoff are obtained from the given data of precipitation. It can be seen that such a procedure is not likely to obtain the best results.

If there are no records of stream flow that can be used as a basis for deriving a rainfall-runoff relationship, but there are available records of precipitation, temperatures, and other meteorological elements, it may be possible to compute volumes of runoff. This method should also be applied to the entire stream basin. To decide the practicability of such a procedure, a rainfall-runoff equation can be set up in the manner of Conkling's (35) equation for ground water storage. This runoff equation would be slightly different from that of ground water storage, and of the many forms possible, the following serves the purpose:

$$\left.\begin{array}{c}\text{Precipitation}\\ \text{plus}\\ \text{Diversions onto the}\\ \text{basin}\end{array}\right\}\text{ equal }\left\{\begin{array}{l}\text{Runoff} = \begin{cases}\text{Surface runoff plus}\\ \text{Ground water runoff}\end{cases}\\ \text{Plus losses by consumptive use}\\ \text{Plus changes in storage} = \begin{cases}\text{Ground water}\\ \text{plus}\\ \text{Soil water}\end{cases}\\ \text{Plus underground outflow}\\ \text{Plus deep seepage}\\ \text{Plus diversions from the basin.}\end{array}\right.$$

All the terms in the above runoff equation except "diversions" have been defined heretofore. By the term "diversion" is meant any artificial transfer of water to or from the basin for any purpose, such as irrigation, public water supply, sewerage from a city whose water supply comes from another basin, and the like.

For example, it is assumed that the above equation would be applicable to an entire surface basin such as the Pomperaug River basin (129). In this case, ground water inflow was prevented by the impervious ridge under the interfluve so that precipitation was the sole source of inflow. However, if the ground water had crossed the interfluve as shown in Figure 166, there would have been some underground inflow and the equation would have to be modified.

The application of the above formula will depend entirely upon the data available and the condition of the watershed under investigation. Some of the elements are usually small and may be neglected. Deep seepage is commonly considered to be negligible; the only circumstance under which it may not be omitted is the dipping of a highly permeable aquifer from a land exposure to an outlet under the ocean or to another watershed. Underground inflow and outflow may frequently be negligible in comparison with the entire flow of the surface watershed, but an investigation should be made to check this situation for possible large losses. Diversions to and from the basin may be determined by a survey of the current use being made of the water of the stream.

The remaining elements in the equation are variables that must be determined or accounted for in some manner. Runoff is the element to be determined for the water inventory. Precipitation can be obtained from the records of the individual stations and computed for the area by means of isohyetals or Thiessen's method. The critical elements are the changes in storage in ground water and soil water, and the consumptive use. Changes in ground water may possibly be determined by a survey of the surface wells (not artesian) in the area, for which records of elevations have been kept by the users, in connection with local water supply. There would be little likelihood of determining any past changes in the soil water, but such changes can be minimized by selecting a yearly period from dry season to dry season so that they can be considered negligible.

Consumptive use is, on the other hand, an important element and must therefore be evaluated with reasonable accuracy. No general formula for deriving year-to-year annual consumptive use from meteorological data alone has yet been derived. Blaney and Morin (13) have published an empirical formula for computing consumptive use under some conditions where the daytime hours, monthly temperatures, and humidity are known. This formula, however, as well as all other methods of computing consumptive use on the basis of heat or temperature, would give results pertinent to full evaporative opportunity such as a water surface or vegetation dependent upon the ground water or capillary fringe. Such formulas would not be applicable in subhumid or semiarid regions where precipitation during much of the time is less than consumptive use.

If, instead of a year-to-year record, only the average annual runoff is desired, the changes in storage of ground water and soil water may be considered negligible and the consumptive use can be computed by some method as that of Lowry and Johnson's which was discussed in

Chapter 8. The average annual runoff may then be computed for the basin, provided other conditions, including ample precipitation to supply consumptive use, are suitable.

This method of estimating runoff entirely from meteorological data should be undertaken only as a last resort and with full realization of the variations in kind and intensity of precipitation and the effect each will have on runoff. The period of accumulation and melting of snow may be dealt with as a unit with respect to time unless the watershed is subject during the winter to alternate snowfall and subsequent loss by melting. Even in regions ordinarily subhumid or semiarid, there are intense storms from which runoff occurs because the rate of precipitation is greater than the infiltration capacity. In such regions the runoff depends upon the intensity of rainfall and the accumulation of snow.

The above methods of computing runoff from precipitation records may be used when averages of long periods are acceptable for use so that individual variations and errors are not important. If, on the other hand, it is desired to obtain stream flow for a short period such as a single flood, the surface discharge should be computed by means of the distribution factors as outlined in Chapter 9. Estimates of ground water flow must be made according to the season and added to the surface runoff. It can readily be seen, however, that the labor involved limits the application of this procedure to short periods of runoff.

The results of an inventory of runoff may be compiled in a table or by means of a hydrograph such as is shown in Figure 110. For record purposes, the former is preferable, but for analysis or presenting the results of analysis the hydrograph may be more desirable.

*Minimum Flow.* The minimum annual runoff, or yield as it is called in water supply usage, is a matter of concern for several purposes. To answer this question a search of all available records of stream flow should be made. To extend the period, precipitation records should be studied within the region to the limits of a homogeneous climate. In some cases it may be justifiable to extend the knowledge of the records by studying tree rings in the region by the methods developed by Douglas and others (70); this procedure may reveal any marked changes in climate. The probability and frequency of the years of observed low precipitation can be computed by methods given in Chapter 5.

*The Problem of Evaporation Losses.* The problem of evaporation losses is of vital importance in arid or semiarid regions where existence depends upon limited water supply. Evaporation should therefore be given careful study. Neither the problem nor the losses can be neg-

lected. Evaporation is a constant deduction from any body of stored water, and must, therefore, be determined before the net volume available for use is known.

Often the only practical solution is to use one of the formulas given in Chapter 8 to compute the evaporation from the meteorological data of the Weather Bureau. Meyer's formula is commonly used for this purpose. This, as well as the others, gives the evaporation in inches per day. However, in view of the low correlation, 0.13, found between evaporation from pans and that from reservoirs, there is little to be gained (except additional labor) by computing daily evaporation. The monthly means average out some of the irregularities and yield results as accurate as can be expected from the present knowledge of evaporation.

To illustrate the procedure of computing evaporation from data of the Weather Bureau, evaporation at Austin, Texas, will be computed from data published in the *Monthly Weather Review* for the year 1933. Meyer's formula is used; it is as follows:

$$E = (0.5 + 0.05W)(e_s - e_d)$$

in which the notation is the same as given heretofore. The data available consist of mean monthly wind velocities in miles per hour, mean monthly temperature, and relative humidity. For convenience in computing, the formula can be modified slightly. The wind factor, $(0.5 + 0.05W)$, remains the same, but air temperatures and the vapor pressures pertaining thereto must be substituted for saturation at water temperature, and saturation at dew point must be replaced by relative humidity. The formula can then be set up as follows:

$$E \doteq (0.5 + 0.05W)(S.V.P.)_a(1 - R.H.)$$

in which $(S.V.P.)_a$ is the saturated vapor pressure at air temperature and $R.H.$ is the relative humidity. The reasons for these substitutions are plain in the light of the definitions of humidity. This formula gives the mean daily evaporation, so each result must be multiplied by the number of days in the month to obtain the monthly total. The computation is given in Table 88.

The computations in Table 88 are clear so that no further explanation is required. There is no satisfactory check on the results because of the lack of suitable observations of evaporation, except a record of 18 years of pan evaporation at Hill Ranch, near Austin, which yielded a mean annual of 67.74 inches. However, the temperatures and wind velocities in Table 88 were both higher than the comparable data at Hill Ranch, which would account for a part, if not all, of the difference

TABLE 88. COMPUTED EVAPORATION, AUSTIN, TEXAS, 1933

| MONTH | WIND *mph* | FACTOR (.5 + .05*W*) | TEMP *F* | *S.V.P* *Inches* | *R.H.* *Per Cent* | *S.V.P.* × (1 − *R.H.*) | EVAPORATION *Mean Day* | INCHES *Month* |
|---|---|---|---|---|---|---|---|---|
| Jan. | 7.9 | 0.895 | 58. | 0.482 | 70. | 0.145 | 0.130 | 4.03 |
| Feb. | 9.1 | .955 | 50. | .360 | 75. | .090 | .086 | 2.41 |
| Mar. | 9.4 | .970 | 64. | .595 | 66. | .202 | .196 | 6.07 |
| Apr. | 9.1 | .955 | 69. | .707 | 57. | .304 | .290 | 8.70 |
| May | 9.2 | .960 | 79. | .989 | 67. | .326 | .313 | 9.72 |
| June | 7.8 | .890 | 80. | 1.022 | 59. | .418 | .373 | 11.20 |
| July | 8.0 | .900 | 85. | 1.201 | 65. | .420 | .378 | 11.74 |
| Aug. | 6.2 | .810 | 84. | 1.163 | 71. | .337 | .273 | 8.47 |
| Sept. | 7.0 | .850 | 83. | 1.127 | 72. | .315 | .268 | 8.04 |
| Oct. | 6.6 | .830 | 74. | 0.838 | 68. | .268 | .222 | 6.88 |
| Nov. | 8.1 | .905 | 62. | .555 | 68. | .176 | .159 | 4.77 |
| Dec. | 9.0 | 0.950 | 60. | 0.517 | 63. | 0.191 | 0.182 | 5.64 |
| | | | | | | | Total | 87.67 |

in annual evaporation. The monthly evaporation values in Table 88 would be equivalent to evaporation from pans and should be reduced by an appropriate coefficient, say 0.70, to obtain the estimated evaporation from a reservoir.

Furthermore, in the use of meteorological data for computing evaporation, the location of the Weather Bureau meteorological station should be carefully checked for similarity of location and exposure, and the data adjusted if necessary, to secure values from an exposure comparable to that of the reservoir. Particularly the height of the instruments should be noted because some stations of the Weather Bureau, being located on buildings, obtain greater wind velocities than would prevail over a reservoir.

The monthly evaporation computed in Table 88 is the unit areal loss. In order to obtain the total loss of water it is necessary to apply the unit loss to the entire area of the reservoir. Since changes in elevation of the water surface entail changes in the area, the actual area should be taken from a curve of reservoir elevation versus area, and the losses computed under operating conditions.

*Distribution of Supply.* The problem of distribution presented here is that of fitting the annual distribution of runoff to the needs of a proposed development. Except as it is modified by snowmelt, runoff follows the distribution of rainfall, some typical examples of which are shown in Figures 17 to 21. Typical distributions of runoff are shown in Figure 110 and the Plates, pages 300 and 301. Snow modifies the distribution of runoff by melting upon the advent of warm weather.

Unfortunately for most purposes the natural distribution does not fit the economic requirements of development. The demand for urban

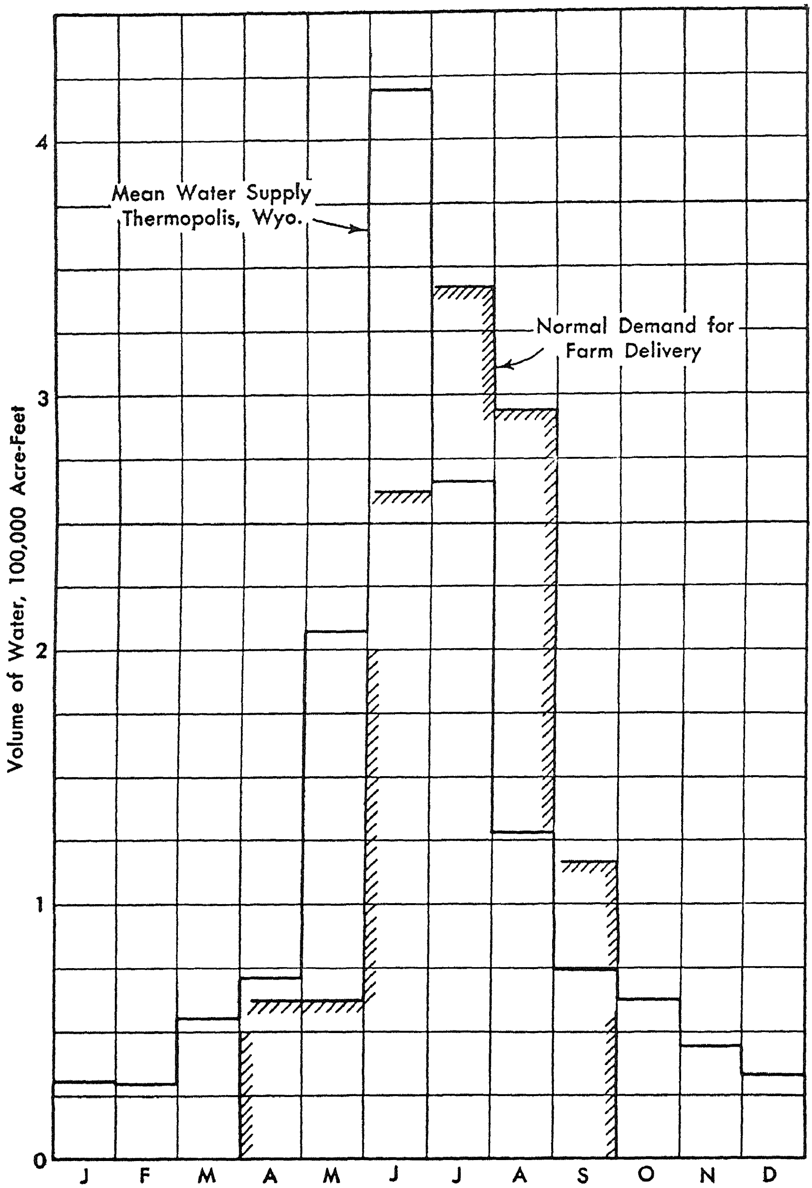



FIGURE 173. Water Supply and Demand, Big Horn River, Wyo.

water supply is probably the most uniform, but even there the monthly demand varies appreciably; for example, the monthly demand ranges from 90 to 124 per cent of the annual mean in Massachusetts cities. The power load on a hydroelectric system varies even more, while the needs for irrigation are very much seasonal in accordance with the demands of growing crops. An interesting example is provided by the

Big Horn River on which the runoff from snowmelt almost but not quite coincides with demands for irrigation; the data are plotted in Figure 173.

Where there exists a marked lack of coordination between natural flow and economic needs the problem of regulation must be solved. For studying and investigating the best means of regulating stream flow, there are several devices.

The first is the hydrograph, which has been introduced in the study of runoff in Chapter 9 and illustrated by Figures 110 and 111. It can be used to show the effect of, or to suggest desirable types of regulation. When a discharge hydrograph has been plotted from stream flow data, there is obtained a rate on one axis and time on the other; then, since the product of a rate and time is a quantity, the area under the hydrograph represents a quantity of water. Then by measurement, one can compute the storage needed in a reservoir to obtain a predetermined discharge and also the volume of water that will be wasted over a spillway.

The hydrograph can be used in a similar manner to compute and show the power available, by using the proper head and constants as multipliers for the rates of flow shown on the hydrograph. Figure 174 illustrates the use of a hydrograph in developing the amount of power available and wastage.

The hydrograph presents a better and more vivid picture than do other devices of what a stream does from day to day, but does not lend itself readily to computation. Because of this, it is little used in making estimates of power. It serves best as a pictorial representation of the power or flow possibilities and it is useful in reports as an aid for visualizing power characteristics.

*The Duration Curve.* The duration curve is an important curve for the calculation of power. Although it may be derived for any unit of time, it is best constructed on the basis of the average daily discharge. This unit of time is small enough to eliminate any appreciable error from using the average flow in the unit of time and is a convenient period for recording the stage and obtaining the average flow. A longer unit admits appreciable errors because, since the average is used for the period, a much smaller flow may have been obtained for a part of the unit period.

To prepare the curve, a list of flows is selected for class limits ranging from minimum to maximum, using again as small a difference as is convenient. These class units of flow should be so selected that the plotted points determine accurately the curve.

The next step is the preparation of a "deficiency table" for each year

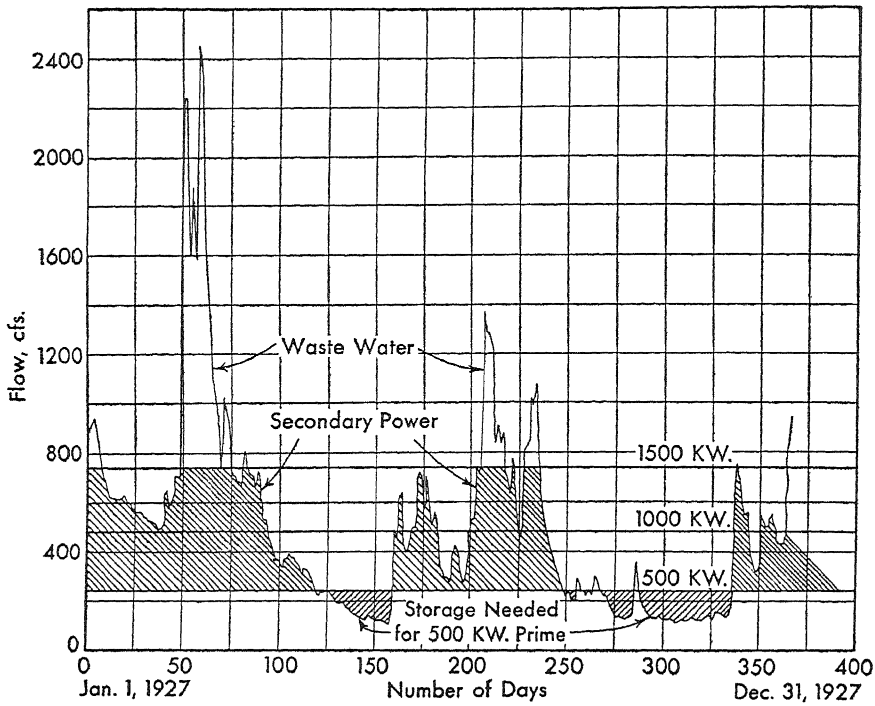



FIGURE 174. Hydrograph of Power Requirements, Ocklocknee River, Near Bloxham, Fla., Jan. 1–Dec. 31, 1927

of record to show the number of days for which the actual flow was less than the designated class limits. The number of days of deficient flow in each year is then summed up for the entire period, and the percentage of total time during which each flow was obtained is computed. These percentages of the total time are used for the abscissas and the selected class limits of flow are used for the ordinates.

The duration curve is used to calculate the power available for various percentages of time. The amount of prime power is one important result that can be obtained from the duration curve. It is commonly taken as the power available for 90 to 100 per cent of the time, or the amount that can be depended upon at all times with whatever pondage may be had. The power obtainable from flows for a smaller percentage of the time is designated "secondary power" and may likewise be computed from the duration curve. Furthermore, the average storage water needed to convert secondary power into prime, or the amount of power that must be purchased or generated by other means, may be calculated from the same curve. Since the abscissas are time, and the ordinates are rates of flow, the product of any abscissa and ordinate is a quantity of water and is represented on the graph by

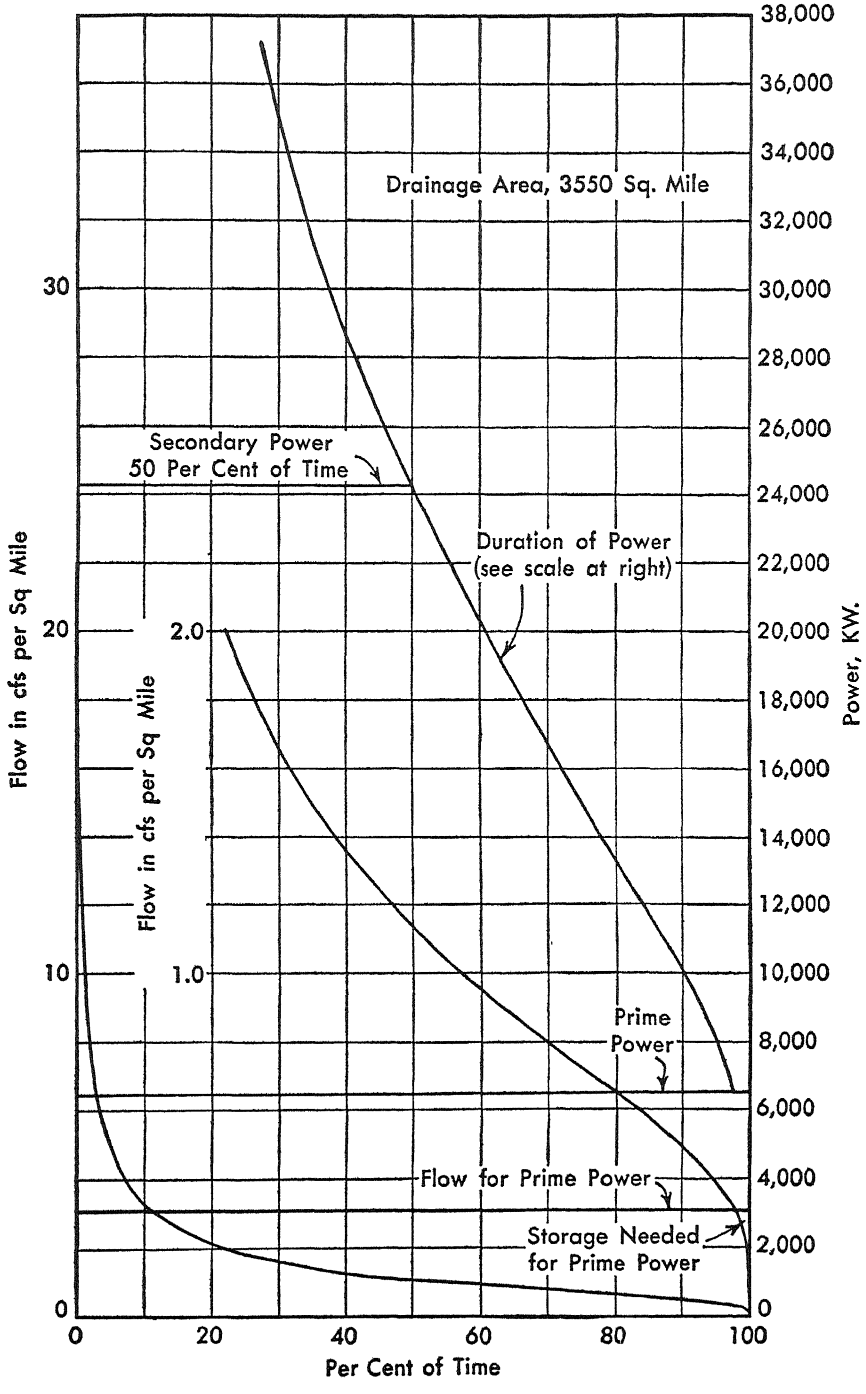



FIGURE 175. Duration Curve, Chattahoochee River, West Point, Ga., Period 1896–1928

area. The area above the curve which is bounded by the abscissa representing the prime flow and the ordinate for time equal to 100 per cent, represents the storage needed to obtain prime power. Likewise the area under any given abscissa and the duration curve represents the amount of energy (not power) obtainable with the natural flow represented by that abscissa. These features are shown on Figure 175.

The storage obtained as above does not necessarily mean storage capacity of the reservoir, but the volume of stored water which may be used and replaced in the reservoir several times. As seen above, the curve was computed with the total length of record of discharge as the basic period. This procedure throws all the annual or more frequent periods of low water together, that is, assumes for a period of $N$ years that all the low water came in one part of that period, which assumption is not, of course, correct. Such a procedure would greatly underestimate the benefit to be obtained from a reservoir. If, on the other hand, one year is assumed as the basic period, the curve becomes an average annual duration curve and has the disadvantage of such averages in that it does not take into account the fluctuations of flow from year to year. In spite of these limitations the duration curve is essential, for studies of power must be based on the average year of flow to compare the output of energy with estimates of cost and possible profit.

Storage capacity, however, must be studied with respect to the worst year of flow. The storage capacity of a reservoir and possibilities of regulation are investigated most advantageously by means of the mass, or as it may more suitably be called, flow summation curve.

*The Flow Summation Curve.* The flow summation curve may be defined as the curve on which the ordinate of any point represents the total amount of water which has flowed past a given section of the stream during the length of time represented by the distance along the abscissa to the same point on the curve. This curve starts with the beginning of the record of stream flow as the origin of coordinate axes where time and quantity are both zero. A convenient unit of time is selected for computing the total quantity of water passing the stream section. A large unit for the quantity of water is needed because over any considerable length of time the summation becomes enormous, and with smaller units the graph soon becomes unwieldy. A unit of acre-foot, day-second-foot, cubic feet, or even gallon, may be used, although the latter two are usually expressed in units of millions or billions. The unit of quantity selected should correspond to the one best adapted for studying the utilization of the water in storage. The unit of time should be selected with regard to the total time involved and should be as

large as possible without introducing appreciable error. For a study of flow over a period of one or two years a unit of five or ten days may be adopted; for a period of record covering ten or twenty years, such as is necessary to secure a reliable duration curve, no unit smaller than a month can well be used. After the quantity of water per unit of time has been computed, each unit quantity is added to the previous summation and is plotted as the ordinate.

This curve has some mathematical properties which should be investigated. In view of its definition and construction, it is always rising (at least in humid climates) since each successive inflow is added to the water previously accumulated. Let $Q$ be the quantity of water accumulated in any period; $T$, the time elapsed from the origin to the end of the period; and $q$, the rate of discharge. Then $Q = Q_2 - Q_1 =$ the quantity of water passed in $(T_2 - T_1)$ time, and

$$q = \frac{Q_2 - Q_1}{T_2 - T_1}$$

or in the notation of calculus,

$$q = \frac{dQ}{dT}.$$

It is thus seen that $q$, the rate of discharge, is the tangent to the curve at any point. Transposing and clearing of fractions,

$$dQ = qdT, \qquad \text{and} \qquad Q = \int qdT.$$

Integrating between the limits, $T_1$ and $T_2$,

$$Q = q(T_2 - T_1), \qquad \text{or} \qquad q = \frac{Q}{T_2 - T_1}$$

equals the slope of a straight line defined by the points $Q_1T_1$ and $Q_2T_2$, which slope is the average discharge for the time $(T_2 - T_1)$.

By introducing a coefficient in the equation $Q = \int qdT$, its curve may be used for other stations or sites on the stream as far up or down stream as the data may be applicable within hydrologic limitations. Let $q'$ be the flow at any point on the stream other than where the data for $q$ were obtained, and let $Q' = \int_{T_1}^{T_2} q'dT$; let $k$ equal the ratio of the respective drainage areas (or other relation). Then

$$q' = \frac{Q'}{T_2 - T_1},$$

and if it is assumed that $Q$ varies with the drainage area,

$$q' = \frac{kQ}{T_2 - T_1} .$$

Therefore, this coefficient may be applied to the results derived from the curve instead of the original data, thereby saving much labor and time.

For studying the effects of storage on the regimen of a stream and determining the flow obtainable, this curve is invaluable. A reservoir with capacity $R$ is assumed, from which it is desired to determine the most suitable outflow, or in other words, to secure the best possible regulation of the available discharge. The rate of inflow is $q$ cubic feet per second and the curve, $Q = \int_{T_1}^{T_2} qdT$ is constructed. The reservoir outflow is to be fixed and as nearly uniform as possible. For fully uniform regulation, the summation curve $Q_x$ of outflow is a straight line passing through the origin and point $Q_1 T_1$, since the amount of water is always proportional to the time. This, however, is rarely possible on account of the limited storage available, and operators must be contented with only partial regulation, the purpose of which is to obtain the most suitable discharge during a natural cycle of high and low flow. This optimum rate of discharge is commonly found by cut and try methods by selecting from a series of given outflows $q_x$, which exhaust the capacity $R$ of the reservoir by the time the curve $Q = \int_{T_1}^{T_2} qdT$, is again parallel to the tangent representing the outflows $q_x$. For convenience, a series of slopes representing a number of possible outflow discharges is laid out on the graph, and the most suitable found by operating parallel rulers or a similar device. An example is shown in Figure 176.

Another method of determining the effects of regulation approaches the solution more directly. Again the reservoir with a capacity of $R$ is assumed. From the stream flow records over several years collected at the reservoir site there has been constructed the flow summation curve $Q = \int_{T_1}^{T_2} qdT$. $Q$ is now moved parallel to itself in a negative direction for a distance of $R$ on the ordinate axis to a new position $Q'$. In other words, another curve $Q'$ is drawn at a distance $R$ beneath the first curve. It is evident that any curve $Q_x$ which can be constructed for the regulated outflow is below the original curve $Q$, because the outflow for the period cannot exceed the inflow. Furthermore, $Q_x$ cannot be below $Q'$ because the reservoir cannot deliver a volume greater than its capacity of water. Therefore, curve $Q_x$ must lie between curves $Q$ and $Q'$. Complete regulation with uniform outflow is shown by a straight

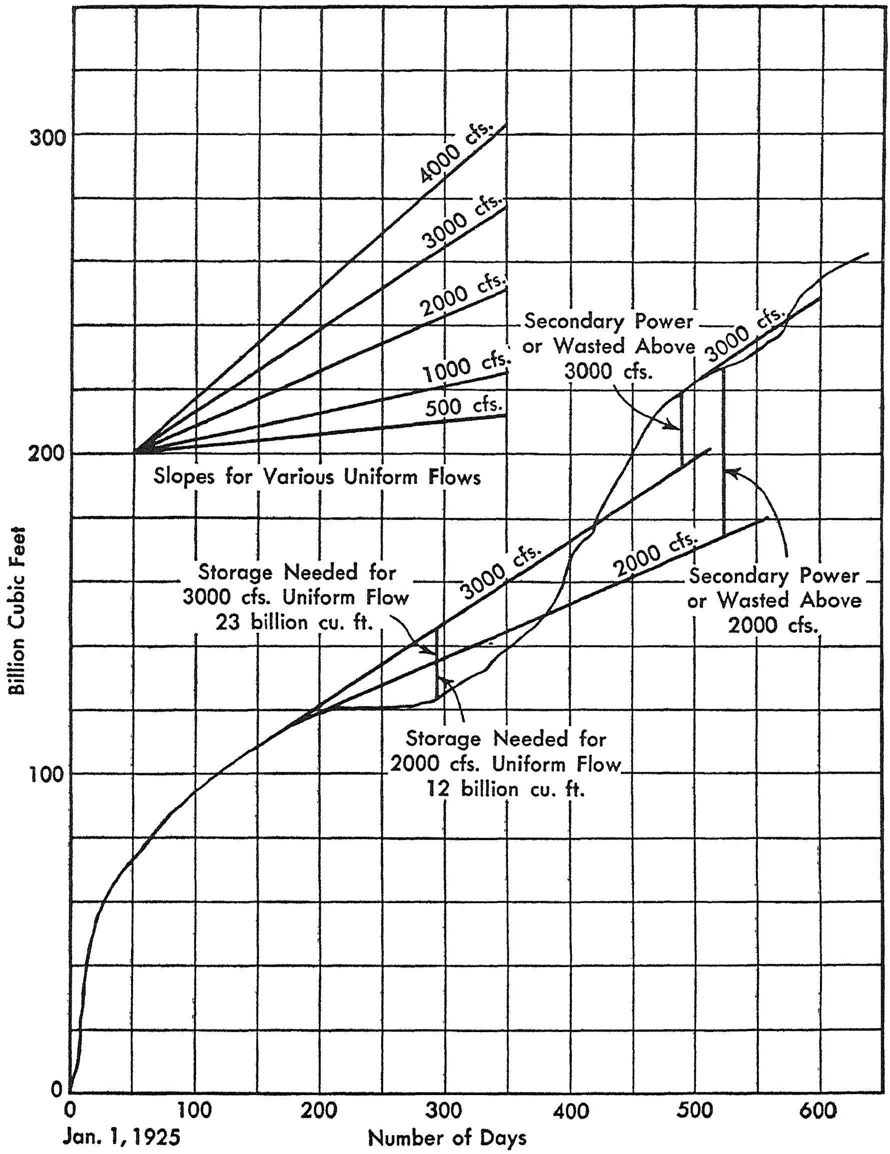



FIGURE 176. Flow Summation Curve, Chattahoochee River, West Point, Ga., Jan. 1–Oct. 2, 1926

line between $Q$ and $Q'$; it may touch but cannot intersect either one. Partial regulation results in a series of common tangents between the limiting curves, drawn so that the discharge represented by the slope is as large as possible over periods of deficient inflow. See Figure 177.

Within the limits between which the data obtained at one station may be adapted to sites up or down stream, the methods outlined in the

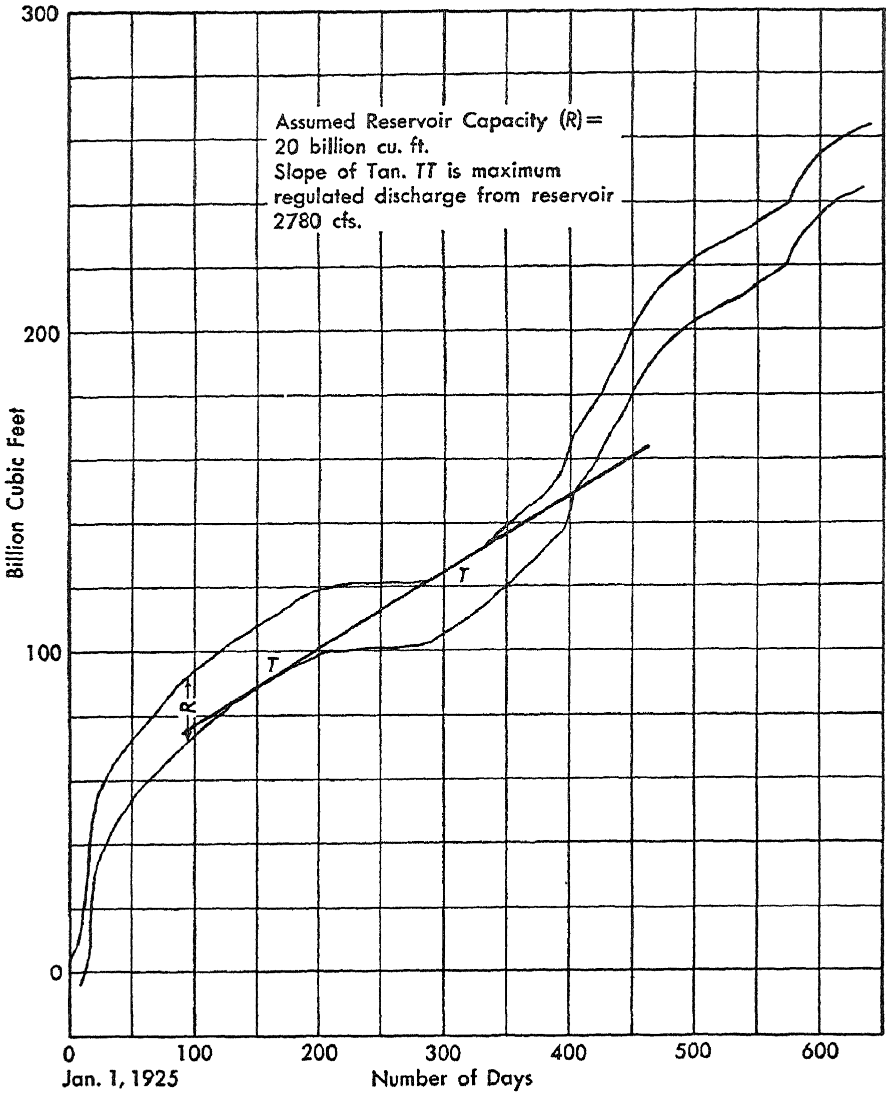



FIGURE 177. Flow Summation Curve, Chattahoochee River, West Point, Ga., Jan. 1–Oct. 2, 1926

preceding paragraph may be used by shifting the results of the computation to sites which it is desired to investigate; this may be done without modifying the data themselves prior to plotting the curve. This may be done by plotting the flow summation curve $Q$ from the data of the gaging station, and then drawing a parallel curve $Q'$ at a distance below which is equivalent to the storage capacity $R$ as modified by the ratio $k$ of the drainage areas (or other suitable factor if the discharge is not proportional to the drainage area). Then $Q' = kQ$.

Then a common tangent such as $TT$ (Figure 177) is drawn, covering the period of most unfavorable natural discharge, and a minimum uniform outflow is obtained which is equivalent to the regulated discharge from the reservoir, as modified by the above factor. When the equivalent regulated flow at the site of the gaging station is found it must then be changed inversely as was the storage capacity, in order to obtain the regulated flow at the site of the reservoir. Thus, if $R$ were divided by $k$ to locate curve $Q'$, the regulated flow must be multiplied by $k$ to make it equivalent to the regulated flow at the reservoir site.

Manifestly, in cases where the gaging station is at the reservoir site no modification is necessary to find the regulated outflow. Where the gaging station is up or down stream from the reservoir site with an appreciable drainage area intervening, the modification outlined in the preceding paragraph is necessary and will be effected by dividing the storage $R$ by the ratio of drainage areas (or other factor) when plotting the curve $Q'$, and multiplying the slope of the tangent by the same ratio to get the uniform flow at the reservoir site. If it is desired to find the effect of regulation at a point downstream from the reservoir, it is first necessary to calculate the regulated outflow for the reservoir site as above, and add to it the minimum estimated natural discharge from the drainage area intervening between the reservoir and the given point on the stream.

*The Flood Frequency Problem.* The flood frequency problem enters in some form the studies of most stream development. However, its chief use is in connection with economic problems, which are many and varied. A particularly important problem is the determination of annual flood losses and benefits from proposed control measures.

Since there is a variety of problems requiring a study of flood frequencies, a number of methods may appropriately be used. Approximate frequencies of the smaller floods, such as would be provided against by coffer dams, may be satisfactorily determined by plotting on Hazen-type logarithmic probability paper the classified data of floods, as was done in Figures 139, 141, 143, and similar graphs. The frequency may be computed by multiplying the percentage of number of floods above a given discharge by the average number of floods per year. For a closer determination of the frequency within a limited range, the method devised by Mr. H. A. Foster as used in Chapter 10 for the Chattahoochee River, that is, using all data above a selected basic stage, may be employed. Wherever the best determination of the frequency of floods is required, a method such as Slade's, based fully on the theory of probability and statistics, should be used. In any case the array of data should consist of all floods above a

selected stage or discharge in order to produce the most reliable results.

*Computation of Annual Flood Loss.* Any development in an area subject to inundation sustains a flood risk. This risk may not be known or understood, yet it exists if the area can be inundated by a flood higher than any previously experienced. The losses caused by floods come intermittently at irregular intervals and in varying amounts, dependent upon the magnitudes of the floods, but the risk is constant, since a disastrous flood may occur in any year. What is needed for economic studies is the evaluation of the flood risk. This risk can be appraised by determining the magnitude of loss to be expected in the case that a given flood event happens and multiplying that loss by the probability that the event will happen.

The damage from a flood event can be determined from a damage survey made subsequently to a flood, or can be estimated on the basis of assumed flood heights and losses to be expected for inundation to those heights. The data for several such floods, actual or assumed, are then plotted to construct a flood damage curve, showing the relationship between stage and flood damage. In addition and of equal importance, there must be developed a flood frequency curve, such as shown in Figures 152 to 163. The flood risk can then be computed by taking data from the flood damage and frequency curves and subjecting them to suitable arithmetic operations. Table 89 illustrates the procedure for computing annual flood losses.

In Table 89, column (1) is of the stages taken in arbitrary but convenient increments, the upper limit being at 53.1 feet since it was the maximum flood stage observed, and the lower limit being 45.1 feet because it was the stage of incipient damage. Column (2) lists the corresponding discharges. Column (3) contains the number of floods to be expected annually and is taken from a frequency curve of momentary peaks, which in this case, was derived from the frequency curve of one-day peaks, shown as curve *C*, Figure 152. The momentary frequencies were derived simply by multiplying the discharge of curve *C* by 110 per cent which is the average ratio of momentary to one-day peaks. Column (4) gives the difference between successive increments of numbers of floods. In column (5) is the damage caused by floods reaching the given stage; the data of the column are taken from a stage damage curve, which is not shown but was constructed from data obtained by a damage survey. Each value in column (6) is the average damage of successive increments, and is taken to be the average loss caused by floods within the same increment stage. Columns (4) and (6) are multiplied together to give the expected annual loss

TABLE 89. COMPUTATION OF ANNUAL FLOOD LOSS, LAWRENCE, MASS.

| STAGE *Ft. msl.* | DISCHARGE *Q cfs* | NO. OF FLOODS PER YR | | DAMAGE | | ANNUAL LOSS |
|---|---|---|---|---|---|---|
| | | *For Q* | *For Interval* | *For Q* | *For Interval* | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 45.1 | 47,700 | .465 | | $ 0 | $ | $ |
| | | | .381 | | 38,500 | 14,669 |
| 47.1 | 77,000 | .0840 | | 77,000 | | |
| | | | .0405 | | 96,000 | 3,888 |
| 48.1 | 92,100 | .0435 | | 115,000 | | |
| | | | .0207 | | 137,500 | 2,846 |
| 49.1 | 108,000 | .0228 | | 160,000 | | |
| | | | .0106 | | 192,500 | 2,041 |
| 50.1 | 124,000 | .0122 | | 225,000 | | |
| | | | .0054 | | 274,500 | 1,482 |
| 51.1 | 141,700 | .00680 | | 324,000 | | |
| | | | .0027 | | 384,500 | 1,038 |
| 52.1 | 158,700 | .00410 | | 445,000 | | |
| | | | .00142 | | 517,500 | 735 |
| 53.1 | 174,000 | .00268 | | 590,000 | | |
| | | | .00268 | | 590,000 | 1,581 |
| all | above | .00 | | 590,000 | | |
| Total | | | | | | $28,280 |

from floods of each increment, the total of which is the annual flood loss or risk of the reach or area.

After the reduction in stage and discharge due to protective measures has been determined, the expected loss from floods, if any, can be recomputed in the same manner as shown in Table 89. The difference between the original annual loss and that obtained after protective works are in operation is the expected annual benefit from the protective works.

For illustrative purposes, it may be desired to show the annual loss graphically. For this purpose the annual losses in column (7) may be plotted against the stages in column (1), or the discharge in column (2). A similar curve can be made of losses to be expected after the completion of the proposed measures of flood protection. This second curve falls below the first one because of the diminished losses, and the area between the two curves represents the total annual loss which can be eliminated by the protective works. This area can be determined by planimetering. The total annual loss determined in this manner may be slightly different from what is determined by the purely arithmetic calculation, because of the curvature of the lines in the graphs which round out the area between the computed points. This difference, however, is well within the probable error of the final result.

*The Rain Frequency Problem.* The rain frequency problem is somewhat similar to the flood frequency problem in its application. It

furnishes desirable general information of the rainfall regimen of a locality. Like flood frequency, its most important application is to economic problems, which, however, are so varied that individual analysis must be made for each case as it arises. The general answer is found in the mathematical expectation, or annual cost, if the risk brought about by rain is a repeating one. As in the case of floods the mathematical expectation of loss, or cost, is the product of the total damage to be expected if a given event occurs and the probability that the event will happen. The expected loss must be estimated from economic data of rainfall damage, while the probability of the event (expressed, for example, as number of days of rainfall per year) can be derived from frequency curves such as those shown in Figures 73 to 91.

*The Intensity-Frequency Problem.* The intensity-frequency problem is essentially one connected with functional design. It arises in design for capacity of storm sewers, land drainage, and interior drainage of flood protective works, in which it is a matter of balancing greater costs against the added benefits of larger capacities for outlets, sewers, drains, or other conduits. The solution consists in determining what intensity and frequency are prevalent, by such means as have been discussed in Chapter 5.

The first step is the construction of intensity-frequency curves in which depth is plotted against time (in minutes for short storms) as was done in Figures 92 and 93. Those curves were envelope curves in which maximum observed amounts of precipitation were used for the given increments of time. Then for situations warranting the control of maximum precipitation for a given frequency, the amounts of precipitation may be taken from the curves and distributed for an assumed storm as shown in the tabulation (Table 90) of rainfall taken from Figure 92 for the 10-year frequency:

TABLE 90. INCREMENTS OF RAINFALL

| INCREMENTS OF TIME *Minutes* | ACCUMULATED RAIN *Inches* | INCREMENTS OF RAIN *Inches* | DISTRIBUTION FOR STORM *Time, min.* | *Rain, inches* |
|---|---|---|---|---|
| 10 | 1.65 | 1.65 | 60 | 0.40 |
| 20 | 1.90 | 0.25 | 10 | 0.10 |
| 30 | 2.10 | 0.20 | 10 | 0.25 |
| 40 | 2.20 | 0.10 | 10 | 1.65 |
| 50 | 2.30 | 0.10 | 10 | 0.20 |
| 60 | 2.40 | 0.10 | 10 | 0.10 |
| 120 | 2.80 | 0.40 | 10 | 0.10 |
| 180 | 3.05 | 0.25 | 60 | 0.25 |

There are, of course, any number of possible distributions of the assumed rainfall through a design storm; the one given above in the

last column provides for some preliminary rain to satisfy partially the high initial infiltration capacity before receiving the greater rates of intense rainfall.

For storm sewers and drainage, however, using a storm of a frequency of two to ten years, it may be advisable to use a storm distribution that is roughly an average of the storms to be expected. This average distribution may be obtained by using curves, such as shown in Figure 55, of accumulated rainfall of a number of storms of the type and intensity desired, and deriving an average distribution. Figure 178 shows the distribution of four storms averaged in this manner. From the mean curve the distribution in Table 91 is taken:

TABLE 91. STORM DISTRIBUTION OF RAINFALL

| HOUR | PORTION OF STORM *Rainfall, Per Cent* | DISTRIBUTION OF 12-HOUR STORM *4.2 inch* |
|---|---|---|
| 1 | 1 | .04 |
| 2 | 4 | .17 |
| 3 | 5 | .21 |
| 4 | 11 | .46 |
| 5 | 14 | .59 |
| 6 | 23 | .97 |
| 7 | 19 | .80 |
| 8 | 9 | .38 |
| 9 | 6 | .25 |
| 10 | 4 | .17 |
| 11 | 2 | .08 |
| 12 | 2 | .08 |
| | 100 | 4.20 inch |

Although little is said of it, the same problem enters studies of erosion. It appears that the more intense and less frequent rains are the worse ones for causing erosion. In that case, the intensity and frequency should be important elements in the study of erosion of land and subsequent sedimentation in connection with reservoirs and other hydraulic works.

*The Design Flood Problem.* This problem enters several types of river improvements wherever a spillway, levee, channel improvement, or conduit is to be constructed. The elements of analysis of the problem have been given heretofore but it remains to discuss the synthesis of a design flood. Primary attention will be given to the spillway flood which is usually the probable maximum flood less such reduction as can be secured by reservoir operation.

The general procedure for determining the probable maximum flood at a given site consists in examining all big storms within the limits of a homogeneous climate and transposing the largest to the basin with whatever modifications appear desirable, and from the modified pre-

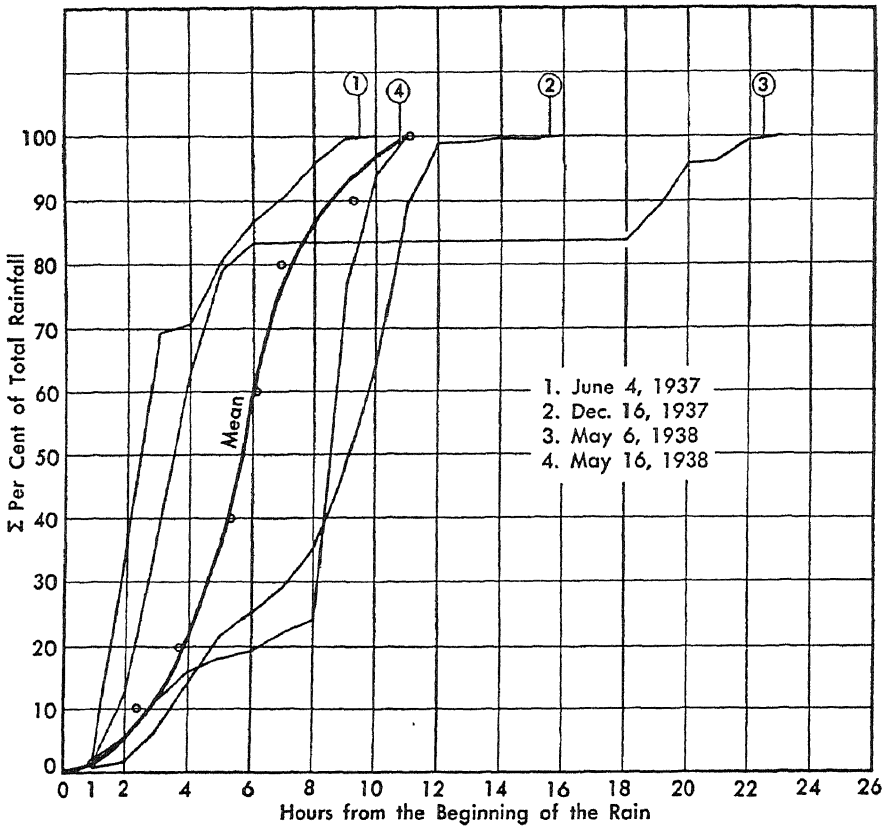



FIGURE 178. Summation of Rainfall, Houston, Texas

cipitation computing the design flood. All storms of intense precipitation should be examined, but only a few of the type likely to produce the most intense rainfall need to be transposed. The selected storms should be thoroughly analyzed from the aspects of depths of precipitation and area with view to determining maximum values. The type to be transposed depends largely upon the size of the drainage area. Hathaway (83) has discussed this matter quite thoroughly.

When the storms have been selected, the next step is to transpose them to the drainage area above the dam site. As pointed out by Showalter (168), the elements of primary importance in making the adjustment of precipitation for the new location are differences in effective precipitable water, and velocity and duration of inflowing warm air. Appreciable differences in topography should also be considered, if there are any within the region of homogeneous climate. The differences in maximum precipitable water can be determined by a search of the records of dew point. Records of this sort are not abundant

so that the task is not arduous, although it may be disappointing. Likewise, data of the differences in wind of the two areas can be found only in meteorological records.

The differences in precipitation of the same storm can be taken as directly proportional to the differences in effective precipitable moisture and volume of inflowing warm moist air.

Then pluviographs should be constructed on the basis of the adjusted precipitation. The arrangement or distribution may follow that found in the original storm, but before selecting a distribution a number of pluviographs along patterns found in the region should be constructed and floods computed from them. The pluviograph producing the worst flood, or highest runoff and largest spillway discharge, should be selected.

The effect of snow on runoff should be investigated if there is snowfall on the basin that can be melted at a time to contribute to the flood. This study should cover depths, water equivalent, and rates of melting, and estimates should be made of the depth of water likely to be contributed to the flood in conjunction with the runoff from the rainfall.

Estimates of the infiltration rates should be made as well as of other losses. Particular attention should be given to the possibility that the ground may be frozen if the basin for which the design flood is desired is in a northern region.

Finally, the estimated snowmelt water is added to the selected pluviograph of rainfall, the infiltration losses, and others if any, are deducted. From the remainder, or effective rainfall, the flood runoff is computed by means of the distribution factors obtained by the method discussed in Chapter 9.

The foregoing procedures are designed to give the probable maximum possible flood for spillway purposes. This flood will probably be several times as large as any flood observed and may in many cases seem unreasonably high, but the object in a spillway design flood is to secure absolute safety. Nothing less is acceptable if human life would be lost by the failure of the dam.

*Computation of a Design Flood.* In order to illustrate the foregoing procedure more clearly, one example of computing a design flood will be worked out in detail. The James River at Buchanan, Va., is the site and the daily distribution factors computed in Chapter 9 are used. The drainage area is 2080 square miles.

It is assumed that a thorough search has been made for the most intense storms with highest rates of rainfall, and that it is found that the worst probable storm is a hurricane that has moved inland. This assumption is not unreasonable for the area of the James River. This

fact eliminates snowmelt as a factor, although spring or winter floods on the James River do occasionally carry snow water. It is assumed that a study of the depth area and depth time curves shows that for the given drainage area the following depths of rainfall should be expected on rare occasions:

| TIME *Hours* | RAINFALL *Inches* |
|---|---|
| 12 | 10.0 |
| 24 | 12.8 |
| 36 | 14.7 |
| 48 | 15.2 |
| 60 | 15.6 |
| 72 | 15.8 |

An infiltration rate of 0.1 inch per hour is assumed; although this is a low rate, it is taken to be conservative in view of the steep slopes on the watershed above the site and the likelihood of saturated soil before the storm.

This type of storm is of short duration as well as high intensity, and for this reason it is desirable to have distribution factors for shorter periods than were computed previously for the James River. Therefore, for this example the daily distribution factors have been broken down into factors for 12-hour periods by using the equations given for such a purpose in Chapter 9. The new factors and other computations are shown in Table 92.

TABLE 92. COMPUTATION OF DESIGN FLOOD, JAMES RIVER AT BUCHANAN, VA.

| RAINFALL | | INFILTRATION | RUNOFF | | DISTR. FACT. | FLOOD FLOW 1000 DSF | | | |
|---|---|---|---|---|---|---|---|---|---|
| *Period 12hr* | *Amount Inches* | *Inches* | *Inch* | 1000 *dsf* | | *For Period* 1 | 2 | 3 | *Total* |
| 1 | 0.5 | 0.5 | 0 | | | | | | |
| 2 | 2.8 | 1.2 | 1.6 | 89.6 | 0.02 | 1.79 | | | 1.79 |
| 3 | 10.0 | 1.2 | 8.8 | 492.8 | .21 | 18.82 | 9.87 | | 28.69 |
| 4 | 1.9 | 1.2 | 0.7 | 39.2 | .29 | 25.98 | 103.40 | 0.78 | 130.16 |
| 5 | 0.4 | 0.4 | .0 | | .15 | 13.44 | 142.9 | 8.22 | 164.56 |
| 6 | 0.2 | 0.2 | .0 | | .095 | 8.51 | 74.0 | 11.38 | 93.89 |
| 7 | | | | | .070 | 6.27 | 46.8 | 5.88 | 58.95 |
| 8 | | | | | .050 | 4.48 | 34.5 | 3.72 | 42.70 |
| 9 | | | | | .030 | 2.69 | 24.65 | 2.74 | 30.08 |
| 10 | | | | | .025 | 2.24 | 14.80 | 1.96 | 19.00 |
| 11 | | | | | .015 | 1.79 | 12.30 | 1.18 | 15.27 |
| 12 | | | | | .010 | 1.34 | 9.87 | .98 | 12.19 |
| 13 | | | | | .010 | .90 | 7.40 | .78 | 9.08 |
| 14 | | | | | 0.005 | .90 | 4.93 | .59 | 6.42 |
| 15 | | | | | | 0.45 | 4.93 | .39 | 5.77 |
| 16 | | | | | | | 2.45 | .39 | 2.84 |
| 17 | | | | | | | | 0.21 | 0.21 |
| Total | 15.8 | 4.7 | 11.1 | 621.6 | 1.000 | 89.60 | 492.80 | 39.20 | 621.60 |

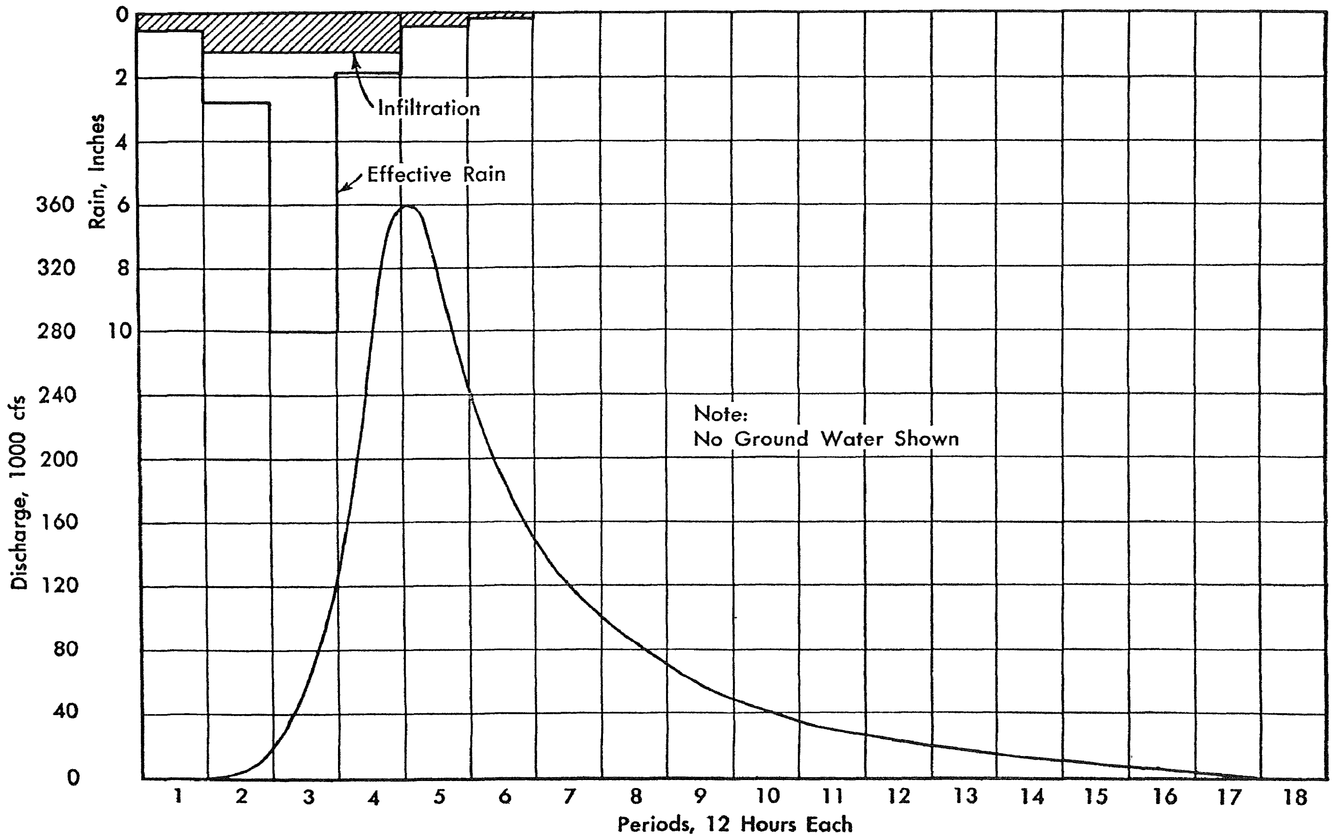



FIGURE 179. Spillway Design Flood, James River, Buchanan, Va.

The computation of a design flood may be accomplished in a number of ways, but Table 92 is representative and convenient. The first column contains the order of 12-hour periods beginning with the first day of rain; the second column lists the rainfall, given in an order that would be expected to cause the highest peak. This sequence is not the only possible arrangement, but it is typical. The depths of infiltration for the 12-hour periods are given in the third column; it can be seen that infiltration absorbs all the rain of the first, fifth, and sixth periods. The next two columns give the runoff in inches and in day-second-feet. The computed distribution factors for 12-hour periods are given in the sixth column; the first factor is set on line with the second time period and first period of runoff. In the seventh, eighth, and ninth columns is the computed flow for the three consecutive periods of rain. The flow is computed by multiplying the runoff by the distribution factors in order, and setting the product on line with the factor in the case of the first period runoff; in the case of the second and third periods, the computed flow is set down one and two lines, respectively. This order of setting the computed flow down a line for each consecutive period of rain is important because it brings on line the flow contributed by each rainfall period to the runoff. Finally, in the last column are the total volumes of runoff for each time period. These totals are obtained by adding the volumes on line. It should be noted that these volumes are in day-second-feet for one-half day periods, and must be multiplied by 2 when plotted for a hydrograph. See Figure 179.

Each column is totaled to give a number of checks for the computations, the most important of which is that the total flood flow in day-second-feet, 621.6, should equal the total of the runoff in the fifth column.

*Final Comment.* In closing this final chapter it is repeated that only a few of the most conspicuous problems have been touched upon and those only lightly, for there are many others. The studies required for utilization of water resources are many and intricate. They can be solved only by individual application of the sound principles of hydrology in each development and use. These problems should not be slighted because the need for water grows yearly and only a logical solution of the problems involved will obtain the best use for limited supplies.

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Agri. Exp. Stat., Agricultural Experiment Station.
A. G. U., American Geophysical Union, Washington, D. C.
Am. Geog. Soc., American Geographical Society, New York, N. Y.
A. M. S., American Meteorological Society, Milton, Mass.
A. S. C. E., American Society of Civil Engineers, New York, N. Y.
Bull., Bulletin.
Civ. Eng., Civil Engineering, American Society of Civil Engineers, New York, N. Y.
Eng. News-Rec., Engineering News-Record, McGraw-Hill Publishing Co., New York, N. Y.
Geog. Rev., Geographical Review, American Geographical Society, New York, N. Y.
Gov. Print., U. S. Government Printing Office, Washington, D. C.
Inter. Ass'n. Hydrol., International Association of Hydrology, Paris, France.
Jour., Journal.
Mass. Inst. Tech., Massachusetts Institute of Technology, Cambridge, Mass.
Mon. Wea. Rev., Monthly Weather Review, U. S. Weather Bureau, Gov. Print., Washington, D. C.
N. Dak. Geol. Sur., North Dakota Geological Survey, Grand Forks, North Dakota.
N. E. W. W. Ass'n., New England Waterworks Association., Boston, Mass.
Pt., Part.
Proc., Proceedings.
Trans., Transactions.
U. S. Dept. Agri., U. S. Department of Agriculture, Washington, D. C.
U. S. G. S., U. S. Geological Survey, Washington, D. C.
Univ , University.
W. S. P., Water Supply Paper.


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